**Electronic Supplementary Materials**

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A Simultaneous Analysis of Turnout and Voting under Proportional Representation: Theory and Experiments

**Aaron Kamm & Arthur Schram**

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## Appendix A: Nash equilibria

### Mandatory voting

We solve for symmetric cut-point equilibria, which implies that voters with a position to the left of $x_L$ vote for the left-wing party, voters between $x_L$ and $x_R$ vote for the central party and voters to the right of $x_R$ vote for the right-wing party.

When voting is mandatory the equilibrium is therefore the solution to the following set of equations

(i) $EU(vote\ left|position\ is\ x_L) = EU(vote\ center|position\ is\ x_L)$

(ii) $EU(vote\ right|position\ is\ x_R) = EU(vote\ center|position\ is\ x_R)$

(iii) $P(vote\ left) = P(voters\ position\ to\ the\ left\ of\ x_L)$

(iv) $P(vote\ right) = P(voters\ position\ to\ the\ right\ of\ x_R)$

(v) $EU(vote\ left|position\ is\ x) = f_L(P(vote\ left), P(vote\ right))$

(vi) $EU(vote\ center|position\ is\ x) = f_C(P(vote\ left), P(vote\ right))$

(vii) $EU(vote\ right|position\ is\ x) = f_R(P(vote\ left), P(vote\ right))$

Here EU is the expected payoff from an action and *P* denotes the probability of a certain event. The functions $f_L$, $f_C$ and $f_R$ compute the expected payoffs by going through all possible election outcomes, computing the resulting payoffs for the voter and weighting them by their respective probabilities given the probability that a vote will be for a certain party.

Numerical solution yields the Nash equilibria depicted in Table A1.

**Table A1: Nash Equilibria Mandatory Voting**

| | CentMand | ExtrMand |
|---|---|---|
| Equilibrium 1 (selected by QRE) | $X_L = –0.65$; $X_R = 3.63$ | $X_L = –5.02$; $X_R = 4.53$ |
| Equilibrium 2 | $X_L = –0.77$; $X_R = 7.94$ | $X_L = –4.07$; $X_R = 6.01$ |
| Equilibrium 3 | $X_L = –3.56$; $X_R = 3.63$ | |

**Notes.** Cells give the Nash equilibrium cut points for party choice when voting is mandatory. Voters with an ideal point to the left of $X_L$ vote for the left-wing party, voters to the right of $X_R$ vote for the right-wing party and voters in between vote for the central party.

The table shows multiple equilibria. One way to refine these is to solve for the quantal response equilibrium letting λ go to infinity along the principal branch of the Multinomial Logit Correspondence (McKelvey and Palfrey 1995). This selects the Nash equilibria shown in the top row.

### Voluntary Voting

We again solve for symmetric cut-point equilibria with cut points $x_L$ and $x_R$. Note that these cut-points are independent of the costs of voting since such costs only influence whether a voter abstains or not but not for which party she will vote if she turns out. The turnout decision is also described by a cut-point where a voter with position x votes if her voting costs are below a threshold $\bar{c}(x)$.

With voluntary voting the equilibrium is therefore the solution to the following set of equations

*(i)* $EU(vote\ left|position\ is\ x_L) = EU(vote\ center|position\ is\ x_L)$

*(ii)* $EU(vote\ right|position\ is\ x_R) = EU(vote\ center|position\ is\ x_R)$

*(iii)* $P(vote\ left) = P(voters\ position\ to\ the\ right\ of\ x_L) * \frac{P(a\ voter\ to\ the\ right\ of\ x_L\ votes)}{P(a\ random\ voter\ votes)}$

*(iv)* $P(vote\ right) = P(voters\ position\ to\ the\ right\ of\ x_R) * \frac{P(a\ voter\ to\ the\ right\ of\ x_R\ votes)}{P(a\ random\ voter\ votes)}$

