

Fight or Flight: Endogenous Timing in Conflicts*

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Abstract: We study a dynamic game in which players compete for a prize. In a waiting game with two-sided private information about strength levels, players choose between fighting, fleeing, or waiting. Players earn a “deterrence value” on top of the prize if their opponent escapes without a battle. We show that this value is a key determinant of the type of equilibrium. For intermediate values, sorting takes place with weaker players fleeing before others fight. Time then helps to reduce battles. In an experiment, we find support for the key theoretical predictions, and document suboptimal predatory fighting.

Keywords: fight-or-flight, contest, sorting, theory, experiment

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1 Introduction

Following Maynard Smith's [1974] seminal contribution, competition for a prize is often modeled as the war of attrition. In this game, players choose the time at which they intend to flee. Time is costly, and players may differ in their opportunity costs. The player who waits the longest wins the prize and both players pay a cost proportional to the time it takes for the losing player to flee. [Maynard Smith \[1974\]](#) refers to this type of interaction as a "display". In a display, no physical contact takes place, or if it does, it does not settle the battle or convey information about which player would win an escalated conflict.

The main contribution of our paper is that we develop and analyze a game in which at any moment, players cannot only wait or flee, but also have the option to actively start a fight. In case of fight, a battle ensues and the stronger player wins the prize while the losing player incurs a loss. This dynamic Fight-or-Flight game allows us to make sense of a much wider variety of competitions. It captures the essence of many types of interactions in which the timing of actions plays a crucial role, such as R&D races, litigation, the launch of political or advertisement campaigns, and firm acquisitions. It also fits situations in the animal kingdom, where animals fight over territory or prey. In all these examples, players can 'flee' (e.g., reduce R&D spending, settle), wait to see if the other gives in, or initiate a fight (e.g., suing the opponent, start a hostile takeover), forcing the other into a battle.

Our dynamic game helps to understand why in some situations players want to wait and see if the other flees without a battle, while in other circumstances both want to act as quickly as possible. To illustrate the former type of situation, consider two political candidates who may wait a long time before they officially announce that they are running for office. If the other flees without a

battle, they avoid the costs of a costly campaign that is required to win a fight. Male elephant seals who contest the right of exclusive access to a harem usually wait a couple of minutes to allow the other to flee without a bloody fight.

In other instances, players want to act as quickly as possible. A firm that wants to expand its market by acquiring a competitor should act quickly, to prevent the prospective target from selling its valuable assets. Another possible interpretation is that, compared to letting the other escape, by winning a fight the player sends a stronger signal about its strength to other players, thereby discouraging other players from ever making a challenge. A firm that drives out another firm by force will deter potential future competitors more than if the other firm left voluntarily. In a lawless society without a state monopoly of violence, people may want to rob each other if they can. In an encounter, the stronger player prefers to act as quickly as possible to avoid that the other flees without losing his money.

Notice that both types of examples are not well described by the war of attrition. In the first type of example it may happen that players fight after a waiting period which is not a possibility in the war of attrition. The war of attrition also does not capture the essence of the second type of interaction. In particular, the war of attrition does not accommodate that strong players decide to fight in a split-second.

In this paper, we analyze the Fight-or-Flight game theoretically and experimentally. Theoretically, we identify a key-parameter, the “deterrence value”, that determines how the competition between two players will unfold. The deterrence value is the amount that a player earns on top of the prize if the other player manages to escape. Our theoretical analysis based on standard preferences yields two main novel insights. First, if the deterrence value is negative all player types will rush and act in a split-second. A negative deterrence value is illustrated by the sale of valuable assets by

a fleeing prospective firm in the takeover example. If the deterrence value is positive, players prefer to avoid the costly fight and wait before they act. In the example where two political candidates engage in a battle for office, the costs to organize a campaign represent a positive deterrence value.

The second insight is that if the deterrence value is positive but not too large, sorting will occur in the dynamic Fight-or-Flight game. That is, the weakest players will flee just before the end. Thus, the dynamic structure helps players to avoid costly fights, in comparison to a static version of the game that is stripped of its time element. These two results cannot be obtained in a standard war of attrition. In that game, players' waiting correlates positively with their strength, and rushing by all types is never observed in equilibrium. Moreover, the dynamic standard war of attrition does not help players to sort and avoid costly fights in comparison to the static version (e.g., see [Hörisch and Kirchkamp \[2010\]](#)).

We also investigate what happens in a behavioral model in which players differ in their degree of risk aversion. This model yields two additional testable implications. First, it predicts that sorting will occur in a wider set of circumstances than in the standard model. Second, it predicts that the more risk averse players flee more frequently before the end.

We test the predictions in an experiment in which we systematically vary the deterrence value and the dynamic/static nature of the game between treatments. Our experimental findings support some of the key features of the theory, at least in terms of its comparative statics. With a negative deterrence value subjects quickly learn to decide in a split-second. With a positive deterrence value, subjects tend to wait much longer and indeed use time to sort. In agreement with the model of heterogeneous risk aversion, we find that endogenous timing reduces the likelihood of costly battles in a wider set of circumstances than predicted by standard theory. Subjects who are classified as more risk averse on the basis of an independent task are indeed the ones that tend to flee more often

early in the game. Thus, while not all results are consistent with the point predictions of the model, in terms of comparative statics behavior often moves in the expected direction.

An interesting finding that deviates from the predictions is that a sizable minority of subjects fight early when the deterrence value is positive. This is the case even after ample time to learn. This finding is in stark contrast with some behavioral findings in related dynamic games. For instance, [Roth *et al.* \[1988\]](#) report that the deadline effect, a striking concentration of agreements in the final seconds of the game, is the most robust behavioral finding in a class of games designed to test axiomatic models of Nash bargaining. [Roth and Ockenfels \[2002\]](#) and [Ockenfels and Roth \[2006\]](#) identify substantial last-minute bidding in second-price auctions. They attribute this phenomenon of sniping to both strategic and naïve considerations of the bidders. We discuss some potential explanations for the anomaly of early fighting in our contest game at the end of the results section.

One feature of our experimental design is that time is discrete but with very short time intervals. This makes it hard for subjects to precisely time their actions, and could be one of the reasons behind the decrease in costly battles in the dynamic games. In a follow-up experiment, we make it easier for subjects to time their action, by making the time intervals longer. Consistent with the theoretical predictions, we no longer observe a decrease in battles compared to the static games when the deterrence value is negative. In other respects, the results closely resemble that of the original experiment.

Our paper contributes to the literature on dynamic games in which players compete for a prize. Several studies compare dynamic with static environments. [Hörisch and Kirchkamp \[2010\]](#) investigate how experimental subjects behave in static and dynamic versions of the war of attrition and some closely related games. Theoretically, the dynamic version of a war of attrition does

not help players to sort, and indeed, they do not observe such a difference in their experiments.¹ Theoretically, in an auction with symmetric interdependent valuations, [Goeree and Offerman \[2003a\]](#) do not find that the efficiency of a dynamic English auction is improved compared to the static second-price auction. In contrast, [Kirchkamp and Moldovanu \[2004\]](#) investigate a setup where a bidder's value is determined by his own signal in combination with the signal of his right neighbor. In this setting, bidders can retrieve valuable information in a dynamic auction process. In an experiment, they find that the efficiency of the English auction is higher than in a second-price auction in which no such information can be retrieved, which accords with theory.²

¹There is a large literature on static contest games. [Carrillo and Palfrey \[2009\]](#) study a contest game that is quite close to our static benchmark. They find that subjects compromise more often than in equilibrium, and they discuss some explanations based on cognitive limitations. [De Dreu et al. \[2016\]](#) investigate a game in which a group of attackers competes with a group of defenders. They find that in-group defense is stronger and better coordinated than out-group aggression. [Oprea et al. \[2011\]](#) show how the matching protocol affects outcomes in continuous time Hawk-Dove games. [Dechenaux et al. \[2015\]](#) provide a survey of the experimental literature on contest games.

²The war of attrition has been applied to various settings, including versions with private information [[Fudenberg and Tirole, 1986](#); [Ponsati and Sákovics, 1995](#)] and applications to public good provision [[Bliss and Nalebuff, 1984](#); [Weesie, 1993](#)]. [Oprea et al. \[2013\]](#) experimentally study war of attrition games with two-sided private information (as in [Fudenberg and Tirole \[1986\]](#)) and observe behavior close to theoretical predictions. More generally, the study of dynamic games reveals novel insights that significantly surpass what we know from the study of static games. Recent contributions include [Potters et al. \[2005\]](#), [Levin and Peck \[2008\]](#), [Ivanov et al. \[2009\]](#), [Kolb \[2015\]](#) and [Agranov and Elliott \[2017\]](#). The recent experimental literature on continuous

The remainder of the paper is organized in the following way. Section 2 introduces the Fight-or-Flight game and presents the theory. Section 3 discusses the experimental design and procedures. Section 4 provides the experimental results and Section 5 concludes.

2 Theory

2.1 Dynamic Fight-or-Flight game

We first describe the dynamic version of the Fight-or-Flight game. In this section, we present a basic version of the game. In section 2.3 and Appendix B we discuss several extensions.

Time is discrete, with a finite number of periods $t = 0, 1, \dots, T$. For each $t < T$, as long as the game has not ended, the two players independently decide to wait, flee (R , for "retreat"), or fight (F). In the final period, players can no longer wait and have to choose F or R . The game ends with at least one player choosing F or R , at which point the action set becomes null. At the start, each player i is privately informed of her fighting ability a_i . It is common knowledge that a_i is independently drawn from a uniform distribution over the unit interval. A player's strategy lists for every ability the number of periods in which she chooses to wait and her choice if play reaches the period in which she wants to act. A player type's strategy $s(a_i)$ is described as (t, A) , where $A \in \{F, R\}$. This means that player i with ability a_i will choose action A (fight or flee) in period t if the other player did not fight or flee earlier.

The game ends as soon as one of the players decides to fight or flee. The outcome can be a *battle* or an *escape*. A battle occurs if the player with the shortest waiting time chooses to fight or if

time experiments shows that outcomes in continuous time may substantially differ from outcomes in discrete time [Friedman and Oprea, 2012; Oprea *et al.*, 2014; Bigoni *et al.*, 2015; Calford and Oprea, 2017].

they both choose to fight at the same time. An escape occurs if the player with the shortest waiting time chooses to flee or if they both choose to flee at the same time. If one of the players chooses to fight and the other chooses to flee at the same time, an escape occurs with probability p and a battle with probability $1 - p$.

Payoffs. In case of a battle, the player with the higher ability receives $v^h > 0$ (the prize) and the other earns $-v^l$, where $v^h, v^l > 0$. In case of an escape, the player who chose to flee earns 0 while the other earns $v^h + k$, the prize plus a *deterrence payoff* k . This deterrence value can be positive or negative. A positive deterrence value captures situations where fighting is costly, so that players prefer to get the prize without fighting for it. A negative deterrence value captures situations in which beating the other generates a higher value compared to when the other escapes. We restrict the analysis to $k > -v^h$, so that if the other escapes this always gives a higher payoff than escaping. As tie breaking rules, we assume that if there is a battle between equally strong players, it is randomly determined (with equal probability) which player receives v^h and which player receives $-v^l$. If both players decided to flee at the same time, it is randomly determined (with equal probability) who earns 0 and who earns $v^h + k$. Alternatively, players could be allowed to share the prize equally in case that they both flee. This would not affect the theoretical analysis if players are risk neutral.

We assume that players maximize their expected utility and do not discount the future. In Appendix B, we analyze the case with discounting, but here our aim is to show how time *per se* affects the ability of players to sort themselves according to their strength. The case without discounting is also relevant for many cases, such as when the cost of waiting is small compared to the prize, the maximum duration of the game is short, or the consumption of the prize happens at

a fixed point in time.³

We allow for the possibility that players are risk averse. To keep the model parsimonious, we assume that each player's utility function is piecewise linear in the payoff x and given by:

$$U(x) = \begin{cases} x & \text{if } x \geq 0 \\ \lambda x & \text{if } x < 0 \end{cases} \quad (1)$$

Here, $\lambda > 0$ captures the degree of risk aversion (for $\lambda > 1$) or risk seeking ($0 < \lambda < 1$). Naturally, when $\lambda > 1$, this specification is also consistent with loss aversion. Our approach does not distinguish between loss and risk aversion.

2.2 Equilibrium

We look for pure-strategy Bayesian Nash equilibria. In this section, we derive equilibria under the assumption that players have threshold strategies, where types below a certain threshold flee and types above that threshold fight. Intuitively, stronger types have more to gain from fighting. We also assume that no type acts after the period in which the strongest type acts. In Appendix A, we show that all equilibrium profiles satisfy these properties.

Negative deterrence value: $-v^h < k < 0$. For a negative deterrence value, the payoff of winning a battle exceeds that of allowing the other to escape. In this case, there is a unique equilibrium outcome in which all players fight or flee immediately. The very strong types will want to fight, and very weak types will want to flee. If the weakest types would flee after $t = 0$, the strongest types have an incentive to fight before that, to avoid that the opponent escapes. But then the weakest types would deviate to fleeing earlier. This implies that the strongest types fight immediately, and the weakest types flee immediately. Any other type will then act immediately as well. Acting later

³By design, discounting also cannot play a role in the experiment.

is costly, because it does not result in fewer battles with stronger types that fight, and gives weaker types the possibility to escape.

With all types acting immediately, let type \tilde{a} be indifferent between fighting and fleeing. All stronger types fight and all weaker types flee. Suppose type \tilde{a} flees. If the opponent is weaker, the expected payoff is $(v^h + k)/2$, and this happens with probability \tilde{a} . If the opponent is stronger, a battle results with probability $1 - p$ and this will always be lost by type \tilde{a} , giving a payoff $-\lambda v^l$. The expected payoff of fleeing is therefore given by:

$$\tilde{a}\frac{1}{2}(v^h + k) + (1 - \tilde{a})(1 - p)(-\lambda v^l). \quad (2)$$

Suppose type \tilde{a} fights. A weaker opponent escapes with probability p , giving a payoff $v^h + k$, and otherwise there is a battle that will be won by type \tilde{a} , giving a payoff v^h . If the opponent is stronger, there will always be a battle that will be lost by \tilde{a} . The expected utility of fighting is then given by:

$$\tilde{a}[p(v^h + k) + (1 - p)v^h] + (1 - \tilde{a})(-\lambda v^l). \quad (3)$$

Since type \tilde{a} is indifferent between fleeing and fighting, it follows that:

$$\tilde{a} = \frac{p\lambda v^l}{\frac{1}{2}v^h + p\lambda v^l + k(p - \frac{1}{2})}. \quad (4)$$

The threshold \tilde{a} is increasing in the probability of an escape p . As p increases, fighting against weaker types becomes less attractive since they become more likely to escape. More types will then flee in equilibrium. The effect of k on \tilde{a} depends on the value of p . For $p < \frac{1}{2}$, an increase in k has a larger impact on the fleeing payoff than on the fighting payoff. This means fleeing becomes more attractive, and more types will flee in equilibrium. For $p > \frac{1}{2}$, the reverse is true.

Positive deterrence value: $k > 0$. With a positive deterrence value, players are better off when the

other manages to escape than when they win a battle. In this case, all the action will be concentrated in the final two periods of the game. Intuitively, sufficiently strong players will wait until the last period, to give other players the option to escape. Fighting should only take place in the last period. Weaker types will then also prefer to wait until at least the penultimate period, since waiting until then gives opponents the option to escape without the risk of ending up in a fight.

Consequently, for $k > 0$ there is a fraction of types that flees at $T - 1$ and a fraction that flees at T . The remaining fraction fights at T . All types that flee have the same payoff independent of the moment that they flee; they always lose a battle with a type that fights and their payoff when the opponent flees is independent of their fighting ability. The equilibrium does therefore not pin down *which* types flee first, only the fraction. To determine the fraction of types that flee, we can assume w.l.o.g. that the weakest types flee at $T - 1$. The equilibrium can then be characterized by two threshold levels \hat{a}_1 and $\hat{a}_2 > \hat{a}_1$. Type \hat{a}_1 is indifferent between fleeing at $T - 1$ and fleeing at T . Type \hat{a}_2 is indifferent between fleeing at T and fighting at T . A fraction of types \hat{a}_1 flees at $T - 1$ and a fraction of types $\hat{a}_2 - \hat{a}_1$ flees at T . Types above \hat{a}_2 fight at T . The values of \hat{a}_1 and \hat{a}_2 are given by:

$$\hat{a}_1 = \frac{\lambda v^l [(v^h - k)(1 - 2p) - 2kp^2]}{(v^h + k + 2(1 - p)\lambda v^l)(\frac{1}{2}v^h - (\frac{1}{2} - p)k)}, \quad \hat{a}_2 = \frac{2(1 - p)\lambda v^l}{v^h + k + 2(1 - p)\lambda v^l}. \quad (5)$$

The fraction of types fleeing at $T - 1$ is positive for values of k below \hat{k} , where

$$\hat{k} = \frac{1 - 2p}{1 - 2p + 2p^2} v^h. \quad (6)$$

For larger values of k , all types wait until the final period. Intuitively, if k is large, it always pays off to wait and give others the option to escape, even if that implies risking a battle with stronger

types. The same is true for larger values of p . If the probability of an escape is large, it becomes more attractive to wait, even if the opponent fights.

