PRICE FORMATION WITH CONFIRMATION BIAS*

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Abstract

This paper proposes a dynamic model of financial markets where some investors are prone to the confirmation bias. Following insights from the psychological literature, these agents amplify signals that are consistent with their prior views. In a model with public information, this assumption provides a unified explanation of a variety of empirically documented phenomena such as bubbles and crashes, momentum and reversals in asset returns, and excess volatility. Implications of our model for quantitative investments are derived: i) optimal trading strategies involve riding bubbles, and ii) both feedback and contrarian trading can be optimal depending on market circumstances. Those market circumstances are shown to depend on biased traders' beliefs (that can be estimated) as well as on the variance of public signals.

Keywords: financial markets, psychological biases, confirmation bias, momemtum, reversal, bubbles, trading strategies, quantitative investments

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Information processing is a major aspect of trading in financial markets. Investors need to learn about the fundamental characteristics of the

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assets that are traded. They also need to understand what other investors will think of these fundamentals in order to better estimate future prices. Economic theory traditionnally assumes that investors enjoy perfect information perception capabilities, that is, investors have an unlimited and unbiased cognition. In contrast, the psychological litterature on judgment and decision making suggests that some individuals are prone to cognitive biases and make systematic mistakes when processing information. [1]

We propose a theory of trading in financial markets that incorporates one of these information processing biases, namely the confirmation bias. A person prone to the confirmation bias tends to interpret ambiguous evidence as consistent with his or her prior views. We model this bias by assuming that a person suffering from the confirmation bias amplifies the signals that are consistent with his or her prior beliefs. In our framework, this bias creates differences of opinion between rational arbitrageurs and biased traders over the interpretation of public information. These differences of opinion in turn motivate trading.[2] In our model, transaction costs limit the use of arbitrage strategies. As a result, the views of both arbitrageurs and biased traders are factored in asset prices. [3]

Our results are twofold and relate to the properties of mispricings and to trading strategies. We show that the presence of traders prone to the confirmation bias creates bubbles and crashes, and induces short-term momentum (i.e. positive short-run return autocorrelation), long-term reversals (i.e. negative long-run return autocorrelation), and excess volatility. Bubbles occur when traders hold optimistic beliefs. [4] This is because optimism increases

[1] See, for example, Kahneman, Slovic, and Tversky (1982) and Gilovich, Griffin, and Kahneman (2002).

[2] This differences-of-opinion type of model is in line with the theoretical analysis of Harris and Raviv (1993).

[3] For simplicity, we base our model on on exogenous transaction costs such as commissions or market impact. Other factors than transaction costs can also limit the willingness to trade of arbitrageurs and would also lead to our conclusions. As analyzed by Wurgler and Zhuravskaya (2002), assets' fundamental risk diminishes the attractiveness of arbitrage when perfect substitutes do not exist. Noise trader risk coupled with short investment horizons as modeled by De Long, Shleifer, Summers, and Waldmann (1990) and by Shleifer and Vishny (1997) are other factors preventing arbitrageurs from fully exploiting unsophisticated investors' erroneous sentiment. Abreu and Brunnermeier (2002)'s synchronization risk is another type of risk borne by arbitrageurs.

[4] We use the terms optimism and pessimism to reflect the fact that traders hold positive

the likelihood that biased traders will in the future exaggerate positive signals. During a bubble, prices are above perfectly rational levels and, from a rational arbitrageur's perspective, the mispricing is expected to worsen in the future. The same logic applies for crashes as they relate to pessimism. Momentum and reversals are a consequence of the fact that beliefs have too much persistence, and of the fact that these beliefs are only corrected in the long run. Excess volatility derives from the fact that biased traders overreact to information when it is consistent with their prior views. These asset pricing patterns are in line with the results of various empirical studies. At weekly, monthly and quarterly horizons, Lo and MacKinley (1988), and Jegadeesh and Titman (1993) find a positive autocorrelation in stock returns. DeBondt and Thaler (1985) and Fama and French (1988) find that stock returns at three- to five-year horizons are negatively correlated. Starting with Leroy and Porter (1981) and Shiller (1981), several contributions report that asset prices are excessively volatile. The present paper offers a parsimonious model based on investors' misperception of public information that is consistent with these stylized facts.[5]

To analyze the implications of our model for quantitative investments, we derive the optimal trading strategy of a short-term rational trader referred to as a hedge fund. We show that a hedge fund has an incentive to "ride" bubbles despite its knowledge of an over-valuation. This is because it correctly anticipates that mispricings will worsen in the future. This is consistent with hedge funds' behavior during the technology bubble as described by Brunnermeier and Nagel (2004) or with Hoare's (a London-based bank) trading behavior during the 1720's South Sea bubble as reported by Temin and Voth (2003). We also show that a hedge fund should short-sell during crashes despite its knowledge of an under-valuation. We finally show

or negative beliefs, respectively, concerning the fundamental value of the asset. Our paper is therefore different from the analysis of Jouini and Napp (2004) among others, that consider optimism or pessimism as a personal and stable psychological trait that leads agent to systematically over- or under- estimate the probability of successes.

[5]The model is parcimonious in the sense that it includes the perfectly rational benchmark as a special case and that departures from perfect rationality are driven by only one parameter. This parameter is denoted by q and measures the severity of the confirmation bias. When q=0, the model corresponds to perfectly rational asset prices and trading strategies. When q¿0, the model corresponds to asset prices and trading strategies with confirmation bias.

that, while a hedge fund in our model uses a positive-feedback strategy more often than a contrarian strategy, both of these trading strategies can be optimal depending on market circumstances: When biased traders are bullish, a hedge fund should adopt a positive-feedback strategy after positive public announcements as well as after strongly negative announcements. The fund should buy after price increases and sell after large price drops.[6] On the contrary, after mildly negative public announcements, a hedge fund should adopt a contrarian strategy, that is, the fund should buy after moderate price drops. When biased traders are bearish, a hedge fund should adopt a positive feedback strategy after both negative and strongly positive public announcements. It should adopt a contrarian strategy after midly positive news. Overall, the two important inputs that drive optimal trading strategies are biased trader's beliefs and the variance of public information. We provide an explicit characterization of these inputs and show how to use them in structuring an investment portfolio.