*(v)* $EU(vote\ left|position\ is\ x) = f_L(P(vote\ left), P(vote\ right))$

(vi) $EU(vote\ center|position\ is\ x) = f_C(P(vote\ left), P(vote\ right))$
*(vii)* $EU(vote\ right|position\ is\ x) = f_R(P(vote\ left), P(vote\ right))$
*(viii)* $P(a\ voter\ with\ position\ x\ votes) =$
$P(costs\ are\ below\ max_i\ [EU\ (vote\ for\ party\ i) - EU(abstain)]$

Compared to the situation with mandatory voting we now have to take into account that voters have different turnout rates depending on their position. Therefore, the probability of, for instance, a left-wing vote is not simply the probability that a voter is to the left of $x_L$, it also has to be weighted by the relative turnout rate of a left-wing voter compared to the average turnout rate in the population.

Solving this set of equations leads to unique equilibria for both specifications of left-wing party positions (Table A2).

**Table A2: Nash Equilibria Voluntary Voting**

| CentVolu | ExtrVolu |
|---|---|
| $X_L = -0.26$; $X_R = 3.26$ | $X_L = -2.85$; $X_R = 3.05$ |

**Notes.** Cells give the Nash equilibrium cut points for party choice when voting is voluntary. If they vote, voters with an ideal point to the left of $X_L$ vote for the left-wing party, voters to the right of $X_R$ vote for the right-wing party and voters in between vote for the central party.

### Convergence of QRE

Finally, to confirm the convergence of the QRE to the Nash equilibria, Table A3 compares the latter to the QRE for λ = 350. In particular, for the QRE we show the position for which a voter is indifferent between parties L and C ($X_L$) and between C and R ($X_R$). The table confirms that the QRE that we derived converge the Nash equilibria described above.

**Table A3: Comparing Nash Equilibria to the Limit QRE**

| | CentMand | | CentVolu | | ExtrMand | | ExtrVolu | |
|---|---|---|---|---|---|---|---|---|
| | $X_L$ | $X_R$ | $X_L$ | $X_R$ | $X_L$ | $X_R$ | $X_L$ | $X_R$ |
| Nash | –0.65 | 3.63 | –0.26 | 3.26 | –5.02 | 4.53 | –2.85 | 3.05 |
| QRE (λ = 350) | –0.63 | 3.55 | –0.25 | 3.26 | –5.01 | 4.53 | –2.83 | 3.05 |

**Notes.** Cells give the equilibrium cut-off points in the Nash equilibrium and the QRE for the limit case of λ = 350. If they vote, voters with an ideal point to the left of $X_L$ vote for the left-wing party, voters to the right of $X_R$ vote for the right-wing party and voters in between vote for the central party.

### Turnout

The equilibrium for the turnout decision is characterized by a function that assigns to each voter position a critical cost for which a voter is indifferent between abstaining and voting. Figure 3 in the main text plots these for our two treatments and illustrates both the Extremist Effect and the Turnout Effect.

## Appendix B: Additional analysis QRE

### Computation of equilibrium for mandatory voting

As a point of departure we use that in the logit equilibrium the probability of voting for party *i* given a position x and costs of voting c is described by the following expression (eq 2 in the main text):[1]

$$\Pr(\text{vote for party } i|x) = \frac{\exp\,(\lambda * EU(vote\ for\ party\ i|x))}{\sum_i \exp\,(\lambda * EU(vote\ for\ party\ i|x))} \quad \text{(B.1)}$$

This implies that the ex-ante probability of voting for a given party is given by:

$$\Pr(vote\ for\ party\ i) = \int_x P\,r(vote\ for\ party\ i|x)\, f(x)dx \quad \text{(B.2)}$$

where f is the distribution of ideal points (a truncated t-distribution with 0.05 degrees of freedom).