The foregoing shows that there can be three types of equilibrium outcomes. If $k < 0$, there is a *rushing equilibrium* in which all types immediately fight or flee. For intermediate positive values of k , there is a *timing equilibrium* in which some types wait until the penultimate period and then flee, while all others wait until the final period and then fight or flee. For high values of k , there is a *waiting equilibrium* in which all types wait until the last period and then fight or flee. While we derived these equilibria under the assumption that players have threshold strategies, in Appendix A we show that no other equilibria exist. The equilibrium outcome is generically unique, except for $k = 0$ or $k = \hat{k}$.

Proposition 1 (Equilibrium).

- (i) If $k < 0$, the unique equilibrium outcome is a 'rushing equilibrium' in which all players act immediately. Players with abilities $[0, \tilde{a}]$ flee at $t = 0$ and players with abilities $(\tilde{a}, 1]$ fight at $t = 0$,
- (ii) If $0 < k < \hat{k}$, the unique equilibrium outcome is a 'timing equilibrium' in which a fraction \hat{a}_1 of types flee in period $T - 1$, a fraction $\hat{a}_2 - \hat{a}_1$ of types flee in period T , and all types above \hat{a}_2 fight in period T .
- (iii) If $k > \max\{\hat{k}, 0\}$, the unique equilibrium outcome is a 'waiting equilibrium' in which types $[0, \tilde{a}]$ flee in period T and types $(\tilde{a}, 1]$ fight in period T , and $\tilde{a} = 1$ for any $v^h < (1 - 2p)k$.

Proof. All proofs are in Appendix A. □

Figure 1 illustrates the equilibrium outcomes. Figure 1a shows equilibrium outcomes for different combinations of the probability of an escape (p) and the deterrence value (k). Figure 1b shows how the threshold values change with k . For $k < 0$, fewer types fight as k increases. A

higher k makes letting the other escape relatively more attractive, and such an escape becomes less likely by fighting. This reverses for positive values of k , with more types fighting as k increases. For higher values of k , fewer types flee early. Fighting becomes relatively more attractive with more weaker types still around. The figure also illustrates how these thresholds change with an increase in p .

To shed light on whether the dynamic time element of the Fight-or-Flight game decreases costly battles, we use a static version of the game as benchmark. In the static game, players choose simultaneously between fight and flee, and the same payoffs result as when players reach the final period of the dynamic game. The Bayesian Nash equilibrium of the static game coincides with the equilibrium of the dynamic game for parameters where all players act in the same period (that is, either case (i) or case (iii) described in Proposition 1).

An interesting feature of the timing equilibrium of the dynamic game is that sorting takes place over time, resulting in fewer battles compared to what happens in the static game. In the dynamic game, the strongest types remain in the game until the last period, while some weaker types flee before any battle may take place. Moreover, a smaller fraction of types will fight; fighting becomes less attractive with fewer relatively weak players remaining.

Proposition 2 (Battles and sorting).

Compared to a static (simultaneous-move) version of the game:

- (i) the frequency of battles is reduced in case of a timing equilibrium and the same in case of a rushing or waiting equilibrium, and*
- (ii) the rate at which the weaker player in a pair manages to escape is increased in case of a timing equilibrium and the same in case of a rushing or waiting equilibrium.*

2.3 Extensions

2.3.1 Heterogeneous risk aversion

A surprising feature of the analysis with a homogeneous population is that the set of deterrence values for which the timing equilibrium materializes does not depend on players' risk aversion. This result changes when the population is heterogeneous in the degree of risk aversion. Intuitively, players that are relatively averse to risks will want to flee earlier. Indeed, a population that is heterogeneous in the degree of risk aversion can sustain a timing equilibrium for a larger set of deterrence values. We show this in a simple framework with two levels of risk aversion and we outline the two main strategic features of this model.

Suppose that a fraction $1 - q$ of the population has a risk aversion parameter λ_1 , and a fraction q has $\lambda_2 > \lambda_1$. A player's value of λ is private information but all players know the distribution. Consider the case where q is very small. In that case, the threshold levels derived assuming homogeneous risk aversion in Section 2.1 are not much affected for the less risk averse types. Fix an equilibrium in which $k > \hat{k}$, so that all types with λ_1 wait until period T .

If λ_2 is such that:

$$\tilde{a}\frac{1}{2}(v^h + k) + (1 - \tilde{a})(1 - p)(-\lambda_2 v^l) < 0, \quad (7)$$

then types with λ_2 and a fighting ability less than or equal to \tilde{a} prefer to flee in period $T - 1$ while types with λ_1 prefer to wait until T . Thus, for the same level of k , we now have a timing equilibrium instead of a waiting equilibrium.

Another feature of this model is that the more risk averse types will be the ones who flee more frequently before the end. To see this, note that for the ability level for which the less risk averse type is indifferent between fleeing in period $T - 1$ and period T , the more risk averse type still

strictly prefers to flee in period $T - 1$. The reason is that the expected payoff of fleeing in period $T - 1$ is not affected by the degree of risk aversion (since there are no negative payoffs), while the expected payoff of fleeing in period T decreases in a player's risk aversion (since the negative payoff when a battle is lost weighs more heavily). In the experiment we will test these two implications of the model with heterogeneous risk aversion.

2.3.2 Other extensions

We also considered some natural extensions of the model. Here we describe the main qualitative features of these extensions. In Appendix B we provide further details of these extensions, as well as some discussion of the pros and cons of discrete versus continuous modelling in waiting games.

So far we simply assumed that the stronger player always wins a battle. A natural possibility is that stronger types are more likely to win, but do not win with certainty. When relative strength correlates sufficiently strongly with winning a battle, the results are qualitatively the same. That is, with a positive deterrence value, all the action will be concentrated in the final two periods. The strongest types still want to fight in the final period, while no type wants to flee before the penultimate period. Likewise, with a negative deterrence value, all types will still act immediately. When the link between relative strength and winning a battle becomes weak, other types of equilibria exist. In the extreme case, where each type has an almost equal chance of winning a battle against any other type, there can be equilibria where all types prefer to fight, possibly at different periods. There can also be an equilibrium in which all types prefer to flee in the last period.

Another natural extension is discounting of future payoffs. Conditional on a discount factor sufficiently close to 1, our main theoretical findings remain qualitatively similar. That is, we find a rushing equilibrium when $k < 0$, a timing equilibrium when $0 < k < \hat{k}$ and a waiting equilibrium

when $k > \max(0, \hat{k})$. Like in the timing equilibrium without discounting, all the action happens in the penultimate and last period. The main difference with the model without discounting is that the thresholds now also depend on the discount factor. When discounting is important, the comparison between the static and dynamic case becomes less clear-cut in terms of welfare: the higher degree of sorting comes at the cost of waiting longer.

A final variant that yields somewhat different predictions is the one where players face a known cost for time. Here, it may happen that weak players decide to drop out earlier than the penultimate period. With a cost of waiting, a more gradual fleeing of types may be observed in equilibrium.

In the experiment, we focus on the variant where time is not costly for two reasons. First, it allows us to investigate in a meaningful way how the dynamic game helps players to avoid costly battles compared to the static game where time plays no role. Second, we think that it is a stronger result if players use time as a sorting device when time is not costly.

3 Experimental design and procedures

3.1 Design

Subjects participated in a laboratory experiment in which they played the Fight-or-Flight game. In all treatments, we set the value of winning a battle to $v^h = 10$ and losing a battle to $-v^l = -10$. The probability of an escape when at the same time one player decided to fight and the other decided to flee was set to $p = 0.1$. Each subject played the game 40 rounds, with random rematching after every round within a matching group of 8 subjects. At the start of each round, the subjects were informed of their fighting ability for that round, which was an integer number from 0,1,2,..., 1000. They knew that each number was equally likely, that each subject faced the same distribution and that draws were independent across subjects and rounds. At the end of a round, each subject was

informed of the outcome, the paired subject's fighting ability, and the resulting payoffs.

We implemented two treatment variations. The first treatment variable was the deterrence variable k , which was either -6, 6, or 12. The second treatment variable concerned the dynamic or static nature of the Flight-or-Fight game. This gives a 3x2 design. Every subject participated in only one of the treatments. In total, 360 subjects participated, with 7 or 8 independent matching groups per treatment. Table 1 presents an overview.

In the *dynamic* Fight-or-Flight game, a 5 second countdown started after all subjects in the laboratory had indicated that they were ready to start. This ensured that subjects knew exactly when the game would start. During the game itself, a clock started counting down from 10 seconds to 0. The program divided the 10 seconds in 50 periods of 200 milliseconds each. Subjects implemented their strategies in real time. For instance a subject could decide to wait for 5 seconds (i.e., for the first 25 periods), and to then choose to fight which would then determine the outcome of the game (unless the other subject had already terminated the game earlier). This way she would implement the strategy $(25, F)$. If subjects let the time run down to 0, they entered the *endgame*, in which they simultaneously decided between fight and flee (with no time constraints, as they decided simultaneously anyway).

Our dynamic game has 50 periods, more than the minimum required to test the theoretical predictions of the model, and short time intervals of 200 ms. Our goal was to have a design that is closer to the examples that motivated our research. A disadvantage is that rational subjects might find it hard to exactly implement equilibrium strategies in our setup. A follow-up experiment with longer time-intervals addresses this concern (see section 4.4).

The *static* version of the game abstracted from the time element and only consisted of the endgame of the dynamic version. That is, in this version of the game subjects were immediately

Table 1: Overview of treatments

	Treatment	N subjects	N matching groups
version	deterrence value (k)		
Dynamic	-6	64	8
Dynamic	6	56	7
Dynamic	12	64	8
Static	-6	56	7
Static	6	64	8
Static	12	56	7

put in the same position as the players of the dynamic game who had both decided to wait until the end of the game. So in the static game both subjects simultaneously chose between fight and flee.

After the main part, we obtained additional measurements. We assessed subjects' risk aversion using the method of [Gächter et al. \[2007\]](#). A subject chooses whether to accept or reject 6 different lotteries. In a lottery, the winning amount is 6 euros. The losing amount varies across lotteries, from 2 till 7. In each lottery, the winning and the losing amount are equally likely. If a subject rejects a lottery, she surely receives 0 euro. At the end of the experiment, 1 of the 6 lotteries is selected at random and played out for actual payment. The number of rejected lotteries is our measure of a subject's degree of risk aversion.

We also measured physical strength. We asked subjects to press a hand dynamometer as hard as they could, following the procedure of [Sell et al. \[2009\]](#). This measurement was obtained twice, and the best attempt was rewarded with 5 eurocents per kilo pushed. Finally, we obtained some self-reported measurements on social dominance and prestige (taken from [Cheng et al. \[2010\]](#)), perceived masculinity, sex, and age.⁴

This design allows us to investigate the predictions summarized in Propositions 1 and 2. In addition, it makes it possible to test the predictions from the behavioral model of heterogeneous risk aversion.

3.2 Procedures

The experiment was computerized and run at CREED (University of Amsterdam). The instructions are in Appendix E. Subjects read the instructions at their own pace. They could only continue after

⁴Perceived masculinity is measured by the answer to the question: “*On a scale from 1 (very feminine) to 7 (very masculine), how would you describe yourself?*”.

correctly answering test questions at the end of the instructions. To ease understanding, we used non-neutral labels such as ‘fight’ and ‘escape’. Subjects were informed that there would be two parts, receiving new instructions at the start of each part.

During the experiment, subjects earned points, where 1 point = €0.70 (\approx \$0.84). To avoid a net loss at the end of the experiment, they received a starting capital of 21 points and any profits or losses would be added to or subtracted from this. At the end of the experiment, one round of the main part was randomly selected for payment. Total earnings averaged €19.09, ranging from €5.30 to €38.20.⁵ A session took approximately 65-75 minutes.⁶

4 Results

In subsections 4.1 and 4.2 we will first consider the testable predictions following from Propositions 1 and 2 respectively. Then, in subsection 4.3 we will turn to decisions at the individual level. All statistical tests comparing treatment differences use matching group averages as the independent unit of observation, unless indicated otherwise.

4.1 Timing of actions

Following Proposition 1, we address the comparative static prediction that the timing of actions is influenced by the deterrence value. Specifically, we expect very quick decisions if the deterrence value is negative and decisions in the final periods if the deterrence value is positive. Figure 2 shows

⁵The payment subjects received consisted of the starting capital and their earnings in the Fight-or-Flight game, the lottery task, and the physical strength task.

⁶In addition to the 40 decision rounds (which lasted around 20 minutes), subjects spent time on the instructions and test questions (25 minutes), the lottery task, questionnaire and physical strength task (15 minutes) and payment of subjects (10 minutes).

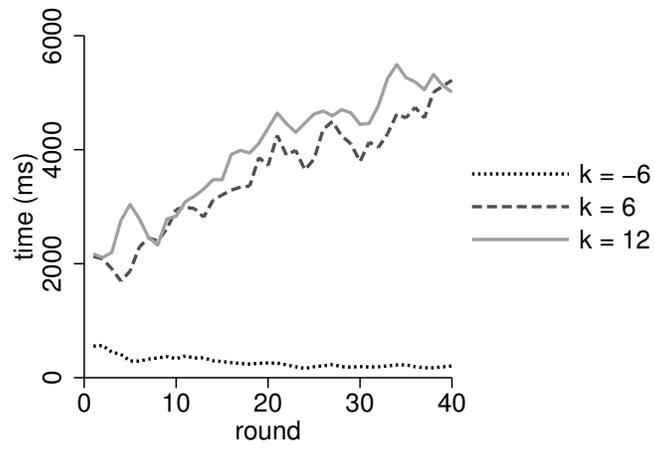


Figure 2: Average waiting time (in ms) before subjects make a decision in the dynamic game, by treatment and round. Lines are moving averages of 3 rounds.

the average elapsed time before subjects made a decision in the dynamic games. As predicted, we observe a clear effect of the deterrence value on the timing of actions. With a negative deterrence value, subjects tend to fight or flee almost immediately. On average, subjects make a decision after 273 ms. When the deterrence value is positive, subjects tend to wait much longer. For $k = 6$, the average elapsed time before making a decision is 3545 ms and for $k = 12$ this is 3973 ms. For both treatments with a positive deterrence value, the average waiting time is significantly longer than for $k = -6$ (Mann-Whitney tests, $p = 0.001$, $N = 15$ for $k = -6$ vs $k = 6$ and $p < 0.001$, $N = 16$ for $k = -6$ vs $k = 12$). While subjects wait slightly longer when $k = 12$ than with $k = 6$, the difference is not statistically significant (Mann-Whitney test, $p = 0.908$, $N = 15$ for $k = 6$ vs $k = 12$). For all three treatments we observe learning effects. When the deterrence value is positive, subjects learn to wait, reflected by the strong positive time trend over the rounds. The reverse holds for the negative deterrence value. In this case, subjects decide increasingly quicker. The average elapsed time is 402 ms in the first 10 rounds and 200 ms in the final 10 rounds. When comparing the average waiting times in the first 10 rounds and final 10 rounds, all time trends are statistically significant (Wilcoxon signed-rank tests, $p = 0.017$, $N = 8$ for $k = -6$, $p = 0.018$, $N = 7$ for $k = 6$ and $p = 0.017$, $N = 8$ for $k = 12$).

Figure 3 gives a more detailed picture of the timing of decisions. The figure plots the distribution of actions for each of the ten seconds plus the endgame (T). The left panels show this for the first 20 rounds and the right panels for the final 20 rounds. Several patterns emerge. First, with a negative deterrence value, we clearly observe rushing: subjects decide almost immediately. None of the matches make it to the endgame and 99.5 percent of all matches end in the first second. In fact, 90 percent of all matches end within the very first 200 ms, i.e. in the first period.⁷ With a positive

⁷Figure A.1 in Appendix C shows the distribution of actions by 200 ms periods.