Central to our model is the assumption that some traders are prone to the confirmation bias. This bias has been extensively documented in the psychological literature following the seminal contribution of Lord, Ross, and Lepper (1979).[7] Bodenhausen (1988) suggests that the confirmation bias is associated with biased assimilation: a person tends to evaluate more positively information consistent with his or her prior views. This bias can also lead to attitude polarization, that is, the same mixed information can lead people with opposite prior views to update their beliefs in opposite directions. Rabin and Schrag (1999) study the consequences of the confirmation bias for beliefs' formation. Evidence of such judgmental bias has been documented in an economic setting by Forsythe, Nelson, Neumann, and Wright (1992). They analyze data from the Iowa Political Stock Market organized during the 1988 U.S. presidential election campaign. They report that, after the third debate, supporters of one candidate were more likely to think that their preferred candidate won the debate. They also show that these be-

[6]Strong public announcements are defined as signals that are extreme enough to shift biased traders'beliefs from bullish to bearish or inversly from bearish to bullish. By opposition, an announcement is characterized as mild when it does not change the direction of biased traders' beliefs. These definitions will be more precisely laid out in Section III.

[7]Other contributions to the study of the confirmation bias include Darley and Gross (1983), Plous (1991), Dougherty, Turban, and Callender (1994), and Dave and Wolfe (2003). Additional references are offred by Rabin and Schrag (1999).

liefs influenced trading behavior. In the present paper, we propose a model that formally incorporates the confirmation bias and studies its influence on trading strategies and on the price formation process in financial markets.

Some related papers study price formation when investors' beliefs are erroneous. For example, Kyle and Wang (1997), Odean (1998), Scheinkman and Xiong (2003), Dumas, Kurshev, and Uppal (2005), Daniel, Hirshleifer, and Subrahmanyam (1998), and Gervais and Odean (2001) incorporate overconfidence and, for the last two papers, the self-serving attribution bias in models of financial markets. We complement this literature by examining the consequences of a different bias, by providing a unified explanation to various phenomena discussed in these papers, and by offering new testable predictions related to the momentum effect. In particular, our model predicts that momentum should be maximal when biased traders' beliefs are the most neutral, and decreases when these beliefs become more and more optimistic or pessimistic.

Next section describes our model. Our results are presented in Section II. Section III concludes. Proofs are in the Appendix.

1 The Model

We model a financial market as a pure exchange economy with one risky asset and a riskless asset whose rate of return is normalized to zero. There are T periods of trading indexed by $t \in \{0, 1, 2, ..., T\}$. For simplicity, we consider that consumption occurs only at period $T+1$ when the asset's cash-flow $\tilde{v}$ is distributed. We consider that $\tilde{v} = m + \sum_{t=1}^{t=T} s_t + \varepsilon$ where m is a constant that represents prior beliefs, s_t is a public signal announced before trading occurs at period t, and ε is a random noise realized at date $T+1$. Note that no signal reaches the market before trading at date 0.[8] The number of shares of the risky asset is normalized to one. There is a continuum of traders with mass normalized to one. A trader pertains to one of two groups of agents. Arbitragers (denoted by A) are in proportion $1-\lambda$, and biased traders (denoted by B) are in proportion λ. Traders are endowed with one unit of the risky asset and no cash.

[8]This assumption is made for simplicity and has no influence on the results.

In order to focus on the speculative aspects of financial markets, we assume that traders are risk neutral.[9] Absent market frictions, risk-neutrality implies that traders would stand ready to submit infinite demands as long as asset prices do not equal their expectation of the asset value. This would prevent the existence of an equilibrium since the market would not clear. In order to avoid this phenomena, we assume that traders incur an exogenous trading cost that is quadratic in the quantity traded and parameterized by $c > 0$.[10] The total cost of trading for trader j at date t with a demand d_t^j is thus equal to cd_t^{j2}, for all t. This cost can be viewed as an explicit transaction cost traders have to pay to submit orders or as a proxy for the imperfect depth of financial markets.[11] This transaction cost creates limits to arbitrage and opens the scope for potential mispricings.[12]

In our model, differences of opinion emerge because all traders do not encode the public signal in the same way. On the one hand, arbitrageurs are perfectly rational in the sense that they are endowed with the actual probability model $\mathbb{P}_A$. For simplicity, we assume that under $\mathbb{P}_A$, we have $(s_1, ..., s_T, \varepsilon) \overset{law}{=} \mathcal{N}\left(0, \left(\Sigma^2 \mathrm{Id}_{T+1}\right)\right)$, where Id_{T+1} is the identity matrix of dimension $T+1$. On the other hand, biased traders are endowed with a different probability model $\mathbb{P}_B$. Upon receiving public signals s_t, biased traders actually see σ_t, for every t. To incorporate the fact that biased traders believe that they correctly perceive signals s_1,s_2,...,s_T when they in fact observe σ_1,σ_2,...,σ_T, we consider that, under $\mathbb{P}_B$, we have $(\sigma_1, ..., \sigma_T, \varepsilon) \overset{law}{=} \mathcal{N}\left(0, \left(\Sigma^2 \mathrm{Id}_{T+1}\right)\right)$.[13]

To incorporate the confirmation bias in our framework, we assume that

[9] This is in the spirit of Harris and Raviv (1993). This isolates our analysis from the influence of trading motives based on risk sharing.

[10] Alternatively, we could ensure existence of an equilibrium by assuming that there is a fixed number of shares available (i.e., short-sales are not allowed) as in Harris and Raviv (1993), or that traders can only trade up to a fixed amount of shares as in Abreu and Brunnermeier (2002). These different modeling frameworks would not affect our results.

[11] Imperfect market depth could be related to inventory or adverse selection risks borne by liquidity providers. See Madhavan (2000) and Biais, Glosten, and Spatt (2005) for surveys of the market microstructure literature dealing with those issues.

[12] Alternative modeling frameworks generating limits to arbitrage include noise trader risk as in De Long, Shleifer, Summers, and Waldmann (1990), short horizons as in Shleifer and Vishny (1997), and synchronization risk as in Abreu and Brunnermeier (2002). Barberis and Thaler (2001) survey this literature.

[13] This implies that, before receiving the first public signal, the biased traders have a correct understanding of the statistical model underlying the financial market. As stated

biased traders amplify public information when it is consistent with their prior beliefs concerning the final cash-flow. Formally, we assume that , under the appropriate probability model $\mathbb{P}_A$, the information perceived by biased traders is:

$$\sigma_t = q s_t \mathbb{1}_{\mu_{t-1} s_t > 0} + s_t \mathbb{1}_{\mu_{t-1} s_t \leq 0}$$

where q measures the severity of the confirmation bias, $\mathbb{1}_{(.)}$ is the indicator function that takes the value 1 if the condition is satisfied and 0 otherwise, and $\mu_{t-1} = \mathbb{E}^B(\tilde{v}|\sigma_1, ..., \sigma_{t-1})$ represents biased traders' beliefs about the expected cash-flow given their information at period $t-1$. It is straightforward to show that $\mu_0 = \mathbb{E}^B(\tilde{v}) = m$ and $\mu_t = \mu_{t-1} + \sigma_t$, for all t. In our model, the departure from perfect rationality is parameterized by q. If q equals 1, we are in the perfectly rational case: biased traders are fully rational (i.e., they correctly perceive the signal s_t, for every t) and there are no differences of opinion. However, when $q > 1$, biased traders are prone to the confirmation bias in that they overreact to information when it is consistent with their prior view.