The expected utility of a vote is obtained by computing the payoff of this vote for all possible configurations of votes by the other four voters, weighted by the ex-ante probabilities. One can capture this in the following expression:

$$EU(vote\ for\ party\ i|x) =$$

$$\sum_{a=0}^{4} \sum_{b=0}^{4-a} \frac{4!}{a!b!(4-a-b)!} P_L^a P_C^b P_R^{4-a-b} (-2 * [x^*(a,b,i) - x]^2),\ i{=}L,C,R \quad \text{(B.3)}$$

where $x$ is the voter's position, $P_j$ is the ex-ante probability of voting for party $j$, $a(b)$ is the number of other voters voting for party $L(C)$ (leaving 4–*a*–b to vote for $R$) and $x^*(a,b,i)$ is the implemented policy given the other voters behavior and the voter voting for party *i*.

Substituting B.3 into B.1 and the result in B.2 yields three expressions (a voting probability for each party), which set-up a fixed point problem for the vector of probabilities. The set of equations was solved numerically and to account for the possibility of equilibrium multiplicity a wide range of initial conditions was checked (more details are available upon request). Because these all converge to the same equilibrium, we tentatively conclude the results likely to be unique.

### Computation of equilibrium for voluntary voting

The case of voluntary voting is slightly more involved. While eq. (B.1) remains the same (B.2) becomes more complex. The reason is that the distribution of positions and voting costs for the voters may be