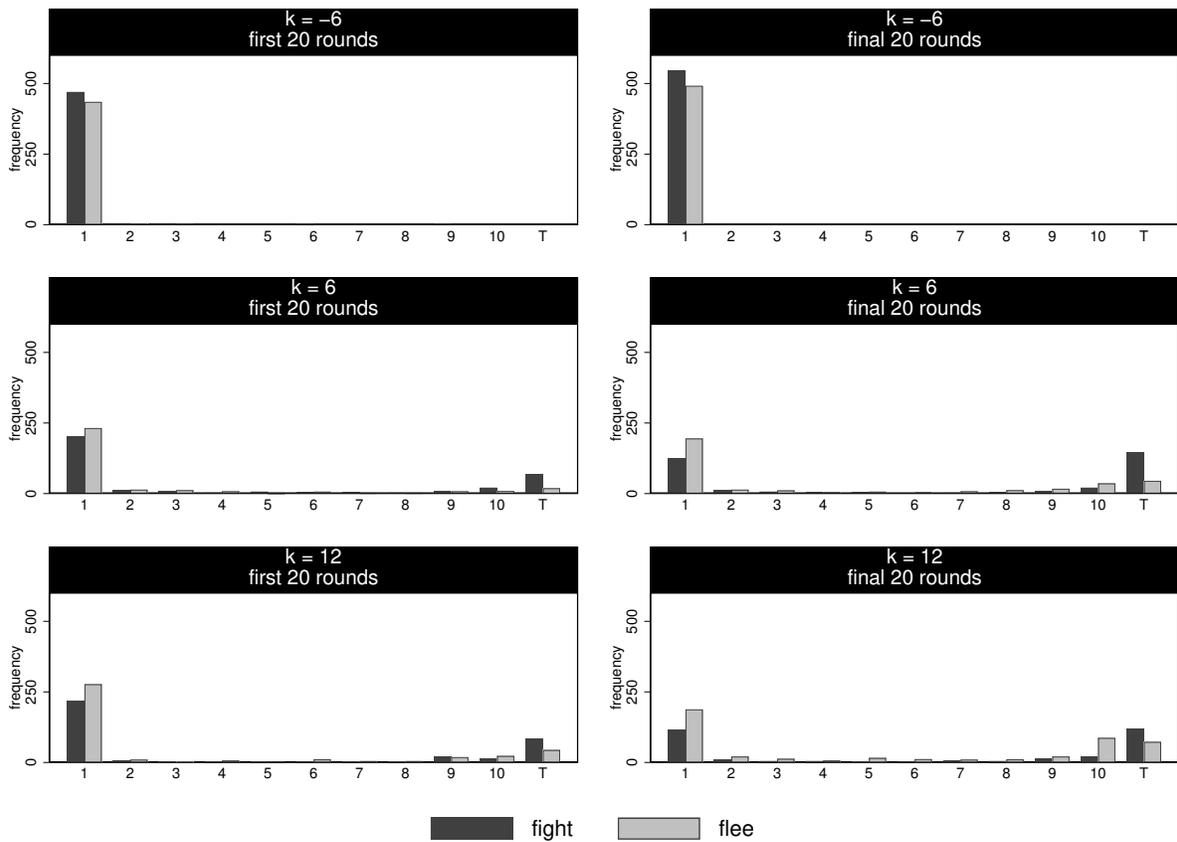


Figure 3: Distribution of decisions over time (seconds) by deterrence value in the dynamic game. Period “T” indicates the endgame. Left panels are for the first 20 rounds, right panels are for the final 20 rounds. Only observations where a player made a decision to fight or flee are included in the graph, i.e. observations where a player was waiting when the other moved are omitted.

deterrence value, most action is at the very beginning and the very end: subjects tend to decide either relatively quickly or wait until the final periods. In the final 20 rounds, a larger fraction of subjects waits until the end. This fraction might be underestimated, because a subject who is willing to wait until the end will only actually reach the end of the game if the paired player is also willing to wait until then. Among those waiting, there are some subjects that flee right before the endgame.

Result 1. *When the deterrence value is negative, players act immediately. When the deterrence value is positive, players are more likely to wait until the end of the game and they learn to wait longer.*

In contrast to the theoretical predictions, some subjects move at the very beginning of the game when the deterrence value is positive. This fraction decreases over time, but even in the final 20 rounds (the right hand panels of Figure 3) we do observe such behavior. This behavior is not in line with the timing equilibrium or waiting equilibrium. We return to this anomaly when we discuss individual behavior (section 4.3). The comparative static results of increasing k are in line with the theoretical predictions though.

4.2 Frequency of battles and sorting

The second main testable prediction -following from Proposition 2- is that endogenous timing helps to avoid costly battles. Specifically, we expect fewer battles in the dynamic games in case of a timing equilibrium, but not in case of a rushing or waiting equilibrium. The left panel of Figure 4 shows the frequency of battles for each treatment (we discuss the results for Experiment 2 in subsection 4.4). We do indeed observe fewer battles in the dynamic treatments compared to the static treatments. The difference varies between 15-26 percentage points depending on the

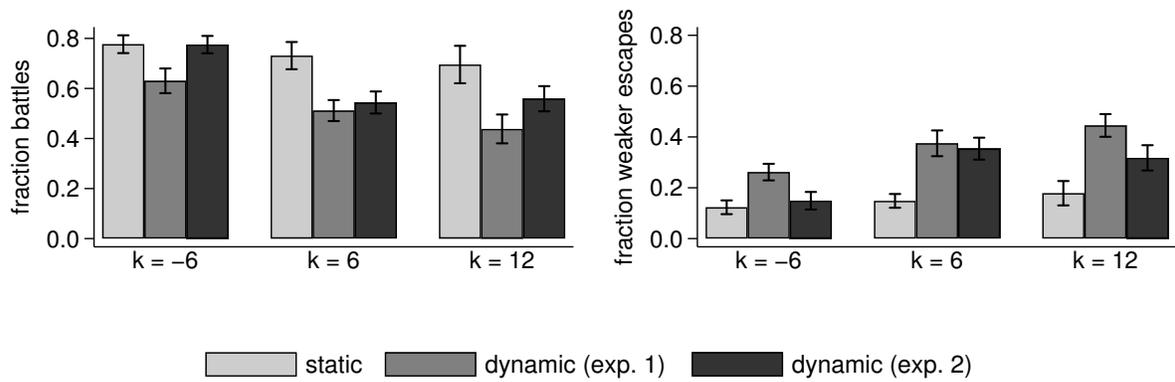


Figure 4: Fraction of battles (left panel) and fraction of times that the weaker player in a pair escapes (right panel). Error bars indicate 95% confidence intervals, based on matching groups as the independent unit of observation.

deterrence value, and is always highly significant ($p < 0.003$ in each case, two-sided Mann-Whitney tests). A regression analysis (Table A.1 in Appendix C, column 1) confirms that there are fewer battles in the dynamic treatments, and this effect is slightly stronger when the deterrence value is positive.

The reduction of battles for $k = 6$ is in line with the comparative static prediction following from Proposition 2. For $k = 6$, the unique equilibrium outcome in the dynamic game is a timing equilibrium, resulting in fewer battles than in the equilibrium of the static game. Even though we observe deviations from the timing equilibrium (in particular, some subjects move at the beginning of the game), we do find that the number of battles is reduced compared to the static case. The observed lower frequency of battles for $k = 12$ is not expected if players are homogeneous in their risk aversion, but is consistent with the comparative static prediction of our version of the model in which players differ in their degree of risk aversion.⁸ In contrast to the theoretical predictions, we also observe a decrease in battles when the deterrence value is negative. This result is, however, partly mechanical; even if all subjects wanted to act immediately, some subjects might be a fraction of a second slower than others, resulting in more escapes.⁹

It is also a possibility that random noise reduces the frequency of battles in the dynamic game.

⁸As for $k = 6$, we also observe deviations from a timing equilibrium when $k = 12$ as a number of subjects move early on in the game. We discuss these deviations in more detail in subsection 4.3.

⁹Of the 15 percentage point difference in battles between static and dynamic games when $k = -6$, 6 percentage points can be attributed to escapes that occur just because the subject who wanted to fight is a fraction slower than the subject who wanted to flee. The remaining 9 percentage points can be attributed to more subjects fighting in the static games.

For instance, if players in the dynamic game choose fight, flee and wait in each period with equal probabilities, while players in the static game choose between flee and fight with equal probabilities, fewer battles will be observed in the static game.¹⁰ However, as we will illustrate in subsection 4.3, the behavior of our subjects is very remote from this random benchmark. Our subjects respond in a sensible way to their private strength parameters. Moreover, in agreement with theory but in contrast to the random benchmark, we find that the dynamic nature matters most for reducing the frequency of battles when $k > 0$.

Also following Proposition 2, we expect that players sort themselves according to their fighting ability in case of a timing equilibrium. The strongest players should wait longer than weaker players, giving weaker players the opportunity to escape. Hence, weaker players should manage to escape more frequently in the dynamic games than the static games if the deterrence value is positive. Our results are in line with this prediction. The right panel of Figure 4 shows how often the weaker subject in a pair escapes. Subjects sort on fighting ability more often in the dynamic than the static game and the increase is larger for dynamic games with a positive deterrence value. For $k = -6$, the weaker player escapes in 12% of the matches in the static game and 26% of the matches in the dynamic game. For $k = 6$ ($k = 12$), the weaker player escapes in 15% (18%) of the matches in the static game and 38% (45%) of the matches in the dynamic game. The diff-in-diff analysis reported in Table A.1 in Appendix C shows that the larger increase for positive deterrence values is statistically significant.¹¹

¹⁰We thank a referee for this insight.

¹¹Figure A.2 in Appendix C shows decision times for weak and strong players separately. It confirms the comparative static prediction that stronger subjects wait longer than weaker subjects if the deterrence value is positive. Moreover, with experience, both weak and strong players learn

Result 2. *There are fewer battles in the dynamic game than in the static game. The dynamic version of the game helps players to sort themselves according to their fighting ability, and this effect is stronger when the deterrence value is positive.*

The reduced number of battles in the dynamic games also positively affects earnings. Figure 5 shows the mean earnings for each treatment and for different levels of fighting ability. As expected, stronger types attain higher earnings. Averaging across all fighting abilities, earnings are higher in the dynamic games than in the static games (Mann-Whitney tests, $p < 0.003$ for all three comparisons). Note that the difference for $k = -6$ is much smaller than the differences for the treatments with a positive deterrence value. Moreover, for $k = -6$ the difference is driven by weaker subjects whereas for the $k > 0$ treatments all types on average benefit from endogenous timing.

4.3 Individual behavior

We start this subsection by considering how actions in the dynamic games depend on fighting ability. Figure 6 plots the fraction of subjects who flee or fight before the endgame, those who were waiting while the other moved, and those who wait until the endgame. We show this for the different deterrence values and for different fighting ability levels (in 10 bins of equal size). In line with the results on decision times discussed in subsection 4.1, no subject waits until the final period when the deterrence value is negative. Only a few subjects (6 percent) are still waiting when the other moves. When the deterrence value is positive, many subjects wait until the endgame, or are waiting when the other moves. Combining those groups, we find that 44 percent of subjects (intend to) wait for both $k = 6$ and $k = 12$. In line with theory, we find in all treatments that weaker players

to wait longer.

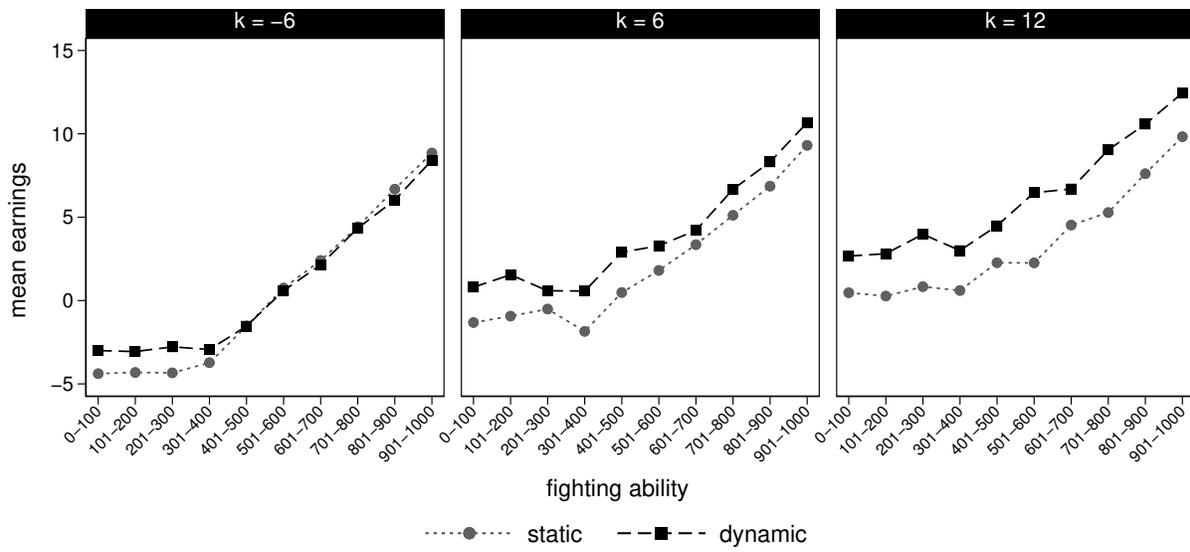


Figure 5: Mean earnings by treatment and fighting ability.

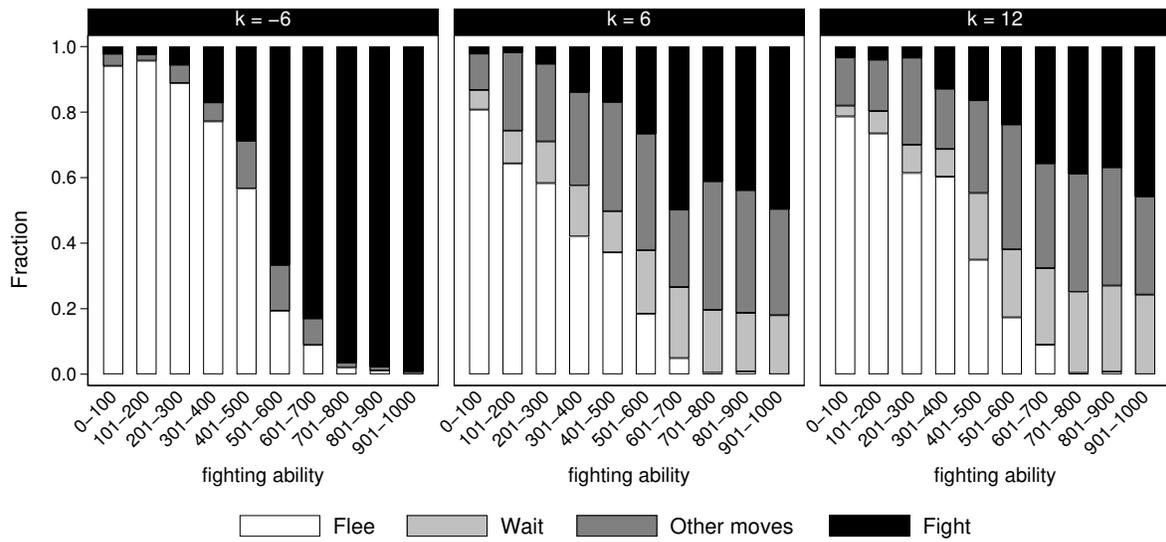


Figure 6: Behavior before the final period in the dynamic game, by deterrence value k and fighting ability a (in 10 bins of equal size). The category “wait” are subjects that made it to the endgame, and “other moves” are subjects who did not make a move before the endgame but the other subject did.

are much more likely to flee and stronger players are much more likely to wait or fight. This pattern clearly shows that the behavior of subjects is far from a random benchmark.

In the Appendix we provide further details on individual strategies. In Appendix D we estimate individual cutoff strategies. We find that most behavior is consistent with the use of cutoff strategies: around 90 percent of all decisions are captured by individual cutoff strategies. There is substantial heterogeneity in the type of cutoff strategies that individuals employ. Although the estimated cutoffs organize the data very well, for a substantial number of subjects the estimated cutoffs are remote from the theoretical prediction.

In subsection 4.2 we reported that sorting was not only observed for $k = 6$ but also for $k = 12$. Although behavior in both treatments does not exactly follow the predictions from the timing equilibrium (notably, some subjects move early on in the game), the finding that subjects sort in $k = 12$ is consistent with the idea that heterogeneous risk aversion enlarges the set of environments for which the timing equilibrium applies. A more direct implication of heterogeneous risk aversion is that the more risk averse players should flee early more often. Table 2 presents panel data probit regressions of how the probability of choosing to flee before the endgame (T) depends on a subject's level of risk aversion, together with some controls. In agreement with the model of heterogeneous risk aversion, more risk averse subjects are more likely to flee before the endgame when $k = -6$ and when $k = 12$, and the effect survives when we combine all three treatments.¹²

An anomaly is the fighting behavior early on in the game when there are benefits of letting the other escape, i.e. when $k > 0$. In this case, fighting early is weakly dominated. Given the observed

¹²When we regress the estimated cutoff fighting ability below which subjects flee before the endgame on risk aversion and other individual characteristics, we obtain qualitatively similar results. The regressions are reported in Table A.2 in Appendix C.