To complete the set up of our model, we consider that, under $\mathbb{P}_B$:

$$s_t = \frac{\sigma_t}{q} \mathbb{1}_{\mu_{t-1}\sigma_t > 0} + \sigma_t \mathbb{1}_{\mu_{t-1}\sigma_t \leq 0}$$

This equation reflects the fact that, despite their erroneous beliefs, biased traders have rational expectations: from their point of view, when an information is consistent with their prior beliefs, the arbitrageurs underreact to this information. Let us point out that the set of information generated by the observation of the signals s coincides with the set of information generated by the biased signals σ. In other words, there is no assymetry of information in our setting. For any randon variable $\tilde{x}$, we will set $E_t^A(\tilde{x}) = \mathbb{E}^A(\tilde{x}|s_1, \ldots, s_t)$.

In our model, the situation is thus the following: there are two types of traders who receive the same public signal. The two types of traders however do not perceive this information in the same way: biased traders amplify signals that are consistent with their prior views. All traders then use Bayes rule to update their beliefs. The two types of traders know that not all the

above, their bias derives from their improper perception of information. Except from that, biased traders maximize their expected utility, update their beliefs using Bayes rule, and have rational expectations.

market participants perceive the information in the same manner but both of them believe that their interpretation is the correct one.[14] This creates differences of opinion across traders offering them a rationale to trade. The next section characterizes price formation in this financial market.

2 Asset Pricing and Trading Strategies with Confirmation Bias

Our model features traders facing a quadratic transaction cost. As a result, asset demands are finite despite risk neutrality and there exists an equilibrium. Standard arguments (presented in the Appendix) show that prices in our financial market are weighted averages of arbitrageurs and biased traders' beliefs given their respective information set. They are given by the following equation for $t \in \{0, \ldots, T\}$

$$\begin{aligned} P_t &= (1-\lambda)\,\mathbb{E}_t^A(\tilde{v}) + \lambda \mathbb{E}_t^B(\tilde{v}) \\ &= m + (1-\lambda)\sum_{i=1}^{i=t} s_i + \lambda \sum_{i=1}^{i=t} \sigma_i. \end{aligned}$$

The confirmation bias affects biased traders' conditional beliefs, $\mathbb{E}_t^B(\tilde{v})$, and thus in turn it impacts prices. To analyze this impact, it is useful to derive asset prices in the benchmark case in which all traders are perfectly rational. Prices in this benchmark are indicated by a star. This benchmark is nested in our model and corresponds to the case where $\lambda = 0$ or $q = 1$. In this case, we have $P_t^* = \mathbb{E}_t^A(\tilde{v}) = m + \sum_{i=1}^{i=t} s_i$.Given the structure of the uncertainty in our model, it is straightforward to show that the following proposition holds.

Proposition 2.1 *When all traders are perfectly rational (that is, $\lambda = 0$ or $q = 1$), market prices are such that:*

[14] We could consider a model where traders assign a non-zero probability to the event that their probabilistic model is incorrect and engage in bayesian learning to appropriately choose the model that is the most plausible. However, since in our setting the asset pays off a cash flow only at the last period, and since agents including biased traders have rational expectations, none of the information that traders could observe (e.g. market prices) would affect the likelihood that their probabilistic model is the correct one.

$$Var^A\left(P_t^*\right) = t\Sigma^2, t \in \{0, \ldots, T\}$$

$$Cov^A\left(P_{t+k}^* - P_t^*, P_t^* - P_{t-k}^*\right) = 0, t \in \{1, \ldots, T\}$$

$$Cov^A\left(P_t^* - P_0^*, \tilde{v} - P_t^*\right) = 0.$$

When there are no biased traders, prices equal the present value of the asset cash-flow conditional on the available information. Public signals are the only factor influencing asset prices. Because we assume that public signals are independent and identically distributed, the variance of prices is just the sum of the public signals' variances, and there is no autocorrelation in returns.

We now study how asset prices are influenced by the fact that some traders tend to amplify public signals that are consistent with their prior views. In order to do so, it is necessary to understand how future prices are related to biaised traders current beliefs. This will be done by introducing an operator $\mathcal{M}$ acting on bounded function defined as:

$$\mathcal{M}f(\mu_t) = \mathbb{E}_t^A\left[f(\mu_{t+1})\right],$$

with

$$\begin{aligned}\mathcal{M}f(x) &= \left[\int_0^{\infty} f(x+qy)e^{-\frac{y^2}{2\Sigma^2}}\frac{dy}{\sqrt{2\pi\Sigma^2}} + \int_{-\infty}^{0} f(x+y)e^{-\frac{y^2}{2\Sigma^2}}\frac{dy}{\sqrt{2\pi\Sigma^2}}\right]\mathbb{1}_{x>0} \\ &+ \left[\int_0^{\infty} f(x+y)e^{-\frac{y^2}{2\Sigma^2}}\frac{dy}{\sqrt{2\pi\Sigma^2}} + \int_{-\infty}^{0} f(x+qy)e^{-\frac{y^2}{2\Sigma^2}}\frac{dy}{\sqrt{2\pi\Sigma^2}}\right]\mathbb{1}_{x<0}.\end{aligned}$$

Applied to the function $f_0(x) = \mathbb{1}_{\{x>0\}}$, the belief operator $\mathcal{M}f_0(x) = \mathbb{1}_{\{x>0\}}$ corresponds to a rational agent expectation of biased traders' future beliefs given the current beliefs of these biased traders.

According to bounded dominated convergence, the belief operator maps the bounded function in the set of continuous functions on the open set $]-\infty,0[\cup]0,+\infty[$. Moreover, it has several properties that will govern the behavior of equilibrium prices. These properties are summarized in the following lemma.

Lemma 2.1 *We have,*

If f is increasing then $\mathcal{M}(f)$ is an increasing function.