different from the ex-ante distribution of these quantities. For instance, extreme voters are more likely to vote and therefore the distribution of ideal points for those who vote has fatter tails than the ex-ante distribution of ideal points. We therefore have to use the expression for the probability of turnout specified in equation (3) of the main text:

$$\Pr(\text{turnout}|x,c) = \frac{\sum_j * \exp\,(\lambda * \Pr(\text{vote for party } j|x,c) * EU(vote\ for\ party\ j|x,c))}{\exp\,(\lambda * EU(abstain|x,c)) + \sum_j \exp\,(\lambda * \Pr(\text{vote for party } j|x,c) * EU(vote\ for\ party\ j|x,c))}$$

[1] For notational convenience we drop the costs of voting c since they do not influence the party choice.

The expression for the expected utility of voting for a specific party (eq. B.3) also becomes more involved since we now have to take abstentions into account and therefore do not know how many other votes will be cast. The expression used is as follows:

$$\left\{\sum_{n=0}^{4}\binom{4}{n}P_V^n(1-P_V)^{4-n}\sum_{a=0}^{n}\sum_{b=0}^{n-a}\frac{n!}{a!\,b!\,(n-a-b)!}P_L^aP_C^bP_R^{n-a-b}(-2*[x^*(a,b,i)-x]^2)\right\}-i*c$$

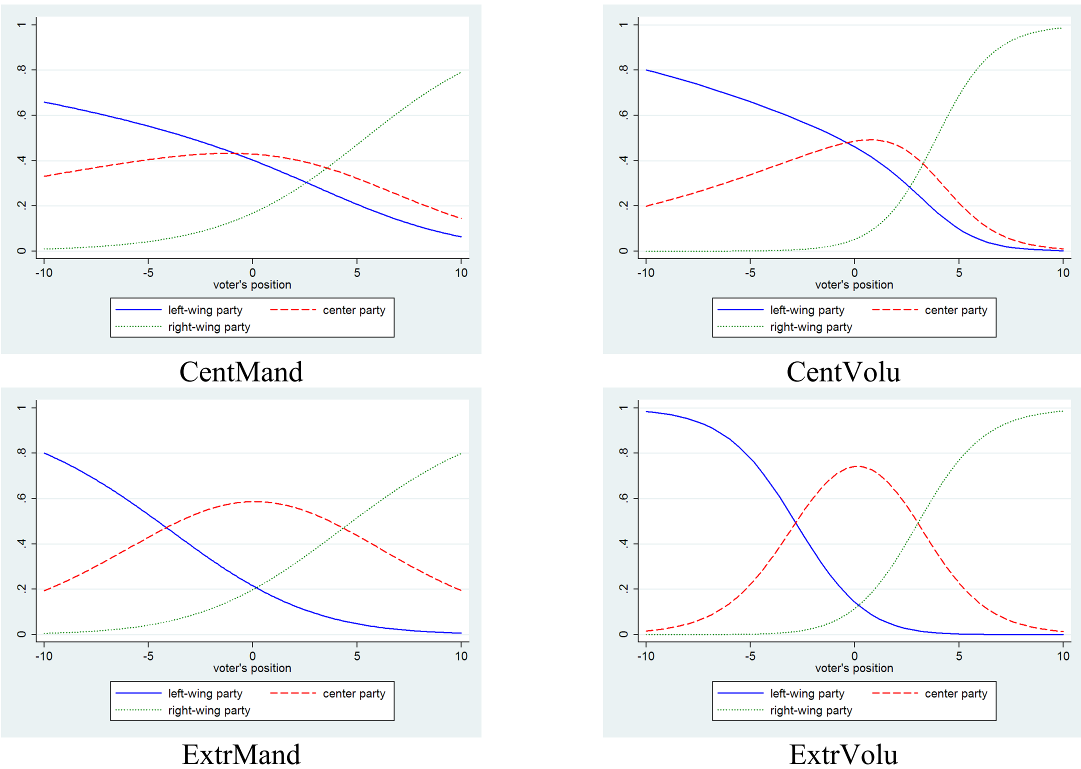
where the variables are defined as before, $n$ is the number of votes cast by other voters and $P_V$ is the ex-ante probability of turning out. The negative term $i*c$ appears since now voting costs matter because voting for party zero (i.e. abstention) avoids them.