Table 2: Fleeing before endgame

	(1) $k = -6$	(2) $k = 6$	(3) $k = 12$	(4) All k
Risk aversion	0.037*** (0.007)	-0.001 (0.018)	0.041** (0.018)	0.026*** (0.009)
Female	-0.025 (0.031)	-0.012 (0.052)	0.003 (0.060)	-0.008 (0.030)
Dominance	0.007 (0.011)	-0.028 (0.023)	-0.001 (0.015)	-0.006 (0.010)
Physical strength	-0.015 (0.019)	-0.004 (0.020)	0.003 (0.020)	-0.002 (0.011)
Fighting ability	-0.980*** (0.003)	-0.881*** (0.034)	-0.889*** (0.016)	-0.935*** (0.013)
Round	0.000 (0.001)	-0.000 (0.001)	-0.000 (0.001)	-0.000 (0.000)
$k = 6$				-0.127*** (0.030)
$k = 12$				-0.112*** (0.022)
Observations	2520	2080	2520	7120

Notes: Panel data probit regressions, with random effects at the subject level. Coefficients are average marginal effects. Dependent variable is a dummy indicating whether the player decided to flee before the endgame or not. Risk aversion is measured as the number of rejected lotteries. Dominance and physical strength are normalized (mean 0 and s.d. 1). Fighting ability takes on values between 0 and 1. Standard errors (clustered at the matching group level) in parentheses. Additional specifications with fewer or more controls are reported in Table A.4 in Appendix C. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

actions in the experiment, the losses of fighting early are substantial. Consider the strongest possible type who wins every fight. This type would earn 14 percent higher expected payoffs by waiting to fight in the endgame if $k = 6$ and 42 percent higher expected payoffs if $k = 12$. Note that fighting early is even more costly for weaker types. One possible reason for why we observe this anomalous behavior is that subjects may need some time to learn. As Figure 3 shows, we do indeed observe less of this behavior in the final 20 rounds compared to the first 20 rounds. Another, more psychological, explanation for fighting early on in the game might be a preference for social dominance. The evidence does not support this. Table 3 shows that the survey measure of social dominance is not a predictor of fighting early. We also do not find an association with physical strength, but we do find that women are more likely to fight early than men.¹³

It may be that some of our subjects start playing the game with a misguided behavioral rule that in contests it generally pays off to strike first. Myerson [1991] proposes that behavior that is apparently suboptimal behavior can sometimes be understood by assuming that observed behavior is optimal in a related but more familiar environment, which he calls a ‘salient perturbation’ (see Myerson [1991]; Samuelson [2001]; Jehiel [2005]). Alternatively, it could be that intuition favors fighting behavior. According to the ‘social heuristics hypothesis’ (e.g., Rand *et al.* [2012, 2014]) applied to our setting, if fighting is typically advantageous, it could become the intuitive response. Note that subjects who fight early on have limited opportunities to learn, since they never experience the benefits of waiting. This could explain why they do not converge fully to waiting until the end of the game.

¹³When we regress the estimated cutoff fighting ability above which subjects fight before the endgame on risk aversion and other individual characteristics, we obtain qualitatively similar results.

The regressions are reported in Table A.3 in Appendix C.

Table 3: Fighting in the first second

	All rounds			Final 20 rounds		
	(1) $k = 6$	(2) $k = 12$	(3) all $k > 0$	(4) $k = 6$	(5) $k = 12$	(6) all $k > 0$
Risk aversion	-0.030** (0.015)	-0.013 (0.018)	-0.020** (0.010)	-0.020 (0.017)	-0.015 (0.013)	-0.018* (0.011)
Female	0.130 (0.100)	0.029 (0.058)	0.066 (0.050)	0.161 (0.111)	0.046 (0.049)	0.097** (0.048)
Dominance	0.002 (0.019)	0.018 (0.021)	0.015 (0.016)	-0.025 (0.021)	0.011 (0.014)	0.006 (0.011)
Physical strength	0.009 (0.054)	0.001 (0.036)	-0.001 (0.029)	0.025 (0.053)	0.022 (0.027)	0.022 (0.025)
Fighting ability	0.415*** (0.067)	0.368*** (0.060)	0.389*** (0.044)	0.272*** (0.072)	0.218** (0.090)	0.242*** (0.048)
Round	-0.004*** (0.001)	-0.005*** (0.001)	-0.005*** (0.001)	-0.002 (0.001)	-0.003* (0.001)	-0.002*** (0.001)
$k = 12$			-0.017 (0.042)			-0.026 (0.036)
Observations	2080	2520	4600	1040	1260	2300

Notes: Panel data probit regressions with random effects at the subject level. Coefficients are average marginal effects. Dependent variable is a dummy indicating whether the player decided to fight in the first second or not. Risk aversion is measured as the number of rejected lotteries. Dominance and physical strength are normalized (mean 0 and s.d. 1). Fighting ability takes on values between 0 and 1. Standard errors (clustered at the matching group level) in parentheses. Additional specifications with fewer or more controls are reported in Table A.5 in Appendix C. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The fact that we observe an approximately equal frequency of early battles when $k = 6$ as when $k = 12$ suggests that this behavior is not due to a separate utility component reflecting (for instance) a desire to control the outcome or a “joy of winning”. If people have a preference to control the outcome, we would expect less early battles when it becomes more costly in $k = 12$.¹⁴ Still, when play has not yet converged to equilibrium, we cannot exclude that early fighting is encouraged by players who experience a “joy of winning” when they beat the other in a battle. In our follow-up experiments reported in Section 4.4, we include some measures of joy of winning to get direct evidence for this possibility.¹⁵

Result 3. *A sizable minority of players acts immediately when the deterrence value is positive. This behavior decreases with experience.*

4.4 Experiment 2

In the dynamic treatments, a period lasted 200 ms. Such short periods can make it hard for participants to precisely time their actions. This could potentially explain why even for $k = -6$ we observe fewer battles and more escapes in the dynamic game compared to the static game. We address this in a follow-up experiment.¹⁶

¹⁴The same argumentation would apply to a distaste for surprise or suspense.

¹⁵Sheremeta [2010], Price and Sheremeta [2011] and Cason *et al.* [2018] all report evidence that joy of winning and risk aversion are important factors in driving subjects’ behavior in contest games. In a second price auction with value uncertainty, Goeree and Offerman [2003b] find that bidders tend to submit bids below the expected value of the object, which suggests that risk aversion may be the stronger force. Sheremeta [2013] provides a survey.

¹⁶We thank a referee for this suggestion.

4.4.1 Experimental design and procedures

The design of Experiment 2 closely follows that of the first experiment. We collected data for all dynamic treatments, using periods of 5 seconds instead of 200 ms, and 4 periods per round (with 40 rounds in total). This gives subjects more scope to time their actions. We also added two items to the survey, measuring subjects' "joy of winning". The first (incentivized) measure is taken from [Sheremeta \[2010, 2018\]](#). In this task, subjects can bid to win a contest with a prize of 0 points. For the second (non-incentivized) measure, subjects indicated how strongly they agreed with the statement: "*I enjoy winning an amount by competing against another person more than I enjoy receiving that same amount without having to compete for it*" (rated on a 7-point Likert scale).

The experiment was run online. Participants were recruited from the same subject pool as for the first experiment (excluding subjects that already participated). As in the first experiment, we included test questions at the end of the instructions. We showed the correct answers after two failed attempts on a question. We did this to prevent that subjects would log out if they had to wait for too long. We kept track of the mistakes they made, so that we can control for this in the analysis.

In total, 168 subjects participated, with 7 matching groups of 8 subjects in each of the three dynamic treatments ($k = -6$, $k = 6$, $k = 12$).¹⁷ Sessions lasted around 60 minutes in total, and earnings varied between €4.20 and €35.70 (€19.00 on average).

4.4.2 Results

Figure 7 shows the timing of actions, which strongly resembles the results of Experiment 1. With a negative deterrence value, virtually all action happens in the first period. With a positive

¹⁷We have some missing data for 5 subjects who lost the connection. If a subject could not be paired in a round because of this, he or she received the maximal payoff.

deterrence value, many subjects wait until later periods. Compared to the first 20 rounds (left panels), more subjects wait in the final 20 rounds (right panels). The mean waiting time does not increase with experience for $k = -6$ and does increase for positive deterrence values (Figure A.3, Appendix C). Moreover, we again observe that some participants act in the first period when $k > 0$. In line with the theoretical predictions, some subjects flee just before the endgame, although some do so in period 2 rather than period 3, and very few fight just before the endgame.

Figure 4 plots the frequency of battles and escapes by the weaker player in both experiments. The results are very comparable to those of Experiment 1. In particular, for positive values of k , the dynamic game leads to a reduction in battles and an increase in escapes by the weaker player compared to the static version. The difference between the static and dynamic game is significant in all those cases (Mann-Whitney test, $p < 0.005$ in all cases). The main difference with Experiment 1 is that for a negative k , there is no reduction in battles or increase in escapes compared to the static games ($p = 0.898$ for battles, $p = 0.368$ for escapes). This supports the idea that in Experiment 1 the decrease in battles and increase in escapes is driven by coordination failures: subjects may have attempted to immediately fight but were not always able to precisely time their action.¹⁸

In Experiment 2, we again observe anomalous early fighting if $k > 0$. The two measures of joy of winning do not explain this early fighting, while the number of mistakes in the test questions and the social dominance score do explain (some) of the anomalous behavior (see Table A.7 in Appendix C). In Experiment 2, we do not replicate the finding that risk aversion correlates with

¹⁸A regression analysis confirms these results. The interaction effects between dynamic timing and positive deterrence values are statistically significant, indicating that the effect of dynamic timing on battles and escapes matters more for $k > 0$ (Table A.6 in Appendix C).

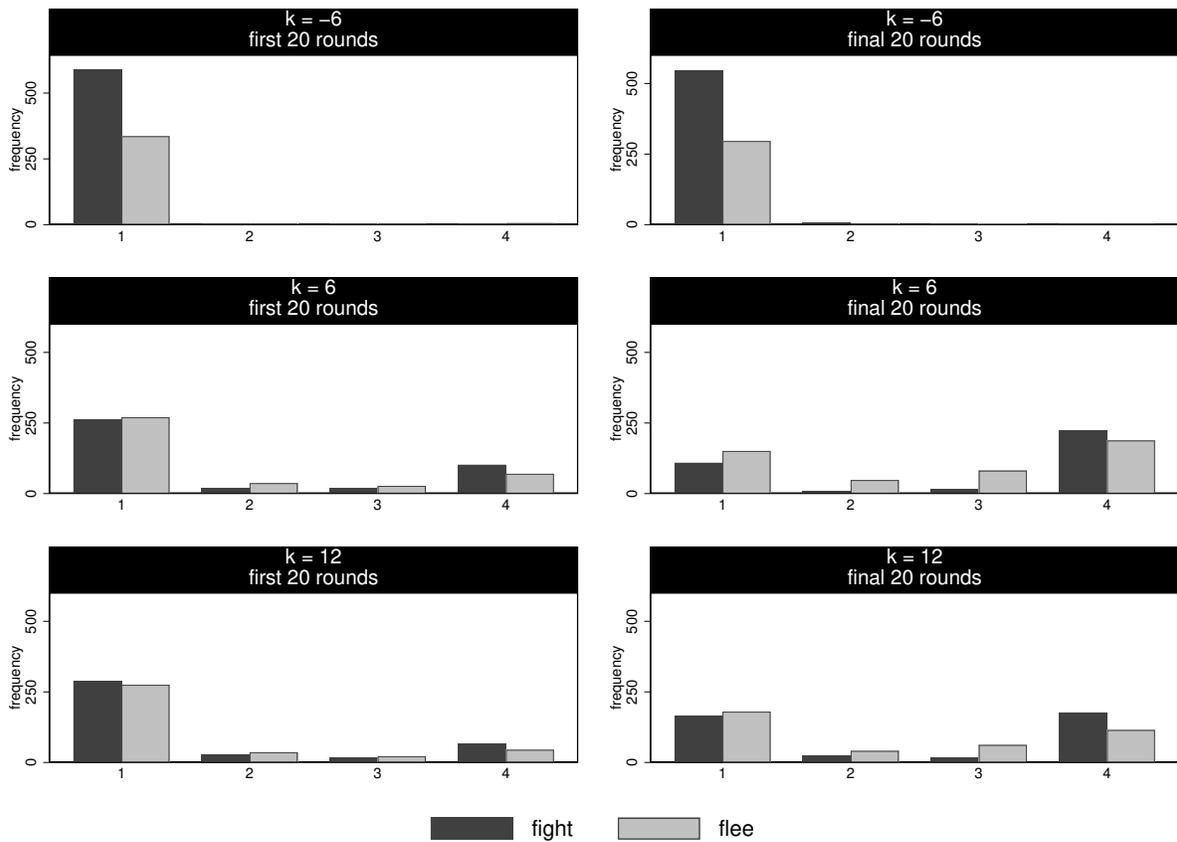


Figure 7: Distribution of decisions over periods by deterrence value in the dynamic game with 4 periods (Experiment 2). Left panels: first 20 rounds. Right panels: final 20 rounds. Only observations where a player made a decision to fight or flee are included (omitting observations where a player was waiting when the other moved).

fleeing before the endgame (see Table [A.8](#) in Appendix [C](#)).¹⁹

5 Concluding Remarks

In this paper, we present a dynamic Fight-or-Flight game that makes sense of a large range of conflicts observed in practice. We highlight the crucial role that the deterrence value plays that players receive when the other player successfully escapes. If it is negative, players will act in a split-second. When it is positive, players will be patient and try to make the other player flee. An interesting feature of the analysis is that if the deterrence value is positive but not too large, sorting will occur. That is, the weakest players will flee just before the end, and thereby avoid costly battles. Thus, this paper clarifies how time can help people reach better outcomes in dynamic games, even when time is not costly. The important role of the deterrence value is confirmed in our experiments. Compared to a static version of the game, players are better able to avoid costly battles.

In the experiment, we find support for a behavioral version of the model that allows for heterogeneous risk aversion. In agreement with this model, sorting occurs for a wider range of situations than predicted by the model with standard preferences. In addition, subjects who appear to be more risk averse in an independent task tend to be the ones that more frequently flee early, although we do not replicate this in the follow-up experiment. We also observe an interesting anomaly. A fraction of the players choose to fight early even in situations where the strategic incentive is to be patient. Our conjecture is that some subjects come to the interaction with a homegrown notion that it generally pays off to strike early in contests. Over time, this costly behavior diminishes but does not disappear.

¹⁹If we combine the data of both experiments, risk aversion is significantly correlated with fleeing before the endgame, and dominance with early fighting. See Tables [A.9](#) and [A.10](#) in Appendix [C](#).

We think that our setup provides a lower limit of the amount of sorting that can be expected in practice. In our game, players manage to sort even though they do not receive any sensory input about the ability of the opponent. In particular when there is a strategic incentive to wait, sensory cues before or during the contest may help players to avoid costly fights. In an actual display, body odor or a high pitched voice may reveal fear and help identify the weaker player (Mujica-Parodi *et al.* [2009], Sobin and Alpert [1999]). A dominant performance in a television show by a candidate running for presidential office may convince a weaker opponent that it is better to flee early. In the future, artificial intelligence may further help players to agree on how they are ranked in terms of ability before they engage in a costly battle. Relevant information about the opponent's ability will also affect players' decisions when the deterrence value is negative. However, in such situations a positive frequency of battles cannot be avoided. Even when information about the opponent helps players to perfectly forecast who will win the fight, the stronger player will still want to catch the weaker player in a battle.

Costly time is another aspect that will encourage a higher proportion of weak types to flee before the end. Also, with costs of time sorting will unfold more gradually, and the weakest types will already flee at the start. We think that extending the analysis in these two directions provides an interesting avenue for future research.

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Online Appendix for: Fight or Flight: Endogenous Timing in Conflicts

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A Proofs of Propositions

Let S_t be the set of feasible actions for a player at time $t < T$. If no player decided to fight or flee at any $t' < t$, then $S_t = \{W, F, R\}$, otherwise S_t is null. $S_T = \{F, R\}$. Here, W indicates wait, F indicates fight, and R indicates flee (retreat). The game ends with at least one player choosing F or R . A pure strategy is then a mapping from each possible date t to S_t , conditional on the player's type. To ease notation, we denote a player's strategy as $s_i(a_i) = (t, A)$, where $A = \{F, R\}$, meaning that player i with ability a_i will take an action (fight or flee) at time t if the other player did not fight or flee before. In what follows, when we describe a strategy, we drop the qualifier "conditional on the other player not fleeing or fighting before."

We first show that equilibrium strategies are monotonic, in the sense that if some type prefers to fight at some point over fleeing at that or any other point, then all stronger types also prefer to fight at some point over fleeing. Let $\tilde{V}_i((t, A), a_i, s_j(a_j))$ be player i 's expected payoff of playing strategy (t, A) given his type a_i and strategy of the opponent (and distribution of possible types of the opponent).

Lemma 1 (Monotonicity of Equilibrium Strategies). *(i) If there is an equilibrium in which there is a period t such that a player with type a_i (strictly) prefers strategy (t, F) to (t', R) for any t' , then any player with type $a_j > a_i$ (strictly) prefers (t, F) to (t', R) for any t' . (ii) Suppose there is an interval of types $a_t = (a_1, a_2)$ that act in period t and let $a_i, a_j \in a_t$. If there is a type a_i that is*

indifferent between (t, F) and (t, R) , then all types $a_j > a_i$ strictly prefer (t, F) to (t, R) and all types $a_j < a_i$ strictly prefer (t, R) to (t, F) .