- *Let f be a bounded function satisfying $f(x) + f(-x) = 1$ for every positive real x. Then, we have for every positive real x*

$$\mathcal{M}f(x) + \mathcal{M}f(-x) = 1.$$

A nice consequence of the previous lemma is that $Mf(x) \geq \frac{1}{2}$ for $x > 0$. Applied to the function $f_0(x)$ defined earlier, this consequence indicates that, if biased traders are currently optimistic, it is more likely that they will also be optimistic in the future (and vice versa when they are pessimistic). This persistence in biased traders' beliefs is at the core of our results on the properties of prices in our model. We are now in a position to state our main result.

Proposition 2.2 *When some traders are prone to the confirmation bias (that is, $\lambda > 0$ and $q > 1$), market prices are such that:*

- *There is excess variance:*

$$Var^A(P_t) > Var^A(P_t^*), \forall t \in \{1, \ldots, T\}$$

- *There is a momentum effect, that is, for every $t \in \{1, \ldots, T\}$ and every $k \leq \max(t, T-t)$,*

$$Cov^A(P_{t+k} - P_t, P_t - P_{t-k}) \geq 0.$$

- *Conditionally on the biased traders' beliefs, the strenght of the momentum effect is inversely related to the strenght of biased traders' beliefs. Namely, for every $k \leq \max(t, T-t)$, the conditional covariance $cov^A_{t-k}(P_{t+k} - P_t, P_t - P_{t-k})$ is a decreasing function of $|\mu_{t-k}|$.*

- *There is a price reversal effect for every t,*

$$cov^A(P_t - P_0, \tilde{v} - P_t) \leq 0$$

Proposition 2.2 is based on the probabilistic model P_A because we take the point of view of an econometrician who is not subject to the confirmation bias. Proposition 2.2 states three fundamental properties of asset prices in our model: i) prices display excessive variance, ii) price changes exhibit momentum, and iii) price evolutions are eventually reversed when the cash-flow is distributed. First, excess variance is characterized by the fact that the variance of prices is greater when there are biased traders than when all traders are rational. This result is consistent with the empirical evidence documented by Shiller (1981), and Mankiw, Romer, and Shapiro (1985) among others. In our model, it derives from the fact that traders who are prone to the confirmation bias amplify some of the public signals. This translates into higher price volatility because of the price pressure they are exerting on the market. As a result, instead of being a function of public signals variance only as in the perfectly rational benchmark, price volatility is linked to other factors such as the strength of the confirmation bias, and the proportion of biased traders. Second, momentum in price changes appears in the fact that positive (negative) price changes are more likely to be followed by (negative) positive price changes. This is in line with the empirical analysis of Jegadeesh and Titman (1993). A comparative-static exercise shows that the momentum effect decreases with the strenght of biased traders' initial beliefs. This constitutes a novel empirical prediction that could be used to test our theory. Finally, price reversal is reflected by the fact that positive (negative) price changes between the first transaction and the current one tend to be followed by negative (positive) changes between the current transaction price and the final cash-flow.

None of the above results is present when all traders are perfectly rational. It is thus clear that our theoretical results are linked to the amplification of public information by biased traders when this information is consistent with their prior views. The present model thus accounts for a variety of empirically documented phenomena using a unique source of irrationality rooted in the psychological evidence concerning individuals' judgments under uncertainty.

In addition to influencing returns, the confirmation bias also affects the level of prices on the market. This is indicated in the following proposition.

Proposition 2.3 *When some traders are prone to the confirmation bias (that is, $\lambda \geq 0$ and $q > 1$), market prices are such that:*

- *The mispricing at date t is:*

$$P_t - P_t^* = \lambda(q-1)\sum_{i=1}^{i=t} s_i \mathbb{1}_{s_i\mu_{i-1}>0}.$$

- *When μ_t is positive, there is a bubble in the sense that for $0 \leq k \leq l$, we have*

$$\mathbb{E}_t^A\left(P_{t+l} - P_{t+l}^*\right) \geq \mathbb{E}_t^A\left(P_{t+k} - P_{t+k}^*\right) > 0$$

- *Alternatively, when μ_t is negative, there is a crash in the sense that for $0 \leq k \leq l$, we have*

$$\mathbb{E}_t^A\left(P_{t+l} - P_{t+l}^*\right) \geq \mathbb{E}_t^A\left(P_{t+k} - P_{t+k}^*\right) > 0$$

Proposition 2.3 shows that the presence of biased traders creates mispricings in the sense that expected prices do not equal the perfectly rational benchmark. This proposition is a reminiscence of the fact that, as underlined by Rabin and Schrag (1999), first impression matters for traders who are prone to the confirmation bias. When their initial beliefs make them optimistic, biased traders amplify good news thus pushing prices above fundamental levels (on the contrary, if their initial beliefs make them pessimistic, biased traders amplify bad news and push prices below fundamental levels). Furthermore, Proposition 2.3 shows that mispricings are expected to worsen instead of being corrected. We interpret this phenomenon as a price bubble or crash depending on the direction of prior beliefs.

Our results suggest that the confirmation bias influence asset pricing in a specific way. We now investigate how a short-term investment strategy can be set up to profit from biased traders' irrationality. To do so, we introduce in the model an additional risk-neutral trader, referred to as a hedge fund, with a negligible mass and a one-period horizon. The negligible mass means that the hedge fund trading behavior does not affect prices. As a result, the pricing results derived above are still valid. The short horizon implies that, at each date, the hedge fund's objective is to maximize next period return.[15]

[15] All of the following results hold as long as the horizon of the hedge fund does not include the date at which the asset cash-flow is distributed.

The question that we aim to address here is to know whether the fund uses a contrarian or a positive-feedback strategy.

Proposition 2.4 below shows how to set up the optimal trading strategy, that is, the strategy that exploits mispricings when they arise.

Proposition 2.4 - *The demand of a short-term trader is:*

$$d_t = \frac{\lambda(q-1)\Sigma}{c\sqrt{2\pi}} \left[2\mathcal{N}\left(\frac{\mu_t}{\Sigma}\right) - 1 \right].$$

- When biased traders are bullish, a short-term trader engages in feedback trading when the public signal is positive or negative enough to alter biased traders' optimism;

- When biased traders are bullish, a short-term trader engages in contrarian trading when the public signal is negative but not enough to alter biased traders' optimism;

- When biased traders are bearish, a short-term trader engages in feedback trading when the public signal is negative or positive enough to alter biased traders' pessimism;

- When biased traders are bearish, a short-term trader engages in contrarian trading when the public signal is positive but not enough to alter biased traders' pessimism;

- A short-term trader adopts a feedback trading strategy more often than a contrarian strategy.