Combining all expressions yields a fixed point problem that was solved numerically. Again a large range of different initial conditions was checked that all converged to the same equilibrium.

**Detailed predictions for the four treatments**

Using the method described, we obtained the logit equilibria for the various treatments of our experiment. Figure B1 shows these.

**Figure B1: Predicted party choice**

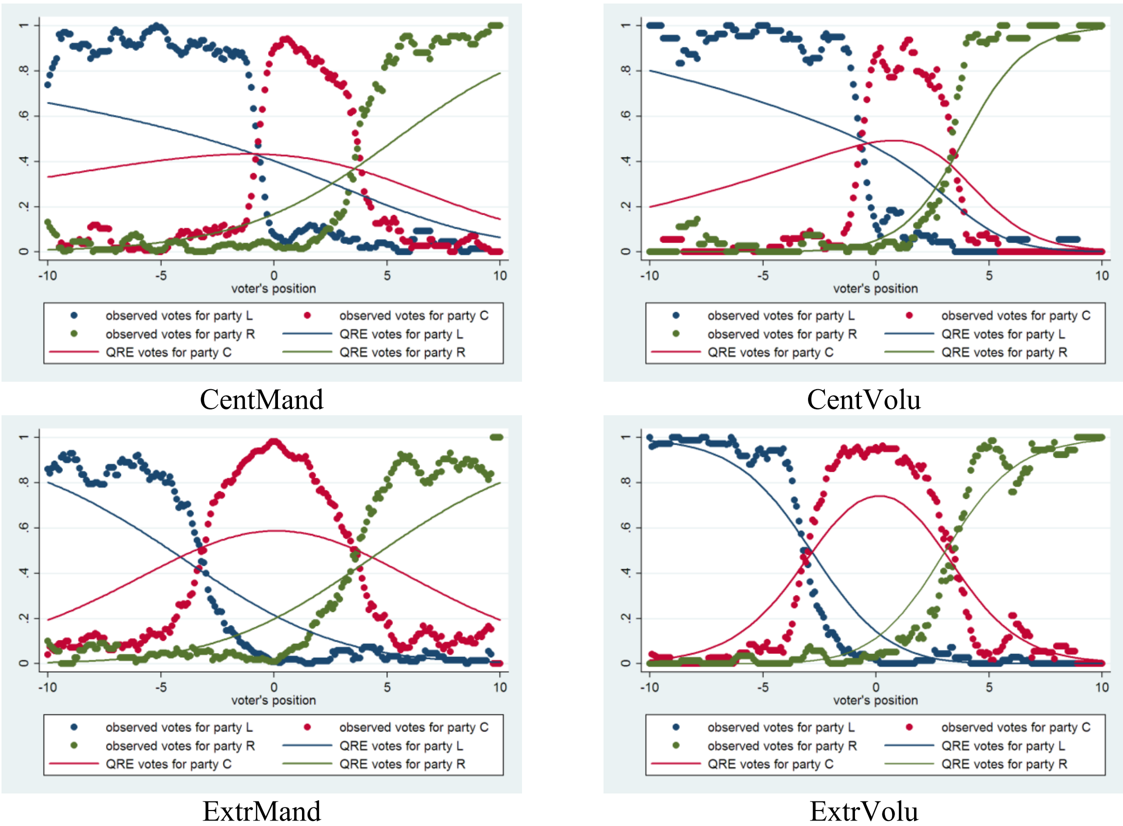



*Notes*. The figures show the predicted probability of voting for each of the three parties as the voter's position varies along the horizontal-axis. The predictions are based on the QRE model with λ=3.7.

**QRE predictions compared to observed behavior**

Figure B2 compares the QRE predictions to observed party choice. Though it shows that voting generally follows the equilibrium pattern, there are also substantial deviations from the QRE prediction. In particular, extreme voters deviate much less from the party that yields the highest expected utility (i.e., the extreme party on their side of the spectrum) than is predicted by the 'noisy' logit equilibrium. Moreover, the slopes of the observed party choice functions are much steeper than predicted. As indicated in the main text, one possible explanation for these deviations is that behavior is less noisy than in the pilot in Kamm (2012) that was used to obtain an out-of-sample estimate of $\lambda$. To investigate this possibility, we explore the parameters that we can estimate from the data from our experiment.

**Figure B2: Predicted compared to observed party choice**

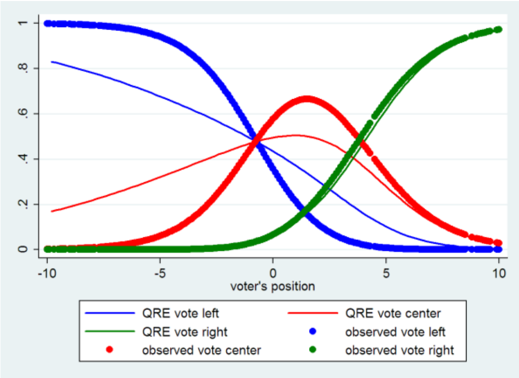



*Notes.* Dots (Lines) show the observed (predicted) probability of voting for each of the three parties as the voter's position varies along the horizontal-axis. The predictions are based on the QRE model with $\lambda$=3.7. To improve readability the observed data are averaged over +/-0.2 the value on the horizontal-axis.