Proof of Lemma 1. Consider two types a' and $a'' > a'$. Suppose type a' prefers (t, F) for some t to (t', R) for any t' . Then it must be that there exists a t such that for all t' ,

$$\Delta(a') \equiv \tilde{V}_i((t, F), a', \cdot) - \tilde{V}_i((t', R), a', \cdot) \geq 0. \quad (8)$$

Fighting in period t instead of fleeing in period t' never decreases and may increase the likelihood of ending up in a battle with types $a \in (a', a'')$. Type a' would lose such a battle and type a'' would win it. If the opponent has ability $a \notin (a', a'')$, then $\Delta(\cdot)$ is affected equally for types a' and a'' . The likelihood of a battle by fighting at t instead of fleeing at t' increases if (i) the opponent fights at t' or after, or (ii) flees at t or after. If \mathcal{A} is the set of types on (a', a'') that fights at t' or after, or flees at t or after, then,

$$\Delta(a'') - \Delta(a') \propto \int_{a \in \mathcal{A}} g(a) da \geq 0,$$

where $g(a)$ is the density function. Using the above fact, equation (8) implies:

$$\Delta(a'') = \tilde{V}_i((t, F), a'', \cdot) - \tilde{V}_i((t', R), a'', \cdot) \geq 0. \quad (9)$$

and the inequality in (9) is strict if either (8) holds with strict inequality or there is a strictly positive mass of types acting at period t' or after.

To show part (ii), note that in this case there is a strictly positive probability of meeting an opponent with an ability between a_j and a_i , and $\Delta(a_j)$ is therefore strictly higher than $\Delta(a_i)$. \square

Proof of Proposition 1. Suppose $-v^h < k < 0$. In this case, winning a battle yields a higher payoff than letting the other escape. There exists a $\varepsilon > 0$ such that all types on $(1 - \varepsilon, 1]$ strictly prefer $(0, F)$ to (t, R) for any t . For $\varepsilon \rightarrow 0$, the likelihood of meeting a stronger type becomes arbitrarily

small for types on that interval, and they win all battles with weaker types. Thus, sufficiently strong types will never flee. In any equilibrium, there must also be a positive fraction of types with strategy (t, R) for some t . If this were not the case, then there exists an $\varepsilon > 0$ such that all types on $[0, \varepsilon)$ strictly prefer $(0, R)$ to (t, F) for any t . For $\varepsilon \rightarrow 0$, the likelihood of meeting a stronger type becomes arbitrarily high for types on that interval, and they lose all battles with stronger types. Thus, sufficiently weak types would deviate to fleeing.

Now let t' be the last period in which a positive fraction of types acts. Denote this set by $\mathcal{A}_{t'} = \{a_i | s_i = (t', R) \cup s_i = (t', F)\}$, and let $\underline{a}_{t'} = \inf \mathcal{A}_{t'}$ and $\bar{a}_{t'} = \sup \mathcal{A}_{t'}$. In that period, there must be a positive fraction of types with (t', R) and a positive fraction of types with (t', F) . If there would be no positive fraction of types fleeing, then for sufficiently small ε , all types on $\mathcal{A}_{t'} \cap [\underline{a}_{t'}, \underline{a}_{t'} + \varepsilon)$ strictly gain by deviating to (t', R) : deviating to fleeing in that period strictly decreases the probability of a battle which the sufficiently weak types in that set would almost surely lose. If there would be no positive fraction of types fighting, then for sufficiently small ε , all types on $\mathcal{A}_{t'} \cap (\bar{a}_{t'} - \varepsilon, \bar{a}_{t'}]$ strictly gain by deviating to (t', F) : deviating to fighting in that period strictly increases the probability of a battle which the sufficiently strong types would almost surely win.

With a positive fraction of types that has strategy (t', R) , there cannot be a period $t < t'$ in which a positive fraction of types has strategy (t, R) . If there were such a period (if there are more, let t be the last of those), then Lemma 1 implies that types with strategy (t, R) must be weaker than types with strategy (t', F) . But then types with strategy (t', F) strictly gain by deviating to (t, F) , since this will not affect the outcome with other types that have strategy (t, F) and strictly decreases the probability of an escape by types with strategy (t, R) (which are weaker).

It then follows that all types must act at $t = 0$. If there is some period $t' > 0$ in which a positive

fraction of types flees, then all types that fight gain by deviating to strategy (t, F) for some $t < t'$. Since in any equilibrium in which a positive fraction of types has strategy (t', R) there must also be a positive fraction of types with strategy (t', F) , it must be that $t' = 0$. The only equilibrium strategies are then $(0, R)$ and $(0, F)$. Lemma 1 then implies that all types below a certain threshold flee, and types above the threshold fight. The threshold is determined by equation (4) in the main text. With these strategies, no player has an incentive to deviate. The equilibrium payoffs for types $a_i < \tilde{a}$ are $\tilde{a}\frac{1}{2}(v^h + k) + (1 - \tilde{a})(1 - p)(-\lambda v^l)$. Fleeing or fighting in some period $t > 0$ would yield payoffs $\tilde{a}(v^h + k) + (1 - \tilde{a})(-\lambda v^l)$. No player deviates if $\tilde{a} \leq p\lambda v^l / [\frac{1}{2}(v^h + k) + p\lambda v^l]$. Substituting for \tilde{a} , we find that this is always satisfied. Types $a_j > \tilde{a}$ clearly have no incentive to deviate to acting later. Acting later does not change the outcomes with other types that fight and increases the likelihood of the weaker types escaping. Finally, no type wishes to deviate to another strategy at $t = 0$. The difference in payoffs between fighting and fleeing (Δ) is strictly increasing for types $a_i < \tilde{a}$ ($\partial\Delta/\partial a_i = \tilde{a}(1 - p)(v^h + \lambda v^l)$) and strictly increasing for $\tilde{a} < a_j$ for any $p > 0$ and constant for $p = 0$ ($\partial\Delta/\partial a_j = (1 - \tilde{a})p(v^h + \lambda v^l)$). Thus, if \tilde{a} is indifferent, then all weaker types strictly prefer to flee and all stronger types (weakly) prefer to fight.

The equilibrium exists for $0 < \tilde{a} < 1$. It is straightforward to verify that this is the case for any $-v^h < k < 0$.

Consider next the case with $0 < k < \hat{k}$. In this case, letting the other escape yields a higher payoff than winning a battle. In equilibrium there has to be a positive fraction of types that for some t prefers (t, R) to (t', F) for any t' . If this were not the case, and no positive fraction flees at some point, then for sufficiently small $\varepsilon > 0$, all types on $[0, \varepsilon)$ strictly gain by fleeing at $t = 0$: this would strictly increase the probability of an escape and they almost surely lose a battle for ε

sufficiently small.

We next show that all types will act in the last two periods. Let t' be the last period in which a positive fraction flees. The strongest types strictly prefer to wait until after t' , if such a period exists. It cannot be that a positive fraction fights after t' , however. If there would be a set of types \mathcal{A}_f fighting after t' , with $\underline{a} = \inf \mathcal{A}_f$, then for $\varepsilon > 0$ sufficiently small, types on $\mathcal{A}_f \cap [\underline{a}, \underline{a} + \varepsilon)$ would strictly gain by fleeing in some period after t' . Thus, it must be the case that the strongest types wait until T , and at least some of the types acting in period T will flee. No type will then fight before T : fighting later does not change the outcome against other types that fight, and gives weaker types the option to escape. Furthermore, no type will act before $T - 1$: if a positive fraction of types would act before $T - 1$, they would strictly gain from waiting until $T - 1$, since no types fight at $T - 1$. Lemma 1 implies that if some types fight at T , then all stronger types must fight too. Any equilibrium can therefore be characterized by the thresholds in equation (5) in the main text.

Under these strategies, no type gains from deviating. All types $[0, \hat{a}_2]$ have the same equilibrium payoff ($\hat{a}_1 \frac{1}{2}(v^h + k) > 0$) and are indifferent between $(T - 1, R)$, (T, R) and (T, F) . If they would flee before $T - 1$ they would earn 0. Fighting earlier is strictly dominated for type \hat{a}_2 since it would give weaker types no option to escape, and therefore also for any weaker type. Types $(\hat{a}_2, 1]$ earn more under strategy (T, F) than under strategy (T, R) , and they also do not want to act earlier, as it would give weaker types no option to escape. If no player acts before T , players will update their beliefs about the opponent's type. With the threshold level \hat{a}_2 , it is easy to show that type \hat{a}_2 is still indifferent between fleeing and fighting, and all stronger (/weaker) types prefer to fight (/flee).

The equilibrium exists if $0 < \hat{a}_1 < \hat{a}_2 < 1$. That $0 < \hat{a}_2 < 1$ is clear from the restrictions on k . After some rewriting, one can show that $\hat{a}_1 < \hat{a}_2$ if $k < v^h$, which always holds for this case as from equation (6) it follows that $k < \hat{k} < v^h$.

Finally, consider the case with $k > \hat{k}$. The analysis is identical to the case for $0 < k < \hat{k}$, except that \tilde{a}_1 is negative. This means that all types like to act at T . Weak types flee and strong types fight, where the threshold is determined as \tilde{a} as in equation (4). Note that for $k(1 - 2p) > v^h$, all types prefer to flee in equilibrium, so we set $\tilde{a} = 1$.

□

Proof of Proposition 2. Part (i). In a simultaneous move game, the threshold type is determined by \tilde{a} . For the rushing and waiting equilibrium, this coincides with the threshold type in the dynamic game and the frequency of battles must be the same in the dynamic and the static games. For the timing equilibrium, a battle occurs when a player with $a_i > \hat{a}_2$ either meets another player with $a_j > \hat{a}_2$ or another player with $a_j \in (\hat{a}_1, \hat{a}_2)$ who does not manage to escape. This means that the frequency of battles in the timing equilibrium is given by $f_{timing}^b = (1 - \hat{a}_2)^2 + 2(1 - \hat{a}_2)(\hat{a}_2 - \hat{a}_1)(1 - p)$. In the static game, a battle occurs if two types with $a_i > \tilde{a}$ meet, or when a type with $a_i > \tilde{a}$ meets a type with $a_j \leq \tilde{a}$ and the weaker type does not manage to escape. This is, the frequency of battles in the static game is given by $f_{static}^b = (1 - \tilde{a})^2 + 2(1 - \tilde{a})\tilde{a}(1 - p)$. A sufficient condition for fewer battles to occur in the timing equilibrium than in the static game (i.e. for $f_{static}^b > f_{timing}^b$ to hold) is that $\tilde{a} < \hat{a}_2$ holds. This requires that $k < \hat{k} \equiv \frac{1-2p}{1-2p+2p^2}v^h$, which is satisfied whenever a timing equilibrium exists. Part (ii) In the static game, the weaker player in a pair manages to escape with frequency $f_{static}^s = \tilde{a}(\tilde{a}\frac{1}{2} + (1 - \tilde{a})p) + (1 - \tilde{a})(\tilde{a}p)$. In the dynamic games, this frequency is the same in case of a rushing or waiting equilibrium. In case of a timing equilibrium, the weaker player in a pair manages to escape with frequency $f_{timing}^s = \hat{a}_1(\hat{a}_1\frac{1}{2} + (1 - \hat{a}_1)) + (\hat{a}_2 - \hat{a}_1)(\hat{a}_1 + (\hat{a}_2 - \hat{a}_1)\frac{1}{2} + (1 - \hat{a}_2)p) + (1 - \hat{a}_2)(\hat{a}_1 + (\hat{a}_2 - \hat{a}_1)p)$. In the proof of part (i), we showed that if a timing equilibrium exists, it must be that $\tilde{a} < \hat{a}_2$.

This implies that $f_{static}^s < \hat{a}_2 (\hat{a}_2 \frac{1}{2} + (1 - \hat{a}_2)p) + (1 - \hat{a}_2)(\hat{a}_2 p)$ and a sufficient condition for $f_{timing}^s > f_{static}^s$ is for $f_{timing}^s > \hat{a}_2 (\hat{a}_2 \frac{1}{2} + (1 - \hat{a}_2)p) + (1 - \hat{a}_2)(\hat{a}_2 p)$ to hold. Rewriting yields that this holds as long as $\hat{a}_1 + \hat{a}_2 + (1 - \hat{a}_2)2p < 2$, which is satisfied as in a timing equilibrium we have that $\hat{a}_1 < \hat{a}_2 < 1$ and $p \leq \frac{1}{2}$. □

B Extensions

B.1 Uncertainty in the likelihood of winning a battle

In the model, we assume that the stronger player always wins a battle. In many cases, there is some uncertainty and weaker players sometimes win battles too. A natural case is one in which the likelihood of winning a battle increases in a player's relative ability compared to the opponent. For instance, the probability that i wins a battle may be determined by:

$$\frac{e^{\mu a_i}}{e^{\mu a_i} + e^{\mu a_j}}, \quad \mu > 0 \quad (10)$$

so that stronger types are more likely to win, and types of similar fighting ability have about equal chances of winning.

For large values of μ , such functions will yield the same qualitative results. That is, with positive deterrence value, all the action will be concentrated in the final two periods. The strongest types will still want to fight in the final period, while no type will want to flee before the penultimate period. Likewise, with a negative deterrence value, all types will still act immediately, provided the strongest types prefer to fight. Naturally, the exact thresholds \tilde{a} , \hat{a}_1 , \hat{a}_2 and \hat{k} will depend on the specifics of the winning function. Stronger types still have more to gain from fighting than weaker types, but the difference decreases. This time, the equilibrium outcome does not only pin down the fraction of types fleeing in the penultimate period, but also the set of types. In the basic setup, all types below \tilde{a}_2 were certain to lose a battle and therefore all had the same payoffs of fleeing and fighting. With a probabilistic chance of losing a battle that depends on relative fighting ability, the weakest types are most likely to lose a battle, and therefore they are the ones fleeing at $T - 1$.

For small values of μ , other types of equilibria exist. As μ becomes small, there is more

randomness in which player wins a battle. In the extreme case, where $\mu \approx 0$, each type has an almost equal chance of winning a battle against any other type. In that case, there can be equilibria where all types prefer to fight (whenever $\frac{1}{2}(v^h - \lambda v^l) > 0$), possibly at different periods. There can also be an equilibrium in which all types prefer to flee in the last period (when $\frac{1}{2}(v^h - \lambda v^l) < 0$).

B.2 Discounting

Another natural extension is to introduce discounting of future payoffs. We assume that a payoff that is received τ periods from some date t is discounted by δ^τ , evaluated at date t . Here, $\delta \in (0, 1]$. To make things interesting, we assume that any payoffs are realized as soon as at least one of the players acts.²⁰

In the following, we show that for δ exceeding some critical level $\delta^* < 1$, the equilibrium outcomes characterized in Proposition 1 continue to exist (with some proper adjustments of the threshold levels \hat{a}_1 and \hat{a}_2). That is, for δ sufficiently close to 1, there exist a rushing equilibrium when $k < 0$, a timing equilibrium when $0 < k < \hat{k}$ and a waiting equilibrium when $k > \min(0, \hat{k})$.

Rushing equilibrium. For $k < 0$, this equilibrium profile, in which all types act immediately, is trivially unaffected by δ ; if a type has no incentive to deviate to fight or flee in a later period for $\delta = 1$, then the type certainly has no incentive to deviate for any $\delta < 1$. The threshold level \tilde{a} is also unaffected. It is also still the case that this equilibrium does not exist for $k > 0$ when $\delta < 1$. If all types would act immediately, the strongest type would gain from acting later; since all other types are supposed to act in period 0, this does not affect the outcome if the opponent fights (which then still happens in period 0), but gives weak opponents the chance to flee.