Proposition 2.4 highlights the fact that one of the crucial dimension of short-term strategies is to track the evolution of biased traders' beliefs. Before the final trading date, if the biased traders are bullish, that is $\mu_t > 0$, the hedge fund establishes a long (short) position because it rationally anticipates that the bullish (bearish) beliefs is likely to persist in the future, that is, the hedge fund rides the bubble or the crash. Such a behavior is in sharp contrast with long-term traders' behavior in our model. These traders indeed sell when biased traders are bullish, and buy when they are bearish. This is because they hold their inventories until the asset cash-flow is distributed. At the final trading date, the hedge fund trades against the mispricing. In addition to this, it is easy to show from Proposition 2.4 that the trading activity and the profit of the hedge fund increase with the intensity of the confirmation bias, and with the proportion of biased traders.

Proposition 2.4 also shows that, even if a short-term trader follows on average a feedback trading strategies, there are times where a contrarian trading strategy is optimal. These times correspond to the cases in which the public signal is inconsistent with biased traders' beliefs but is not strong enough to change their views. For example, a contrarian strategy is optimal when biased traders are bullish and the public signal is mildly negative.

Deriving the optimal demand of the hedge fund is straightforward. We therefore concentrate on the characteristics of the hedge fund strategy: is the fund using a contrarian or a positive-feedback strategy? To do so, we focus on the trading behavior of a given period t. Consider that biased traders are optimistic, that is, $\mu_{t-1} > 0$ (the logic described below also applies when they are pessimistic). The most recent return is $P_t - P_{t-1} = (1-\lambda)s_t + \lambda\sigma_t$ which is positive (negative) if s_t is positive (negative). When s_t is positive, biased traders beliefs remain positive ($\mu_{t-1} + s_t > 0$) in which case the fund is buying ($d_t > 0$). After a good news at date t, there is a positive return and the fund engages in positive-feedback trading. When s_t is negative, there are two cases. First, if $s_t < -\mu_{t-1}$, biased traders beliefs become negative in which case the fund is selling ($d_t < 0$). Again, this case corresponds to positive-feedback trading since, after a very bad news at date t, the price declines and the fund is going short. Second, if $-\mu_{t-1} < s_t < 0$, biased traders beliefs remain positive despite the bad news in which case the fund is buying ($d_t > 0$). In this case, contrarian trading occurs: the fund is buying after a price decline.
Given this discussion, positive-feedback trading occurs in two scenarii: either when $s_t > 0$ or when $s_t < -\mu_{t-1}$.The ex-ante probability that the fund uses a positive-feedback strategy is therefore $\frac{1}{2} + \mathcal{N}(\frac{-m}{\Sigma})$. This probability is greater than $\frac{1}{2}$ which indicates that the hedge fund uses a positive-feedback strategy more often then a contrarian strategy.

3 Conclusion

This paper proposes a theory of price formation based on the premise that some traders are prone to the confirmation bias. We model this cognitive bias by considering that people prone to the confirmation bias tend to amplify signals that are consistent with their prior views. We show that, in the context of financial markets, this bias creates excess volatility, momentum and reversals. In addition, when biased traders are bullish (because of overall positive prior signals), the market experiences a bubble meaning that prices are above fundamentals, and that the mispricing is expected to worsen in the future. A comparative-static exercise shows that the momentum effect is negatively related to the strenghts of biased traders' beliefs. This prediction could be the basis for an empirical test of our model. One way to proceed could be to compare momentum after IPOs and momentum for more established stocks. To the extent that investors have less information (and thus weaker views) about newly listed stocks, our theory predict that these stocks should experience a stronger momentum effect.

We highlight the implications of our model for quantitative investments by deriving the optimal strategy of a short-term trader. We analytically indicate how to structure a portfolio in order to profit from biased traders' misperception. In particular, we show that most of the time, a short-term trader adopts a feedback strategy. However, we also characterize market conditions in which a contrarian trading strategy is optimal. This corresponds to the cases where the public signal is inconsistent with biased traders' beliefs but is not strong enough to change their views. For example, a contrarian strategy is optimal when biased traders are bullish and the public signal is mildly negative. Overall, our model highlights the fact that, to successfully benefit from short-term strategies, traders should track the evolution of biased traders' beliefs that are at the origin of the momentum effect and of the other mispricings. Our model provides an explicit characterization of these biased traders' beliefs and their evolution and may thus be helpful in structuring dynamic trading strategies bound to profit from the momentum effect.

In future work, it would be interesting to estimate the parameters of our model (in particular the proportion of biased traders on the market, λ, and the magnitude of the bias, q) in order to evaluate the abnormal returns generated by a strategy based on the confirmation bias.

4 Appendix

Equilibrium

To derive the equilibrium in our financial market, we start by writing the budget constraint of trader j at date t, $t \in \{0, 1, ..., T\}$:

$$-\left[d_t^j P_t + \frac{c}{2}\left(d_t^j\right)^2\right] - \sum_{s=1}^{s=t-1}\left(d_s P_s + \frac{c}{2}\left(d_s^j\right)^2\right) + \left[d_t P_t + \frac{c}{2} d_t^2 + \sum_{s=1}^{s=t-1}\left(d_s P_s + \frac{c}{2}\left(d_s^j\right)^2\right)\right] = 0$$

where d_t^j is the demand of trader j at date t, and P_t is the price at date t. The first term in the budget constraint is the cost of establishing a position at date t, the second term reflects the reimbursement (or the proceeds) from the loan necessary to establish the position at the previous dates, the third term is the new loan necessary to finance all these operations. The sum of these three terms is zero because traders are assume to receive no income (we thus consider self-financed strategies).

At date $T+1$, the final wealth of trader j, W_j, is:

$$W_j = \sum_{t=0}^{t=T}\left[d_t^j\left(v - P_t\right) - \frac{c}{2}\left(d_t^j\right)^2\right] + v$$

Since traders only consumme at the last date $T+1$, their objective is to maximize their expected final wealth conditional on their information. To solve for the optimal demands, we start by solving the following program of trader j at period T:

$$\max_{d_T^j} \mathbb{E}_T^X\left(\sum_{t=0}^{t=T-1}\left[d_t^j\left(v-P_t\right) - \frac{c}{2}\left(d_t^j\right)^2\right] + d_T^j\left(v - P_T\right) - \frac{c}{2}\left(d_T^j\right)^2 + v\right),\ \forall X \in \{A, B\}$$

It is straighforward to check that the objective function is concave in d_T^j. The first order condition is thus necessary and suffisant to characterize the optimal demand at date T:

$$d_T^j = \frac{\mathbb{E}_T^X(v) - P_T}{c}$$

Proceeding backward, we obtain that:

$$d_t^j = \frac{\mathbb{E}_t^X(v) - P_t}{c},\ \forall t \in \{0, 1, ..., T\}$$

At date t, $t \in \{0, 1, ..., T\}$, the market clearing condition is given by:

$$\int_0^{(1-\lambda)} \frac{\mathbb{E}_t^A(v) - P_t}{c} dj + \int_{(1-\lambda)}^1 \frac{\mathbb{E}_t^B(v) - P_t}{c} dj = 0$$

where 0 corresponds to the fact that no new share is being issued on the market.