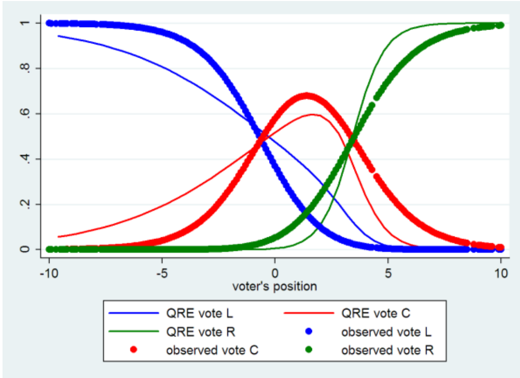
**QRE estimated on observed behavior**

Combining the data from the party choice and turnout decisions, we estimate the noise level that yields the quantal response equilibrium that best fits the observed data. We allow for different levels of noise in the party choice and turnout decisions. The reason for doing so is that given the relatively high rates of observed turnout, a model with a single noise parameter would not be able to explain party choice very well since the noise parameter needs to be very low (implying a lot of noise) to explain the turnout rates. This would conflict with our observation that party choice is not very noisy. Indeed, if we estimate a model with a single noise parameter we find an ML estimate of $\lambda$=3.3. This is very close to the noise level taken from Kamm (2012) with $\lambda$=3.7 that we used thus far. Splitting the noise levels, yields an estimate of the noise parameter in the turnout decision of $\lambda_T$=1.8. This is similar to the noise levels observed in other experiments on turnout where estimated noise levels vary between 1.25 and 2.5 depending on the subjects' experience (Goeree and Holt 2005; Grosser and Schram 2010). The ML parameter for party choice is estimated to be $\lambda_P$=8.2 in our data. Hence, we observe much more noise in the turnout decision than in the party choice. A likelihood ratio test reveals that the model with two distinct noise parameters significantly improves the fit (p-value<0.01).

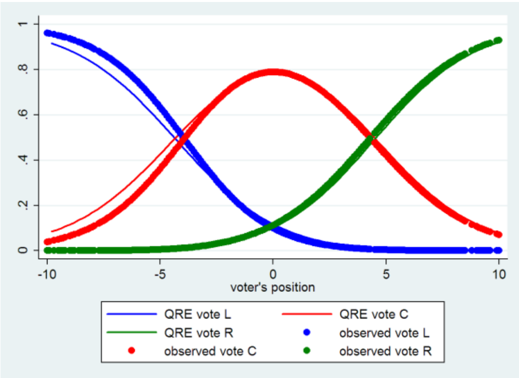
**Figure B3: New predicted compared to observed party choice**



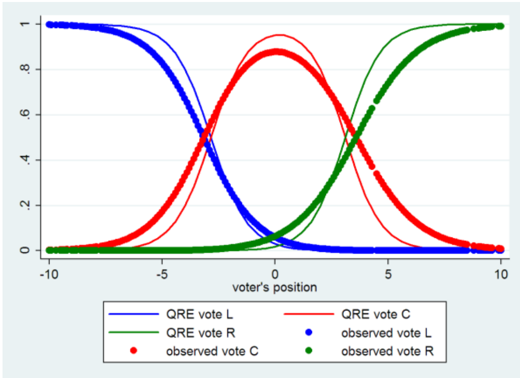

CentMand




CentVolu




ExtrMand



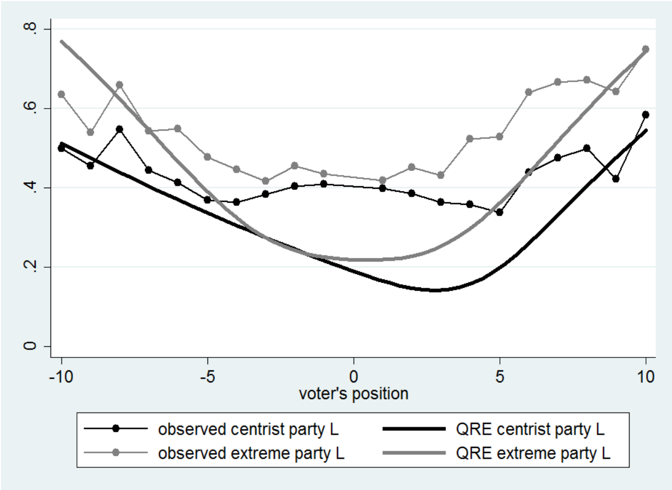

ExtrVolu

*Notes.* The thick (thin) lines show the estimated (predicted) probability of voting for each of the three parties as the voter's position varies along the horizontal axis. The predictions are based on the two-parameter QRE model estimated on observed behavior.

Figure B3 compares the new QRE predictions with the estimated party choice. The estimated party choice is obtained from a multinomial logit regression of party choice on voter position (cf. Appendix G). We find that with an extreme left-wing party the new predictions fit the observed party choice quite well. When the left-wing party is muted we again find that quantal response underestimates the probability of voting sincerely for the left-wing party. As conjectured in the main text, this may be attributed to the powerful heuristic of voting sincerely.