Timing equilibrium. In the timing equilibrium without discounting, all the action happens in

²⁰If all payoffs are realized at T or later, discounting just implies a rescaling of the payoffs.

the penultimate and last period. If the equilibrium exists for $\delta < 1$, the new thresholds \hat{a}'_1 and \hat{a}'_2 will depend on the discount factor, as type $a = \hat{a}'_1$ must be indifferent between fleeing in period $T - 1$ and fleeing one period later. For any type, the payoff of fleeing at $T - 1$ is (evaluated at $t = 0$):²¹

$$\delta^{T-1} \frac{1}{2}(v^h + k) \hat{a}'_1. \quad (11)$$

For any type $a \leq \hat{a}'_2$, the expected payoff of fleeing at T (again evaluated at $t = 0$) is:

$$\hat{a}'_1 \delta^{T-1}(v^h + k) + (\hat{a}'_2 - \hat{a}'_1) \delta^T \frac{1}{2}(v^h + k) + (1 - \hat{a}'_2) \delta^T (1 - p)(-\lambda v^l). \quad (12)$$

For type $a = \hat{a}'_2$, the expected payoff of fighting at T is (again evaluated at $t = 0$):

$$\hat{a}'_1 \delta^{T-1}(v^h + k) + (\hat{a}'_2 - \hat{a}'_1) \delta^T (v^h + pk) + (1 - \hat{a}'_2) \delta^T (-\lambda v^l), \quad (13)$$

Using (11), (12) and (13), and given that type $a = \hat{a}'_1$ must be indifferent between fleeing at $T - 1$ and T , and type $a = \hat{a}'_2$ must be indifferent between fleeing in period T and fighting in period T , gives:

$$\hat{a}'_1 = \frac{((1 - 2p)(v^h - k) - 2p^2k)\lambda v^l \delta}{\delta \lambda v^l ((1 - 2p)v^h - (2p^2 - 2p + 1)k) + (p\lambda v^l - (\frac{1}{2} - p)k + \frac{1}{2}v^h)(v^h + k)}, \quad (14)$$

and:

$$\hat{a}'_2 = \frac{((1 - 2p)(v^h - k) - 2p^2k)\lambda v^l \delta + p\lambda v^l(v^h + k)}{\delta \lambda v^l ((1 - 2p)v^h - (2p^2 - 2p + 1)k) + (p\lambda v^l - (\frac{1}{2} - p)k + \frac{1}{2}v^h)(v^h + k)}, \quad (15)$$

while $\hat{k} = \frac{1-2p}{1-2p+2p^2}v^h$ remains as in (6).

To consider whether this timing equilibrium profile can be sustained with discounting, note that the most profitable deviation before $T - 1$ is to take some action at $t = 0$ (rather than for some

²¹As we assume exponential discounting, the analysis is unaffected by the period at which (future) payoffs are evaluated.

$0 < t < T - 1$). A deviation to fleeing at $t = 0$ is never profitable as it gives a payoff of zero instead of a positive discounted payoff. On the other hand, some types will have positive expected payoffs of fighting at $t = 0$. All types that fight will gain or lose the same from deviating to $t = 0$ when they are paired with a type that flees. However, when paired with another type that fights, weaker types have a larger probability of losing a battle, and by deviating to $t = 0$ they incur any losses $-v^l$ earlier on (and thus discounted less), making it less attractive to deviate compared to stronger types. Hence, of all the types that fight at T , type $a = 1$ has the most to gain from fighting at an earlier date.

Type $a = 1$ prefers not to deviate to fighting at $t = 0$ as long as

$$\hat{a}'_1 \delta^{T-1} (v^h + k) + (\hat{a}'_2 - \hat{a}'_1) \delta^T (v^h + pk) + (1 - \hat{a}'_2) \delta^T v^h \geq v^h, \quad (16)$$

and not to deviate to fighting at $t = T - 1$ as long as

$$\hat{a}'_1 \delta^{T-1} (v^h + k) + (\hat{a}'_2 - \hat{a}'_1) \delta^T (v^h + pk) + (1 - \hat{a}'_2) \delta^T v^h \geq \delta^{T-1} (v^h + \hat{a}'_1 pk). \quad (17)$$

At $\delta = 1$, the inequalities in (16) and (17) are strict. The LHS of (16) and both sides of (17) are clearly continuous in δ , so that there exists $\delta^* < 1$ such that (16) and (17) hold for any $\delta > \delta^*$.

Note also that there cannot be an equilibrium in pure strategies in which all players act before T . In that case, in the last period in which a positive fraction of types act, the strongest type would strictly prefer to deviate to fight at a later period (this would not change the period in which payoffs are realized, gives types that flee a higher chance to escape, and does not change the outcome if the opponent fights).

Waiting equilibrium. As for the rushing equilibrium, the threshold \tilde{a} in (4) is unaffected by discounting. In the waiting equilibrium, the highest type has again the strongest incentive to deviate

to fight in period 0 (and deviating to fleeing in an earlier period is never profitable for any type).

The strongest type has no incentive to deviate to fight at $t = 0$ as long as:

$$\tilde{a}\delta^T(v^h + pk) + (1 - \tilde{a})\delta^T v^h \geq v^h. \quad (18)$$

Solving this, yields that $\delta \geq \left[\frac{v^h}{v^h + \tilde{a}pk} \right]^{1/T}$ should hold for the waiting equilibrium to exist, and note that $[v^h/(v^h + \tilde{a}pk)]^{1/T} < 1$.

Given that all equilibrium outcomes as described in Proposition 1 still exists for sufficiently large discount factors, the results in Proposition 2 remain valid as well. In particular, compared to the static case, a timing equilibrium results in fewer battles and weaker players are more likely to escape. In this case, whether the dynamic version improves welfare is less clear-cut, as the reduction in battles comes at the cost of waiting longer.

B.3 Cost of waiting

A variant that yields somewhat different predictions is the one where players face the same known cost c for time. Here, it may happen that weak players decide to drop out earlier than the penultimate period, as illustrated in the following example. Consider the case in which $T = 2$, $k = 10$, $p = 1/2$, and $\lambda = 1$, while players incur a waiting cost of $c = 5$ per period. Then it is straightforward to show that there is an equilibrium where low types with abilities in the interval $[0, 3/13]$ do not wait and flee immediately at $t = 0$, types in $(3/13, 5/13]$ flee at $t = 1$, types in $(5/13, 9/13]$ flee at $t = 2$ while types in $(9/13, 1]$ fight at $t = 2$. Thus, with a cost of waiting, a more gradual fleeing of types may be observed in equilibrium.

B.4 Continuous time

With continuous time, the ‘Rushing’ and ‘Waiting’ equilibria described in Proposition 1 remain unaffected. When the deterrence value is negative players still want to move immediately and when

the deterrence value is sufficiently large all players will still move as late as possible. However, when the deterrence value is positive but not too large (as in part (ii) of Proposition 1), there would be no equilibrium in pure strategies in a continuous time model due to an open-set problem. In the ‘timing equilibrium’ with discrete time, the weakest types flee just before the end. In a continuous time model, if the weakest types with abilities $[0, a_1^*]$ flee in some period $T - \varepsilon$, then any type in this interval would like to deviate and flee in $T - \frac{1}{2}\varepsilon$, however small ε is chosen. This also holds in the ‘Perfectly Continuous time’ protocol that is discussed in [Calford and Oprea \[2017\]](#) and [Simon and Stinchcombe \[1989\]](#).²² [Calford and Oprea \[2017\]](#) show that behavior in experiments can be closer to either ‘Perfectly Continuous time’ or ‘Perfectly Discrete time’ equilibria, depending on the magnitude of players’ inertia.

Our experimental implementation involves discrete time with short (200 ms) periods. We conjecture that under ‘Perfectly Continuous time’, behavior will be very close to the behavior that we observe in our current experiments. Although the periods are short, most of the actions are predicted to be made at the start of (very near) the end of the game, so that it should still be relatively easy for participants to time their decisions to flee or fight (a prediction that is supported by the data).

²²For further discussion about when discrete time can be more appropriate to model timing in games, see [Fudenberg and Tirole \[1985\]](#) and [Levin and Peck \[2003\]](#).

C Supplementary figures and tables

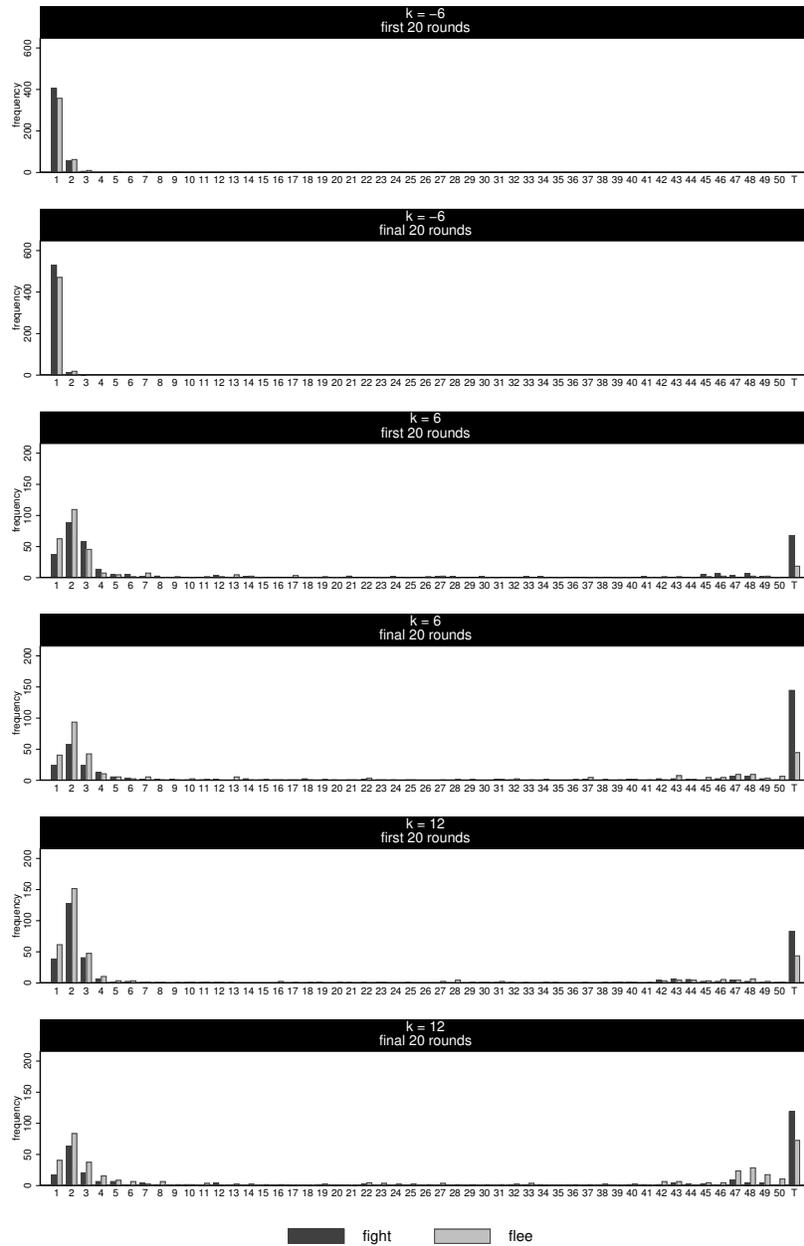


Figure A.1: Distribution of decisions over time periods by deterrence value in the dynamic game. Each period is a 200 ms interval, period “T” is the endgame. First, third and fifth panel are for the first 20 rounds, second, fourth and sixth panel are for the final 20 rounds.

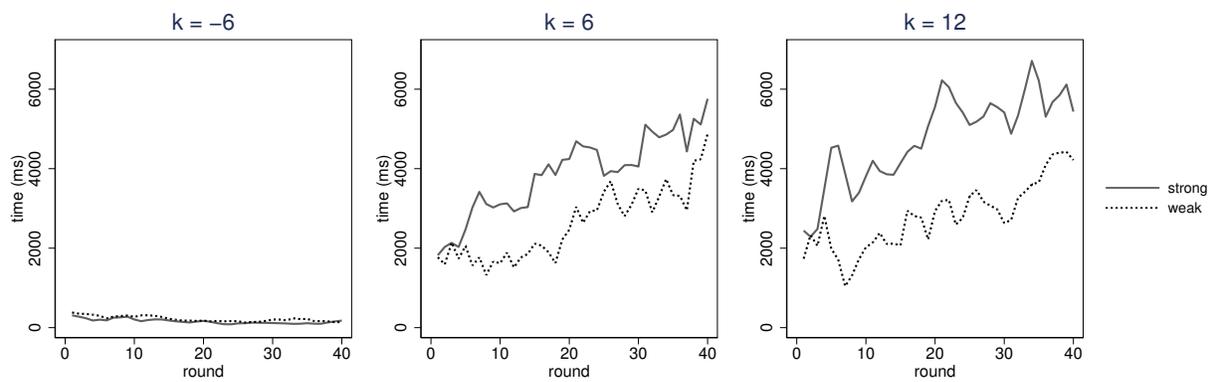


Figure A.2: Average waiting time (in ms) before subjects make a decision in the dynamic game, by treatment, type and round. 3-round moving average. Strong types have fighting ability ≥ 750 , weak types have fighting ability ≤ 250 .

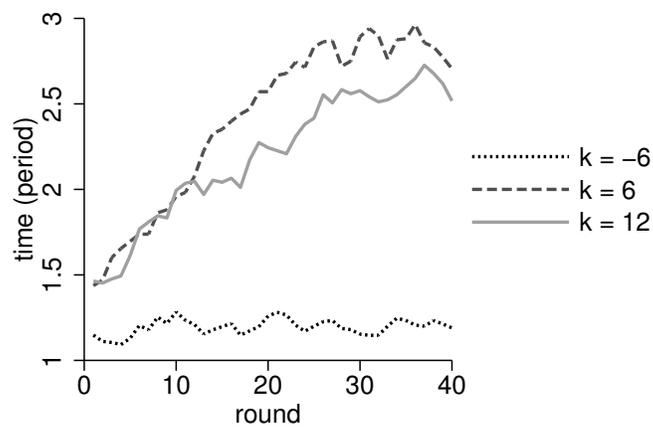


Figure A.3: Average waiting time (in periods) before subjects make a decision in the dynamic games of Experiment 2, by treatment and round. Lines are moving averages of 3 rounds.

Table A.1: Treatment differences in battles and sorting

	(1) Battle occurs	(2) Weaker escapes
$k = 6$	-0.046 (0.039)	0.025 (0.028)
$k = 12$	-0.081* (0.040)	0.055* (0.029)
Dynamic	-0.146*** (0.039)	0.139*** (0.028)
$k = 6 \times$ Dynamic	-0.073 (0.055)	0.088** (0.040)
$k = 12 \times$ Dynamic	-0.111* (0.055)	0.128*** (0.040)
Constant	0.777*** (0.028)	0.123*** (0.021)
Observations	45	45
R^2	0.747	0.847

Notes: OLS regressions. Unit of observation is a matching group. Dependent variable in column (1) is the fraction of battles and in column (2) the fraction of matches where the weaker player in a pair managed to escape. All independent variables are dummies.

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A.2: Fleeing before endgame: based on estimated cutoff strategies

	(1) $k = -6$	(2) $k = 6$	(3) $k = 12$	(4) All k
Risk aversion	31.31* (13.30)	25.46 (21.43)	45.72* (20.52)	32.70*** (10.45)
Female	-12.17 (49.33)	41.74 (80.05)	2.89 (89.27)	2.38 (42.01)
Dominance	12.41 (18.08)	-36.85 (24.82)	13.73 (15.70)	3.45 (11.91)
Physical strength	-5.11 (28.27)	29.30 (27.13)	-7.30 (22.56)	0.70 (14.91)
$k = 6$				-130.56*** (30.53)
$k = 12$				-91.55*** (29.65)
Constant	321.36*** (28.87)	182.48** (62.62)	188.89*** (27.25)	312.51*** (27.83)
Observations	63	52	63	178
R^2	0.10	0.06	0.09	0.15

Notes: OLS regressions. Dependent variable is the estimated cutoff c_1 , indicating the estimated strength below which a subject flees before the endgame (see Appendix D).

Risk aversion is measured as the number of rejected lotteries. Dominance and physical strength are normalized to mean zero and a standard deviation of 1. Standard errors

(clustered at the matching group level) in parentheses. * $p < 0.10$, ** $p < 0.05$, ***

$p < 0.01$

Table A.3: Fighting before endgame: based on estimated cutoff strategies

	(1) $k = -6$	(2) $k = 6$	(3) $k = 12$	(4) All k
Risk aversion	19.80 (15.25)	1.71 (31.26)	-6.16 (28.92)	7.04 (13.78)
Female	5.33 (38.33)	-171.23 (116.98)	-129.28 (72.43)	-92.85* (49.28)
Dominance	0.06 (13.74)	18.68 (27.39)	-44.17 (26.41)	-17.64 (14.96)
Physical strength	12.95 (28.69)	-35.97 (51.76)	-34.43 (48.05)	-15.66 (26.51)
$k = 6$				320.95*** (52.20)
$k = 12$				342.10*** (49.86)
Constant	394.30*** (36.68)	835.61*** (94.89)	853.71*** (70.72)	461.10*** (35.82)
Observations	63	52	63	178
R^2	0.05	0.06	0.08	0.34

Notes: Linear regressions. Dependent variable is the estimated cutoff c_3 , indicating the estimated strength above which a subject fights before the endgame (see Appendix D).