¿From this market clearing condition, we derive the following pricing equation:

$$P_t = (1-\lambda)\,\mathbb{E}_t^A(v) + \lambda \mathbb{E}_t^B(v).$$

Proof of Lemma 2.1

- Let f increasing. Remember that

$$\begin{aligned}
\mathcal{M}f(x) &= \mathbb{E}^A\left(f(\mu_{t+1})|\mu_t = x\right)\\
&= \mathbb{E}_t^A\left(f(x+\sigma_{t+1})\right)\\
&= \mathbb{E}^A\left(f(x + q s_{t+1}\mathbb{1}_{x s_{t+1}\geq 0} + s_{t+1}\mathbb{1}_{x s_{t+1}\leq 0})\right).
\end{aligned}$$

  We conclude by noting that that the function $x \to x + qy\mathbb{1}_{xy\geq 0} + y\mathbb{1}_{xy\leq 0}$ is increasing for every real y.

- Take $x > 0$ and a bounded function f such that $f(x) + f(-x) = 1$. We have

$$\begin{aligned}
&\mathcal{M}f(x) + \mathcal{M}f(-x)\\
=\ & \mathcal{M}f(x) + \left[\int_0^\infty f(-x+qy)e^{-\frac{y^2}{2\Sigma^2}}\frac{dy}{\sqrt{2\pi\Sigma^2}} + \int_{-\infty}^0 f(-x+y)e^{-\frac{y^2}{2\Sigma^2}}\frac{dy}{\sqrt{2\pi\Sigma^2}}\right]\mathbb{1}_{x\leq 0}\\
+\ & \left[\int_0^\infty f(-x+y)e^{-\frac{y^2}{2\Sigma^2}}\frac{dy}{\sqrt{2\pi\Sigma^2}} + \int_{-\infty}^0 f(-x+qy)e^{-\frac{y^2}{2\Sigma^2}}\frac{dy}{\sqrt{2\pi\Sigma^2}}\right]\mathbb{1}_{x\geq 0}\\
=\ & \mathcal{M}f(x) + \left[\int_{-\infty}^0 f(-x-qy)e^{-\frac{y^2}{2\Sigma^2}}\frac{dy}{\sqrt{2\pi\Sigma^2}} + \int_0^\infty f(-x-y)e^{-\frac{y^2}{2\Sigma^2}}\frac{dy}{\sqrt{2\pi\Sigma^2}}\right]\mathbb{1}_{x\leq 0}\\
+\ & \left[\int_{-\infty}^0 f(-x-y)e^{-\frac{y^2}{2\Sigma^2}}\frac{dy}{\sqrt{2\pi\Sigma^2}} + \int_0^\infty f(-x-qy)e^{-\frac{y^2}{2\Sigma^2}}\frac{dy}{\sqrt{2\pi\Sigma^2}}\right]\mathbb{1}_{x\geq 0}\\
=\ & 1.
\end{aligned}$$

□

Proof of Proposition 2.2.

Before proceeding with the proof of Proposition 2.2, we will state a technical lemma claiming that biased traders' beliefs at time t are unconditionally positively correlated with the true signal s_t.

Lemma 4.2 *Let f be a bounded increasing function. We have for every t,*

$$\mathbb{E}(s_t f(\mu_t)) \geq 0.$$

Proof of Lemma 4.2: Let us denote F_t the distribution function of μ_t. We have

$$\begin{aligned}
\mathbb{E}(s_t f(\mu_t)) &= \mathbb{E}(s_t f(\mu_{t-1} + \sigma_t)) \\
&= \int_{\mathbb{R}} \mathbb{E}\left(s_t \mathbb{1}_{\{x s_t \geq 0\}} f(x + q s_t) + s_t \mathbb{1}_{\{x s_t \leq 0\}} f(x + s_t)\right) dF_{t-1}(x) \\
&= \int_{\mathbb{R}} \mathbb{E}\left(s_t \mathbb{1}_{\{x s_t \geq 0\}} (f(x + q s_t) - f(x - s_t))\right) dF_{t-1}(x).
\end{aligned}$$

We conclude by noting that the function $s \to s(f(x + qs) - f(x - s))$ is positive for any increasing function f.

Now, we come back to the proof of the excess variance in our model. To do this, we express P_t in terms of P_t^*,

$$\begin{aligned}
P_t &= P_t^* + \lambda \sum_{i=1}^{t} (\sigma_i - s_i) \\
&= P_t^* + \lambda (q-1) \sum_{i=1}^{t} s_i \mathbb{1}_{\{\mu_{i-1} s_i \geq 0\}}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\text{(4.1)} \quad \operatorname{var}(P_t) &= \operatorname{var}(P_t^*) + 2\lambda(q-1) \operatorname{cov}\left(P_t^*, \sum_{i=1}^{t} s_i \mathbb{1}_{\{\mu_{i-1} s_i \geq 0\}}\right) \\
&+ \lambda^2 (q-1)^2 \operatorname{var}\left(\sum_{i=1}^{t} s_i \mathbb{1}_{\{\mu_{i-1} s_i \geq 0\}}\right).
\end{aligned}$$

We focus on the second term and show that it is positive. Indeed,

$$\begin{aligned}
\operatorname{cov}\left(P_t^*, \sum_{i=1}^{t} s_i \mathbb{1}_{\{\mu_{i-1} s_i \geq 0\}}\right) &= \mathbb{E}\left[\left(\sum_{i=1}^{t} s_i\right)\left(\sum_{i=1}^{t} s_i \mathbb{1}_{\{\mu_{i-1} s_i \geq 0\}}\right)\right] \\
&= \sum_{i=1}^{t} \mathbb{E}\left(s_i^2 \mathbb{1}_{\{\mu_{i-1} s_i \geq 0\}}\right) + 2 \sum_{1 \leq i < j \leq t} \mathbb{E}\left(s_i s_j \mathbb{1}_{\{\mu_{j-1} s_j \geq 0\}}\right).
\end{aligned}$$