Risk aversion is measured as the number of rejected lotteries. Dominance and physical strength are normalized to mean zero and a standard deviation of 1. Standard errors

(clustered at the matching group level) in parentheses. * $p < 0.10$, ** $p < 0.05$, ***

$p < 0.01$

Table A.4: Fleeing before endgame: including fewer or more controls

	(1) All k	(2) All k	(3) All k
Risk aversion	0.026*** (0.009)	0.024*** (0.008)	0.027*** (0.009)
Female	-0.008 (0.030)		-0.010 (0.035)
Dominance	-0.006 (0.010)		-0.007 (0.010)
Physical strength	-0.002 (0.011)		-0.003 (0.011)
Prestige			0.010 (0.011)
Masculinity			0.002 (0.013)
Fighting ability	-0.935*** (0.013)	-0.931*** (0.013)	-0.935*** (0.013)
Round	-0.000 (0.000)	-0.000 (0.000)	-0.000 (0.000)
$k = 6$	-0.127*** (0.030)	-0.141*** (0.029)	-0.122*** (0.030)
$k = 12$	-0.112*** (0.022)	-0.111*** (0.021)	-0.111*** (0.021)
Observations	7120	7360	7040

Notes: Panel data probit regressions, with random effects at the subject level. Reported are average marginal effects. Dependent variable is a dummy indicating whether the player decided to flee before the endgame or not. Risk aversion is measured as the number of rejected lotteries. Dominance, physical strength, prestige and masculinity are normalized to mean zero and a standard deviation of 1. Fighting ability takes on values between 0 and 1. Standard errors (clustered at the matching group level) in parentheses.

For reference, column (1) is identical to column (4) in Table 2. * $p < 0.10$, ** $p < 0.05$,

*** $p < 0.01$

Table A.5: Fighting in the first second: including fewer or more controls

	(1) All $k > 0$	(2) All $k > 0$	(3) All $k > 0$
Risk aversion	-0.018* (0.011)		-0.017 (0.011)
Female	0.097** (0.048)		0.109* (0.058)
Dominance	0.006 (0.011)	0.004 (0.009)	0.004 (0.011)
Physical strength	0.022 (0.025)		0.019 (0.024)
Prestige			-0.004 (0.011)
Masculinity			0.012 (0.021)
Fighting ability	0.242*** (0.048)	0.222*** (0.051)	0.245*** (0.050)
Round	-0.002*** (0.001)	-0.002*** (0.001)	-0.002*** (0.001)
$k = 12$	-0.026 (0.036)	-0.026 (0.031)	-0.022 (0.037)
Observations	2300	2320	2280

Notes: Panel data probit regressions, with random effects at the subject level. Reported are average marginal effects. Dependent variable is a dummy indicating whether the player decided to fight in the first second or not. Risk aversion is measured as the number of rejected lotteries. Dominance, physical strength, prestige and masculinity are normalized to mean zero and a standard deviation of 1. Fighting ability takes on values between 0 and 1. Standard errors (clustered at the matching group level) in parentheses. For reference, column (1) is identical to column (6) in Table 3. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A.6: Treatment differences in battles and sorting (Experiment 2)

	(1) Battle occurs	(2) Weaker escapes
$k = 6$	-0.046 (0.036)	0.025 (0.027)
$k = 12$	-0.081** (0.037)	0.055* (0.028)
Dynamic	-0.001 (0.037)	0.026 (0.028)
$k = 6 \times \text{Dynamic}$	-0.186*** (0.052)	0.180*** (0.040)
$k = 12 \times \text{Dynamic}$	-0.135** (0.053)	0.113*** (0.040)
Constant	0.777*** (0.026)	0.123*** (0.020)
Observations	43	43
R^2	0.681	0.766

Notes: OLS regressions. Unit of observation is a matching group. Dependent variable in column (1) is the fraction of battles and in column (2) the fraction of matches where the weaker player in a pair managed to escape. Based on data from the static treatments (Experiment 1) and the dynamic treatments of Experiment 2. All independent variables are dummies. Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A.7: Fighting in the first period (Experiment 2)

	All rounds			Final 20 rounds		
	(1) $k = 6$	(2) $k = 12$	(3) all $k > 0$	(4) $k = 6$	(5) $k = 12$	(6) all $k > 0$
Risk aversion	0.040*** (0.013)	0.015 (0.016)	0.022* (0.013)	0.030** (0.015)	0.005 (0.012)	0.012 (0.013)
Female	-0.083*** (0.031)	0.063 (0.052)	-0.007 (0.035)	-0.048* (0.026)	0.118** (0.055)	0.032 (0.032)
Dominance	0.041*** (0.013)	-0.017 (0.024)	0.015 (0.017)	0.041*** (0.009)	0.012 (0.020)	0.027** (0.012)
Joy of winning (incentivized)	0.011 (0.008)	0.028 (0.022)	0.015 (0.014)	0.018* (0.010)	0.022 (0.018)	0.013 (0.010)
Joy of winning (self-report)	0.009 (0.011)	0.030 (0.027)	0.013 (0.014)	-0.001 (0.005)	0.024 (0.020)	0.007 (0.009)
Mistakes instr.	0.018*** (0.004)	0.009 (0.010)	0.014** (0.007)	0.010 (0.007)	0.011 (0.008)	0.012** (0.006)
Fighting ability	0.428*** (0.087)	0.527*** (0.073)	0.478*** (0.061)	0.237*** (0.071)	0.363*** (0.089)	0.297*** (0.059)
Round	-0.007*** (0.001)	-0.006*** (0.001)	-0.007*** (0.001)	-0.000 (0.002)	-0.003 (0.002)	-0.002 (0.001)
$k = 12$			0.037 (0.051)			0.065 (0.047)
Observations	2160	2120	4280	1080	1060	2140

Notes: Panel data probit regressions, with random effects at the subject level. Reported are average marginal effects. Dependent variable is a dummy indicating whether the player decided to fight in the first period or not. Risk aversion is measured as the number of rejected lotteries. Dominance and self-reported joy of winning are normalized to mean zero and a standard deviation of 1. Joy of winning is measured as the amount of points invested in a contest for a prize of 0 points. Fighting ability takes on values between 0 and 1. Standard errors (clustered at the matching group level) in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A.8: Fleeing before endgame (Experiment 2)

	(1) $k = -6$	(2) $k = 6$	(3) $k = 12$	(4) All k
Risk aversion	0.008 (0.010)	0.023 (0.016)	0.004 (0.014)	0.007 (0.009)
Female	0.034 (0.073)	-0.030 (0.044)	0.117** (0.055)	0.053 (0.038)
Dominance	-0.024 (0.018)	0.018 (0.024)	0.011 (0.016)	0.002 (0.012)
Joy of winning	-0.003 (0.013)	0.023 (0.017)	-0.034*** (0.008)	-0.010 (0.009)
Joy of winning (self-report)	0.010 (0.021)	0.017** (0.008)	0.000 (0.012)	0.009 (0.010)
Mistakes instr.	0.013 (0.010)	-0.013 (0.009)	-0.009 (0.008)	-0.001 (0.006)
Fighting ability	-0.919*** (0.027)	-0.817*** (0.038)	-0.817*** (0.036)	-0.850*** (0.021)
Round	-0.002 (0.001)	-0.002 (0.001)	-0.003*** (0.001)	-0.002*** (0.001)
$k = 6$				-0.028 (0.034)
$k = 12$				-0.011 (0.029)
Observations	1885	2160	2120	6165

Notes: Panel data probit regressions, with random effects at the subject level. Reported are average marginal effects. Dependent variable is a dummy indicating whether the player decided to flee before the endgame or not. Risk aversion is measured as the number of rejected lotteries. Dominance and self-reported joy of winning are normalized to mean zero and a standard deviation of 1. Joy of winning is measured as the amount of points invested in a contest for a prize of 0 points. Fighting ability takes on values between 0 and 1. Standard errors (clustered at the matching group level) in parentheses.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A.9: Fighting early in the game (Experiments 1 and 2 combined)

	(1) Exp. 1	(2) Exp. 2	(3) Both exp.
Risk aversion	-0.016* (0.009)	0.005 (0.013)	-0.007 (0.008)
Female	0.064** (0.026)	0.036 (0.030)	0.053*** (0.019)
Dominance	0.007 (0.011)	0.032*** (0.011)	0.018** (0.008)
Fighting ability	0.241*** (0.047)	0.291*** (0.058)	0.262*** (0.038)
Round	-0.002*** (0.001)	-0.002 (0.001)	-0.002*** (0.001)
$k = 12$	-0.023 (0.035)	0.072* (0.044)	0.017 (0.028)
Exp. 2			0.002 (0.028)
Observations	2300	2140	4440

Notes: Panel data probit regressions, with random effects at the subject level. Reported are average marginal effects. Based on the final 20 rounds of the dynamic games where $k > 0$. Dependent variable is a dummy indicating whether the player decided to fight in the first second (Experiment 1) or first period (Experiment 2). Risk aversion is measured as the number of rejected lotteries. Dominance is normalized to mean zero and a standard deviation of 1. Fighting ability takes on values between 0 and 1. Standard errors (clustered at the matching group level) in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A.10: Fleeing before endgame (Experiments 1 and 2 combined)

	(1) Exp. 1	(2) Exp. 2	(3) Both exp.
Risk aversion	0.025*** (0.008)	0.008 (0.009)	0.017*** (0.006)
Female	-0.004 (0.018)	0.047 (0.036)	0.014 (0.019)
Dominance	-0.006 (0.009)	0.000 (0.012)	-0.005 (0.008)
Fighting ability	-0.935*** (0.013)	-0.849*** (0.020)	-0.897*** (0.012)
Round	-0.000 (0.000)	-0.002*** (0.001)	-0.001*** (0.000)
$k = 6$	-0.127*** (0.030)	-0.026 (0.033)	-0.083*** (0.023)
$k = 12$	-0.113*** (0.021)	-0.017 (0.028)	-0.072*** (0.019)
Exp. 2			-0.060*** (0.019)
Observations	7120	6165	13285

Notes: Panel data probit regressions, with random effects at the subject level. Reported are average marginal effects. Dependent variable is a dummy indicating whether the player decided to flee before the endgame or not. Risk aversion is measured as the number of rejected lotteries. Dominance is normalized to mean zero and a standard deviation of 1. Fighting ability takes on values between 0 and 1. Standard errors (clustered at the matching group level) in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

D Individual cutoffs

In this appendix, we estimate cutoff strategies at the individual level. We use a grid search (with intervals of 2 ‘fighting ability points’) to find a combination of cutoffs that maximizes the number of accurately classified observed actions. In the exercise, we assume the following cutoff strategies. For each individual, we estimate three cutoffs c_1 , c_2 and c_3 , where $0 \leq c_1 \leq c_2 \leq c_3 \leq 1000$. Figure A.4 shows the assumed cutoff strategies. We assume that individuals flee before the endgame if they draw a fighting ability $a_i < c_1$, they wait until the endgame and then flee if $c_1 \leq a_i \leq c_2$, they wait until the endgame and then fight if $c_2 < a_i \leq c_3$ and fight before the endgame if $c_3 < a_i$. These assumptions nest the risk-neutral equilibrium cutoff strategies. Moreover, the assumed cutoff strategies are in line with how subjects (on average) base their actions on their fighting ability (see Figure 6).

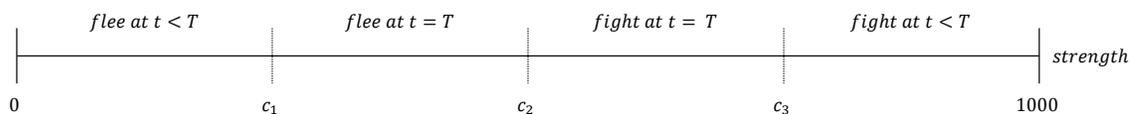


Figure A.4: Assumed cutoff strategies for the empirical model.

Figure A.5 shows the estimated cutoffs c_1 and c_3 . Because of the learning effects we observe, we base the estimates on the final 20 rounds of the experiment. Note that below c_1 subjects should flee before the endgame, between c_1 and c_3 they wait until the endgame, and above c_3 they fight before the endgame. This means that those in the top-left corner $(c_1, c_3) = (0, 1000)$ always wait until the endgame, those in the top-right corner $(c_1, c_3) = (1000, 1000)$ always flee before the endgame and those in the bottom-left corner $(c_1, c_3) = (0, 0)$ always fight before the endgame. Those on the 45 degree line $c_1 = c_3$ never wait until the endgame.

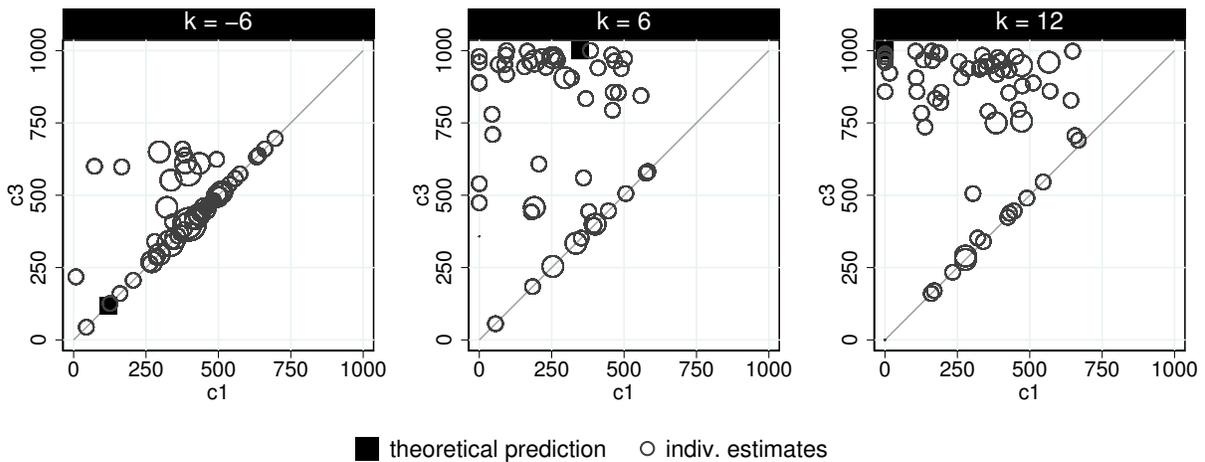


Figure A.5: Estimated individual cutoffs (Experiment 1), based on the final 20 rounds. Open circles represent individual estimates, the solid (orange) circle represents the theoretical prediction assuming risk neutrality.

The left panel of Figure A.5 shows the estimates for $k = -6$. In this case, rushing is predicted and under risk neutrality $c_1 = c_3 \approx 119$. Qualitatively, the results are in line with this prediction. For most subjects, we estimate a cutoff strategy with $c_1 \approx c_3$ as most circles lie on the 45 degree line or very close to it. In contrast to the risk neutral prediction, most estimated cutoffs lie somewhat higher on the 45 degree line than predicted, meaning that subjects flee more often. Of course, this is in line with subjects being risk averse.

The middle and right panel of Figure A.5 show the estimates for positive deterrence values. In these cases, fighting before the endgame is weakly dominated and $c_3 = 1000$ in equilibrium. In line with these predictions, we see that most estimates lie close to $c_3 = 1000$. There is heterogeneity though, and we observe a substantial number of estimates that remote from the theoretical prediction. For some subjects, the estimates lie on the 45 degree line, indicating that those subjects never wait

Table A.11: Estimated cutoffs (Experiment 1)

Treatment	Mean cutoff			Correctly classified	$c_1 = c_3$
	c_1	c_2	c_3		
Dynamic $k = -6$	393	393	444	0.96	0.78
Dynamic $k = 6$	256	359	750	0.88	0.21
Dynamic $k = 12$	304	430	780	0.88	0.16

until the endgame. For 21 percent of the subjects in $k = 6$ we estimate $c_1 = c_3$ while this is 16 percent for $k = 12$.

Table A.11 summarizes the estimation results. Besides the estimated average individual cutoffs, the table also lists what fraction of observed actions are correctly classified by the estimated cutoff strategies. The cutoff strategies capture observed behavior very well: between 88 and 96 percent of all actions are correctly classified.

E Instructions

The experimental instructions are reproduced below. All treatment dependent text is given in italics, preceded by the relevant treatment variable(s) between braces. In the quiz questions, all numbers (strengths, seconds) were generated randomly for each subject.²³

E.1 Experiment 1

Welcome!

Welcome to this experiment on decision-making. Please read the following instructions carefully. You will also receive a handout with a summary. If you have any questions at any time, please raise your hand. An experimenter will assist you privately.

Today's experiment consists of 2 parts. At the beginning of each part, you will receive new instructions. You will spend most time on the first part. Your decisions in one part have no influence on the proceedings or earnings of the other part.

Your decisions and those of other participants will determine your earnings. Your earnings will be paid to you privately at the end of today's session. All your earnings will be denoted in points.

At the end of the experiment, each point that you earned will be exchanged for 70 eurocents.

You will be given a starting capital of 21 points. Any profits or losses you make today will be added to or subtracted from this starting capital.

Part 1: Decisions and Payoffs

This part consists of 40 rounds. In each round you will be randomly paired with another par-

²³Icons used in the instructions are made by Freepik from www.flaticon.com.

ticipant in the laboratory. Therefore, in each round you will (most likely) be paired with a different participant than in the previous round. You will never learn with whom you are paired. At the end of the experiment, one of the rounds of Part 1 will be randomly selected for payment. Your earnings for Part 1 will be completely determined by what happened in this round.

Description of the situation and possible earnings

In each round, there is a prize of 10 points to get for one of you. If a Fight occurs between you and the other participant, the strongest participant will earn the prize of 10 points and the weaker participant will lose 10 points.

{k = -6} Each of you can also try to avoid a fight by attempting to flight. If a participant manages to flight, there is No Fight, and the participant who flights guarantees him- or herself 0 points (instead of winning or losing 10 points depending on his or her strength). The participant that did not flight then automatically receives the prize of 10 points minus a cost of 6 points because he or she let the other get away (thus earning 4 points in total).

{k = 6} Each of you can also try to avoid a fight by attempting to flight. If a participant manages to flight, there is No Fight, and the participant who flights guarantees him- or herself 0 points (instead of winning or losing 10 points depending on his or her strength). The participant that did not flight then automatically receives the prize of 10 points plus an additional 6 points because he or she did not have to fight for the prize (thus earning 16 in total).

{k = 12} Each of you can also try to avoid a fight by attempting to flight. If a participant manages to flight, there is No Fight, and the participant who flights guarantees him- or herself 0 points (instead of winning or losing 10 points depending on his or her strength). The participant

that did not fight then automatically receives the prize of 10 points plus an additional 12 points because he or she did not have to fight for the prize (thus earning 22 in total).

{dynamic, $k = -6$ } If both of you flight at the same time, it will be randomly determined who wins the prize at a cost and who flights. So one of you will earn 4 points and the other will earn 0 points, and each possibility is equally likely.

{dynamic, $k = 6$ } If both of you flight at the same time, it will be randomly determined who wins the prize together with the bonus and who flights. So one of you will earn 16 points and the other will earn 0 points, and each possibility is equally likely.

{dynamic, $k = 12$ } If both of you flight at the same time, it will be randomly determined who wins the prize together with the bonus and who flights. So one of you will earn 22 points and the other will earn 0 points, and each possibility is equally likely.

{static, $k = -6$ } If both of you attempt to flight, it will be randomly determined who wins the prize at a cost and who flights. So one of you will earn 4 points and the other will earn 0 points, and each possibility is equally likely.

{static, $k = 6$ } If both of you attempt to flight, it will be randomly determined who wins the prize together with the bonus and who flights. So one of you will earn 16 points and the other will earn 0 points, and each possibility is equally likely.

{static, $k = 12$ } If both of you attempt to flight, it will be randomly determined who wins the prize together with the bonus and who flights. So one of you will earn 22 points and the other will earn 0 points, and each possibility is equally likely.

When will a fight occur?

{dynamic} *In each round, there will be a clock that counts down from 10 seconds to 0. At any point during this countdown, you and the other participant have the option to choose fight or flight. You have also the option to wait and thereby postpone your decision. The computer checks every fifth of a second if a decision has been made by one or both of you.*

{static} *In each round, you and the other participant have the option to choose fight or flight. You will make this decision simultaneously with the other participant, without knowing what the other participant chooses.*

{dynamic} *If both of you decided to fight, or one of you decided to fight while the other is still waiting to make a decision, a Fight occurs.*

{static} *If both of you decided to fight, a Fight occurs.*

{dynamic} *If both of you decided to flight, or one of you decided to flight while the other is still waiting to make a decision, there is No Fight.*

{static} *If both of you decided to flight, there is No Fight.*

{dynamic} *If one of you decided to flight at the same time that the other decided to fight, a fight occurs 90% of the time. In the other 10% of the time, the person deciding to flight manages to avoid a fight. So on average, the person that attempts to flight will get away 1 out of 10 times that you end up in such a scenario.*

{static} *If one of you decided to flight and the other decided to fight, a fight occurs 90% of the time. In the other 10% of the time, the person deciding to flight manages to avoid a fight. So on average, the person that attempts to flight will get away 1 out of 10 times that you end up in such a*

scenario.

The possible scenarios are illustrated in the figure below.

{dynamic}



{static}



{dynamic} **What will happen if both of you waited until 10 seconds have passed?**

{dynamic} *It is possible that after 10 seconds none of you has made a decision to fight or flight. In that case, you are forced to make a decision to fight or flight. You will make this decision simultaneously with the other participant, without knowing what the other participant chooses. Your decision together with the decision of the other participant will then determine whether or not a Fight occurs, according to the same rules as above.*

Part 1: Strength and Information

Strength

At the start of each round, each participant will be informed of her or his strength in that round.

- A participant's strength will be a random number between 0 and 1000 (0 and 1000 are also possible). Each of these numbers is equally likely.
- In each round, every participant is assigned a new (and independent) strength. Therefore, the different participants (most likely) have different strengths in a round, and the same participant (most likely) has different strengths across rounds.
- At the start of a round, each participant is only informed about her or his own strength.
- It is very unlikely that both players have the same strength, but if this happens it will be randomly determined who is the stronger player.

Information at the end of a round

At the end of a round, each participant will be informed of the outcome, the other participant's strength and the resulting payoffs.

On the next screen you will be asked to answer some control questions. Please answer these questions now.

Decision screen

Below you can test how the decision screen works. You can do this by clicking on "show example" below. You can do this as many times as you like by clicking on "show example" again.

If you understand the screen, click on "go to practice questions" to continue.

Practice questions

Please answer the following questions:

In each round, you are matched with:

The same participant

A randomly determined participant

In each round, your strength is:

The same

Randomly determined

The following decisions are imaginary and do not indicate what you should do in the experiment. The numbers are randomly drawn.

Consider a round in which your strength is 889 and the other has a strength of 181.

{dynamic} *You choose Fight after 7 seconds, before the other makes a decision.*

{static} *You choose Fight, the other chooses Flight. The other does not manage to get away, so a FIGHT occurs.*

If this round is selected for payment:

How much would you earn? ___ points

How much would the other earn? ___ points

Consider a round in which your strength is 889 and the other has a strength of 181.

{dynamic} *The other chooses Flight after 7 seconds, before you make a decision.*

{static} *You choose Fight, the other chooses Flight. The other manages to get away, so there is NO FIGHT.*

If this round is selected for payment:

How much would you earn? ___ points

How much would the other earn? ___ points

Consider a round in which your strength is 912 and the other has a strength of 130.

{dynamic} *Both of you don't make a decision within 10 seconds. You have to make a decision simultaneously. You choose Flight, the other chooses Fight. You do not manage to get away, so a FIGHT occurs.*

{static} *You choose Flight, the other chooses Fight. You do not manage to get away, so a FIGHT occurs.*

If this round is selected for payment:

How much would you earn? ___ points

How much would the other earn? ___ points

Consider a round in which your strength is 912 and the other has a strength of 130.

{dynamic} *Both of you don't make a decision within 10 seconds. You have to make a decision simultaneously. Both of you choose Flight, so there is NO FIGHT. It is randomly determined that you are the one who wins the prize without a Fight.*

{static} *Both of you choose Flight, so there is NO FIGHT. It is randomly determined that you are the one who wins the prize without a Fight.*

If this round is selected for payment:

How much would you earn? ___ points

How much would the other earn? ___ points

{dynamic} *Both of you don't make a decision within 10 seconds. You have to make a decision simultaneously. You choose Flight, the other chooses Fight. What is the chance that you can get away?*

{static} *You choose Flight, the other chooses Fight. What is the chance that you can get away?*

___ %

End of instructions

You have reached the end of the instructions. You can still go back by using the menu above.

If you are ready, click on 'continue' below. If you need help, please raise your hand.

E.2 Experiment 2

Welcome

Welcome to this experiment on decision-making. Please read the following instructions carefully. If you have any questions, or if you experience technical difficulties during the experiment, contact the experimenter via Zoom or email.

Today's experiment consists of 2 parts. At the beginning of each part, you will receive new instructions. You will spend most time on the first part. Your decisions in one part have no influence

on the proceedings or earnings of the other part.

Earnings

Your decisions and those of other participants will determine your earnings. Your earnings will be paid to you via bank transfer shortly after today's session. You will only receive your payment if you finish all parts of the experiment. Please enter your IBAN below:

IBAN:

IBAN:

Please enter your IBAN twice to make sure that you entered it correctly.

All your earnings will be denoted in points.

At the end of the experiment, each point that you earned will be exchanged for 70 eurocents.

You will be given a starting capital of 21 points. Any profits or losses you make today will be added to or subtracted from this starting capital.

Part 1

This part consists of 40 rounds. In each round you will be randomly paired with another participant. Therefore, in each round you will (most likely) be paired with a different participant than in the previous round. You will never learn with whom you are paired. At the end of the experiment, one of the rounds of Part 1 will be randomly selected for payment. Your earnings for Part 1 will be completely determined by what happened in this round.

Description of the situation and possible earnings

In each round, there is a prize of 10 points to get for one of you. If a fight occurs between you

and the other participant, the strongest participant will earn the prize of 10 points and the weaker participant will lose 10 points.

$\{k = -6\}$ Each of you can also try to avoid a fight by choosing *FLIGHT*. If a participant manages to flee, there is no fight, and the participant who chose *FLIGHT* guarantees him- or herself 0 points (instead of winning or losing 10 points depending on his or her strength). The participant that did not choose *FLIGHT* then automatically receives the prize of 10 points minus a cost of 6 points because he or she let the other get away (thus earning 4 points in total).

$\{k = 6\}$ Each of you can also try to avoid a fight by choosing *FLIGHT*. If a participant manages to flee, there is no fight, and the participant who chose *FLIGHT* guarantees him- or herself 0 points (instead of winning or losing 10 points depending on his or her strength). The participant that did not choose *FLIGHT* then automatically receives the prize of 10 points plus an additional 6 points because he or she did not have to fight for the prize (thus earning 16 points in total).

$\{k = 12\}$ Each of you can also try to avoid a fight by choosing *FLIGHT*. If a participant manages to flee, there is no fight, and the participant who chose *FLIGHT* guarantees him- or herself 0 points (instead of winning or losing 10 points depending on his or her strength). The participant that did not choose *FLIGHT* then automatically receives the prize of 10 points plus an additional 12 points because he or she did not have to fight for the prize (thus earning 22 points in total).

$\{k = -6\}$ If both of you choose *FLIGHT* at the same time, it will be randomly determined who wins the prize at a cost and who flees. So one of you will earn 4 points and the other will earn 0 points, and each possibility is equally likely.

$\{k = 6\}$ If both of you choose *FLIGHT* at the same time, it will be randomly determined who

wins the prize together with the bonus and who flees. So one of you will earn 16 points and the other will earn 0 points, and each possibility is equally likely.

{k = 12} If both of you choose FLIGHT at the same time, it will be randomly determined who wins the prize together with the bonus and who flees. So one of you will earn 22 points and the other will earn 0 points, and each possibility is equally likely.

When will a fight occur?

Each round consists of 4 time periods. In each time period, you have 5 seconds to make a decision. You and the other participant have the option to choose FIGHT or FLIGHT. Except for the final time period, you also have the option to wait and thereby postpone your decision to a later time period.

If both of you choose FIGHT in the same time period, or one of you chooses FIGHT in a time period while the other decided to wait in that time period, a fight occurs.

If both of you choose FLIGHT in the same time period, or one of you chooses FLIGHT in a time period while the other decided to wait in that time period, there is no fight.

If one of you chooses FLIGHT in the same time period that the other chooses FIGHT, a fight occurs 90% of the time. In the other 10% of the time, the person who chose FLIGHT manages to avoid a fight. So on average, the person that chose FLIGHT will get away 1 out of 10 times that you end up in such a scenario.

The possible scenarios are illustrated in the figure below.

Strength

At the start of each round, each participant will be informed of her or his strength in that round.



- A participant's strength will be a random number between 0 and 1000 (0 and 1000 are also possible). Each of these numbers is equally likely.
- In each round, every participant is assigned a new (and independent) strength. Therefore, the different participants (most likely) have different strengths in a round, and the same participant (most likely) has different strengths across rounds.
- At the start of a round, each participant is only informed about her or his own strength.
- It is very unlikely that both players have the same strength, but if this happens it will be randomly determined who is the stronger player.

At the end of a round, each participant will be informed of the outcome, the other participant's strength and the resulting payoffs.

Decision screen

Below you can test how the decision screen works. You can do this by clicking on "show example" below. You can do this as many times as you like by clicking on "show example" again. If you understand the screen, click on "go to practice questions" to continue.

Question 1

In each round, you are matched with:

the same participant

a randomly determined participant

Question 2

In each round, your strength is:

the same

randomly determined

Question 3

The following decisions are imaginary and do not indicate what you should do in the experiment.

The numbers are randomly drawn.

Consider a round in which your strength is 925 and the other has a strength of 4. You choose FIGHT in time period 2, while the other was still waiting.

If this round is selected for payment:

How much would you earn? ___ points

How much would the other earn? ___ points

Note: enter the earnings without the starting capital of 21 points. You can enter losing an amount by using a minus sign. For example, losing 10 points should be entered as -10.

Question 4

The following decisions are imaginary and do not indicate what you should do in the experiment.

The numbers are randomly drawn.

Consider a round in which your strength is 925 and the other has a strength of 4. The other chooses FLIGHT in time period 2, while you were still waiting.

If this round is selected for payment:

How much would you earn? ___ points

How much would the other earn? ___ points

Note: enter the earnings without the starting capital of 21 points. You can enter losing an amount by using a minus sign. For example, losing 10 points should be entered as -10.

Question 5

The following decisions are imaginary and do not indicate what you should do in the experiment. The numbers are randomly drawn.

Consider a round in which your strength is 14 and the other has a strength of 854. Both of you make a decision in time period 3. You choose FLIGHT, the other chooses FIGHT. You do not manage to get away, so a fight occurs.

If this round is selected for payment:

How much would you earn? ___ points

How much would the other earn? ___ points

Note: enter the earnings without the starting capital of 21 points. You can enter losing an amount by using a minus sign. For example, losing 10 points should be entered as -10.

Question 6

The following decisions are imaginary and do not indicate what you should do in the experiment. The numbers are randomly drawn.

Consider a round in which your strength is 14 and the other has a strength of 854. Both of you make a decision in time period 4. Both of you choose FLIGHT, so there is no fight. It is randomly determined that you are the one who wins the prize without a fight.

If this round is selected for payment:

How much would you earn? ___ points

How much would the other earn? ___ points

Note: enter the earnings without the starting capital of 21 points. You can enter losing an amount by using a minus sign. For example, losing 10 points should be entered as -10.

E.3 Lottery task

In this task, you are presented 6 lotteries, and for each lottery you decide whether to accept or reject it. In each lottery the winning amount is fixed at 6 points. The losing amount varies between lotteries. At the end of the experiment, one of the 6 choices is selected at random, and your choice for this lottery together with a coin flip of a fictitious coin will determine your outcome.

If you rejected the selected lottery, your outcome will certainly be 0 points.

If you accepted the selected lottery, the computer will flip the coin, and the outcome will be heads with 50% probability and tails with 50% probability. If the outcome is heads, you lose the stated amount of the lottery. If the outcome is tails, you win 6 points.

Choice	Lottery	Accept	Reject
1	lose 2 points (if heads turns up) or win 6 points (if tails turns up)	<input type="radio"/>	<input type="radio"/>
2	lose 3 points (if heads turns up) or win 6 points (if tails turns up)	<input type="radio"/>	<input type="radio"/>
3	lose 4 points (if heads turns up) or win 6 points (if tails turns up)	<input type="radio"/>	<input type="radio"/>
4	lose 5 points (if heads turns up) or win 6 points (if tails turns up)	<input type="radio"/>	<input type="radio"/>
5	lose 6 points (if heads turns up) or win 6 points (if tails turns up)	<input type="radio"/>	<input type="radio"/>
6	lose 7 points (if heads turns up) or win 6 points (if tails turns up)	<input type="radio"/>	<input type="radio"/>

You have to choose for each of the lotteries whether you accept or reject it.

If you have any questions, please raise your hand and one of us will come to your desk.

E.4 Joy of winning measure

In this task you will be randomly and anonymously matched with another participant. You and the other participant will choose how much to bid in order to be a winner. The reward is worth 0 points to you and the other participant. You may bid any number of points between 0 and 4 (including increments of 0.1 points). After both participants have made their decisions, your earnings will be calculated as follows.

$$\text{Earnings} = 0 - \text{your bid}$$

After both participants make their bids, the computer will make a random draw based on these bids and determine who the winner is. At the end of the experiment, the computer will display the results of this task (that is, whether you won or not), and will calculate your earnings.

The more you bid, the more likely you are to be the winner. The more the other participant bids, the less likely you are to be the winner. Specifically, for each 0.1 points that you bid you will receive one lottery ticket. At the end of today's experiment, the computer will draw randomly one ticket among the tickets purchased by you and the other participant. The owner of the drawn ticket will be the winner. Thus, your chance of receiving the reward is given by the number of points you bid divided by the total number of points you and the other participant bid.

$$\text{Your chance of winning} = \frac{(\text{your bid})}{(\text{your bid} + \text{the other participant's bid})}$$

If both participants bid zero, the winner is randomly determined.

Please make your decision below.

Your bid: ____