Now, consider $i < j$. We have,

$$\begin{aligned}\mathbb{E}\left(s_i s_j \mathbb{1}_{\{\mu_{j-1}s_j \geq 0\}}\right) &= \mathbb{E}\left(s_i \mathbb{E}_i(s_j \mathbb{1}_{\{\mu_{j-1}s_j \geq 0\}})\right) \\ &= 2(q-1)\frac{\Sigma}{\sqrt{2\pi}}\mathbb{E}(s_i \mathbb{P}_i(\mu_{j-1} > 0)).\end{aligned}$$

Let us define $f_0(x) = \mathbb{1}_{\{x>0\}}$. Note that f_0 is a bounded increasing function. We have

$$\begin{aligned}\mathbb{P}_i(\mu_{j-1} > 0) &= \mathbb{E}_i(f_0(\mu_{j-1})) \\ &= \mathcal{M}^{j-i-1} f_0(\mu_i).\end{aligned}$$

Finally, we obtain

$$\text{cov}\left(P_t^*, \sum_{i=1}^t s_i \mathbb{1}_{\{\mu_{i-1}s_i \geq 0\}}\right) = \sum_{i=1}^t \mathbb{E}\left(s_i^2 \mathbb{1}_{\{\mu_{i-1}s_i \geq 0\}}\right) + 4(q-1)\frac{\Sigma}{\sqrt{2\pi}} \sum_{1 \leq i < j \leq t} \mathbb{E}(s_i \mathcal{M}^{j-i-1} f_0(\mu_i)).$$

We conclude by applying Lemma 4.2 since $\mathcal{M}^{j-i-1} f_0$ is a bounded increasing function.

In order to show the momentum effect, we focus on the case $k = 1$ without loss of generality. Let us take $t \in \{0, \ldots, T\}$, we have

$$\begin{aligned}\text{cov}(P_{t+1} - P_t, P_t - P_{t-1}) &= \text{cov}(\lambda\sigma_{t+1} + (1-\lambda)s_{t+1}, \lambda\sigma_t + (1-\lambda)s_t) \\ &= \lambda^2 \text{cov}\left(\sigma_{t+1}, \sigma_t\right) + \lambda(1-\lambda)\text{cov}\left(\sigma_{t+1}, s_t\right).\end{aligned}$$

We will prove that the two terms of the right-hand side are positive. Indeed, we have

$$\begin{aligned}\text{cov}\left(\sigma_{t+1}, s_t\right) &= \mathbb{E}\left(\sigma_{t+1} s_t\right) \\ &= q\mathbb{E}\left(s_t s_{t+1} \mathbb{1}_{\{\mu_t s_{t+1} \geq 0\}}\right) + \mathbb{E}\left(s_t s_{t+1} \mathbb{1}_{\{\mu_t s_{t+1} \geq 0\}}\right) \\ &= 2(q-1)\frac{\Sigma}{\sqrt{2\pi}}\mathbb{E}\left(s_t \mathbb{1}_{\{\mu_t \geq 0\}}\right).\end{aligned}$$

Now

$$\begin{aligned}\mathbb{E}\left(s_t \mathbb{1}_{\{\mu_t \geq 0\}}\right) &= \mathbb{E}\left(s_t \mathbb{1}_{\{\mu_{t-1}+\sigma_t \geq 0\}}\right) \\ &= \mathbb{E}\left(s_t \mathbb{1}_{\{\mu_{t-1}+qs_t \geq 0\}} \mathbb{1}_{\{\mu_{t-1}s_t \geq 0\}}\right) + \mathbb{E}\left(s_t \mathbb{1}_{\{\mu_{t-1}+s_t \geq 0\}} \mathbb{1}_{\{\mu_{t-1}s_t \leq 0\}}\right) \\ &= \mathbb{E}\left(s_t \mathbb{1}_{s_t \geq 0\}} \mathbb{1}_{\{\mu_{t-1} \geq 0\}}\right) + \mathbb{E}\left(s_t \mathbb{1}_{\{\mu_{t-1}+s_t \geq 0\}} \mathbb{1}_{\{\mu_{t-1} \leq 0\}}\right)\end{aligned}$$

$$
\begin{aligned}
&+ \quad \mathbb{E}\left(s_t \mathbb{1}_{\{-\mu_{t-1}<s_t<0\}} \mathbb{1}_{\{\mu_{t-1}\geq 0\}}\right)\\
&= \quad \mathbb{E}\left(s_t \mathbb{1}_{\{\mu_{t-1}+s_t\geq 0\}}\right)\\
&= \quad \frac{\Sigma}{\sqrt{2\pi}}\mathbb{E}(e^{-\frac{\mu_{t-1}^2}{2\Sigma^2}})\\
&\geq \quad 0.
\end{aligned}
$$

Thus, $\operatorname{cov}(\sigma_{t+1}, s_t) \geq 0$.
On the other hand,

$$
\operatorname{cov}(\sigma_{t+1}, \sigma_t) = \mathbb{E}(\sigma_{t+1}\sigma_t) - \mathbb{E}(\sigma_{t+1})\mathbb{E}(\sigma_t).
$$

We have, for every t,

$$
\begin{aligned}
\mathbb{E}(\sigma_t) &= (q-1)\mathbb{E}(s_t \mathbb{1}_{\{\mu_{t-1}s_t\leq 0\}})\\
&= (q-1)\frac{\Sigma}{\sqrt{2\pi}}(2\mathbb{P}(\mu_{t-1}>0)-1).
\end{aligned}
$$

and

$$
\begin{aligned}
\mathbb{E}(\sigma_{t+1}\sigma_t) &= \mathbb{E}(\mathbb{E}_t(\sigma_{t+1})\sigma_t)\\
&= (q-1)\frac{\Sigma}{\sqrt{2\pi}}\mathbb{E}(\sigma_t(2\mathbb{1}_{\{\mu_t\geq 0\}}-1))
\end{aligned}
$$

But,

$$
\begin{aligned}
\mathbb{E}(\sigma_t \mathbb{1}_{\{\mu_t\geq 0\}}) &= (q-1)\mathbb{E}(s_t \mathbb{1}_{\{\mu_{t-1}s_t\geq 0\}} \mathbb{1}_{\{\mu_t\geq 0\}}) + \mathbb{E}\left(s_t \mathbb{1}_{\{\mu_t\geq 0\}}\right)\\
&= (q-1)\mathbb{E}(s_t \mathbb{1}_{\{\mu_{t-1}\geq 0\}} \mathbb{1}_{\{s_t\geq 0\}}) + \mathbb{E}\left(s_t \mathbb{1}_{\{\mu_t\geq 0\}}\right)\\
&= (q-1)\frac{\Sigma}{\sqrt{2\pi}}\mathbb{P}(\mu_{t-1}>0) + \frac{\Sigma}{\sqrt{2\pi}}\mathbb{E}(e^{-\frac{\mu_{t-1}^2}{2\Sigma^2}}).
\end{aligned}
$$

Finally,

$$
\operatorname{cov}(\sigma_t, \sigma_{t+1}) = (q-1)\frac{\Sigma^2}{2\pi}\left(2\mathbb{E}(e^{-\frac{\mu_{t-1}^2}{2\Sigma^2}}) + (q-1)\left(1-(2\mathbb{P}(\mu_{t-1}>0)-1)(2\mathbb{P}(\mu_t>0)-1)\right)\right).
$$

The latter quantity is positive since

$$
|(2\mathbb{P}(\mu_{t-1}>0)-1)(2\mathbb{P}(\mu_t>0)-1)| \leq 1.
$$

Furthermore, conditionally to the information set at time $t-1$, computations above give that

$$\mathrm{cov}_{t-1}(P_{t+1}-P_t, P_t-P_{t-1}) = f(|\mu_{t-1}|),$$

where f is defined on $(0,+\infty)$ by

$$f(x) = \lambda(q-1)\frac{\Sigma^2}{\pi}e^{-\frac{x^2}{2\Sigma^2}} + \lambda^2(q-1)^2\frac{\Sigma^2}{\pi}\mathcal{N}\left(\frac{-x}{\Sigma}\right).$$

Obviously, f is a decreasing function. Moreover, $\lim_{x\to+\infty} f(x)=0$ and f attains its maximum at $x=0$.

We finally focus on the price reversal effect as measured by $\mathrm{cov}(P_t - P_0, \tilde{v}-P_t) = \mathrm{cov}(P_t, \tilde{v}-P_t)$.
According to 4.1, we have

$$\begin{aligned}
\mathrm{var}(P_t) &= \mathrm{var}(P_t^*) + 2\lambda(q-1)\mathrm{cov}\left(P_t^*, \sum_{i=1}^{t} s_i \mathbb{1}_{\{\mu_{i-1}s_i\geq 0\}}\right) \\
&+ \lambda^2(q-1)^2\mathrm{var}\left(\sum_{i=1}^{t} s_i \mathbb{1}_{\{\mu_{i-1}s_i\geq 0\}}\right) \\
&\geq \mathrm{var}(P_t^*) + \lambda(q-1)\mathrm{cov}\left(P_t^*, \sum_{i=1}^{t} s_i \mathbb{1}_{\{\mu_{i-1}s_i\geq 0\}}\right) \\
&= \mathrm{cov}\left(v, P_t\right)
\end{aligned}$$

because $\mathrm{cov}\left(P_t^*, \sum_{i=1}^{t} s_i \mathbb{1}_{\{\mu_{i-1}s_i\geq 0\}}\right)$ is positive. □

Proof of Proposition 2.3

We start by recalling the analytical expression of the mispricing, as measure by the difference $P_t - P_t^*$. We have, for every t,

$$\begin{aligned}
P_t - P_t^* &= \lambda\sum_{i=1}^{t}(\sigma_i - s_i) \\
&= \lambda(q-1)\sum_{i=1}^{t} s_i \mathbb{1}_{s_i\mu_{i-1}>0}
\end{aligned}$$

Therefore, for $0 \leq k \leq l$,

$$P_{t+l} - P^*_{t+l} = P_{t+k} - P^*_{t+k} + \lambda(q-1) \sum_{i=k+1}^{l} s_{t+i} \mathbb{1}_{\{s_{t+i}\mu_{t+i-1} > 0\}}$$

Taking expectations conditionally on the information available at time t, we obtain

$$\mathbb{E}_t \left(P_{t+l} - P^*_{t+l}\right) = \mathbb{E}_t \left(P_{t+k} - P^*_{t+k}\right) + \lambda(q-1) \frac{\Sigma}{\sqrt{2\pi}} \sum_{i=k+1}^{l} \left(2\mathbb{P}_t(\mu_{t+i-1} > 0) - 1\right).$$

Now,

$$\mathbb{P}_t(\mu_{t+i-1} > 0) = \mathcal{M}^{i-1} f_0(\mu_t),$$

where $f_0(x) = \mathbb{1}_{x \geq 0}$. According to Lemma 2.1, we have that $\mathcal{M}^{i-1} f_0(\mu_t) \geq \frac{1}{2}$ if and only if $\mu_t > 0$ which concludes the proof. □

Proof of Proposition 2.4

The trading stategy of a one-period hedge fund is

$$d_t = \frac{1}{c} \left(\mathbb{E}^A_t (P_{t+1} - P_t)\right).$$

Because,

$$P_{t+1} = P_t + (1-\lambda) s_{t+1} + \lambda \sigma_{t+1},$$

we get

$$\begin{aligned} d_t &= \frac{\lambda}{c} \mathbb{E}^A_t(\sigma_{t+1}) \\ &= \frac{\lambda(q-1)\Sigma}{c\sqrt{2\pi}} \left[2\mathbb{P}_t(\mu_{t+1} > 0) - 1\right] \\ &= \frac{\lambda(q-1)\Sigma}{c\sqrt{2\pi}} \left[2\mathbb{P}_t(s_{t+1} > -\mu_t) - 1\right] \\ &= \frac{\lambda(q-1)\Sigma}{c\sqrt{2\pi}} \left[2\mathcal{N}\left(\frac{\mu_t}{\Sigma}\right) - 1\right] \end{aligned}$$

which implies that $d_t > 0$ if $\mu_t > 0$.
In order to study the behavior of a short term trader, we assume without loss of generality that we are at time t and $\mu_t > 0$.

- If the public signal $s_{t+1} \geq 0$, the variation of price $P_{t+1} - P_t = \lambda s_{t+1} + (1-\lambda) q s_{t+1}$ is positive and the biased traders'belief $\mu_{t+1} = \mu_t + q s_{t+1} > 0$ which implies that $d_{t+1} > 0$. The short term trader engages in feeback trading.

- If the public signal $s_{t+1} \leq -\mu_t$, the variation of price $P_{t+1} - P_t = s_{t+1}$ is negative and the biased traders' belief $\mu_{t+1} = \mu_t + s_{t+1} < 0$ which implies that $d_{t+1} < 0$. The short term trader engages in feeback trading.

- If the public signal $-\mu_t \leq s_{t+1} \leq 0$, the variation of price $P_{t+1} - P_t = s_{t+1}$ is negative but the biased traders' belief $\mu_{t+1} = \mu_t + s_{t+1} > 0$ which implies that $d_{t+1} > 0$. The short term trader engages in contrarian trading.

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