Finally, Figure B4 shows the revised QRE predictions for the turnout decision. While these new estimates naturally improve over the predictions using out-of-sample parameter estimates, it still is not able to capture the relatively moderate degree to which centrist voters vote less than extreme voters.

**Figure B4: Estimated turnout rates**




*Notes.* The figure compares the observed and predicted turnout rates in CentVolu and ExtrVolu as the voter's position varies along the x-axis. For the observed behavior data are averaged over +/-0.2 of the value on the horizontal-axis and the predictions are based on the two-parameter QRE model estimated on observed behavior.

## Appendix C: Strategic voting and the polarization effect

**Higher polarization under voluntary voting**

We use the equilibria derived using the methods described in Appendix B to isolate the polarization effect. First, we show that polarization is larger under voluntary voting than in the mandatory case. To compare polarization across these two environments, we use two measures. The first is simply the vote share of the central party. The second is the Dalton polarization index (e.g., Dalton 2021). Denoted by *D*, this is defined by:

$$D = \sqrt{\sum_{i=1}^{3} \gamma_i [(\zeta_i - \bar{\zeta})/10]^2} \text{ , with } \bar{\zeta} = \sum_{i=1}^{3} \gamma_i \frac{\zeta_i}{\sum_1^3 \zeta_i},$$

where $\gamma_i$ is party i's vote share, $\zeta_i$ denotes i's position in the policy space, and $\bar{\zeta}$ is the average position of parties, weighted by their vote shares. Note that *D* increases with polarization, is equal to 0 when all parties have the same ideal point and equal to 1 when half the votes are for a party at position -10 and the other half for a party at +10.

The values of these measures for the Nash equilibrium are shown in Figure 3 of the main text. Table C1 (left panel) shows the values of these two measures in the QRE introduced in the main text, with λ = 3.7.

**Table C1: Polarization**

| | **muted left-wing party** | | **extreme left-wing party** | |
|---|---|---|---|---|
| | Vote share C | Dalton index | Vote share C | Dalton index |
| mandatory | 0.368 | 0.363 | 0.451 | 0.363 |
| voluntary | 0.283 | 0.384 | 0.259 | 0.407 |

**Notes.** Cells show the vote share of the central party or the Dalton index in the QRE (with λ = 3.7) under mandatory and voluntary voting.

Both measures indicate higher polarization under voluntary voting than under mandatory voting. The effect is larger with an extreme left-wing party than when this party has a muted ideal point. The difference in polarization is the cumulative result of the two effects distinguished between in the main text, the polarization effect (voters who vote do so for more extreme parties) and the extremist effect (voters who support extreme parties are more likely to cast a vote). We first isolate the former.

**Measuring the polarization effect**

We consider the hypothetical situation where voters choose a party with the probabilities of the QRE for the voluntary voting case, but we assume that everybody casts a vote. This allows us to decompose the change in vote distribution between mandatory and voluntary voting in the two effects that we are interested in. The difference between the vote distribution in the equilibrium of the mandatory case with this hypothetical voluntary scenario is what we have called the polarization effect. As explained in the main text, this is caused by the changes in strategic voting that voluntary voting yields (see below). The difference in vote distribution between the hypothetical voluntary case and the voluntary voting environment that takes equilibrium turnout decisions into account is what we have dubbed the extremist effect. Table C2 summarizes the result of this analysis, using the same measures as in Table C1.

**Table C2: Isolating the polarization effect**

| | muted left-wing party | | extreme left-wing party | |
|---|---|---|---|---|
| | Vote share C | Dalton index | Vote share C | Dalton index |
| mandatory | 0.368 | 0.363 | 0.451 | 0.363 |
| polarization effect | –0.044 | 0.010 | –0.089 | 0.021 |
| hypothetical voluntary | 0.324 | 0.373 | 0.362 | 0.384 |
| extremism effect | –0.041 | 0.011 | –0.103 | 0.023 |
| voluntary | 0.283 | 0.384 | 0.259 | 0.407 |

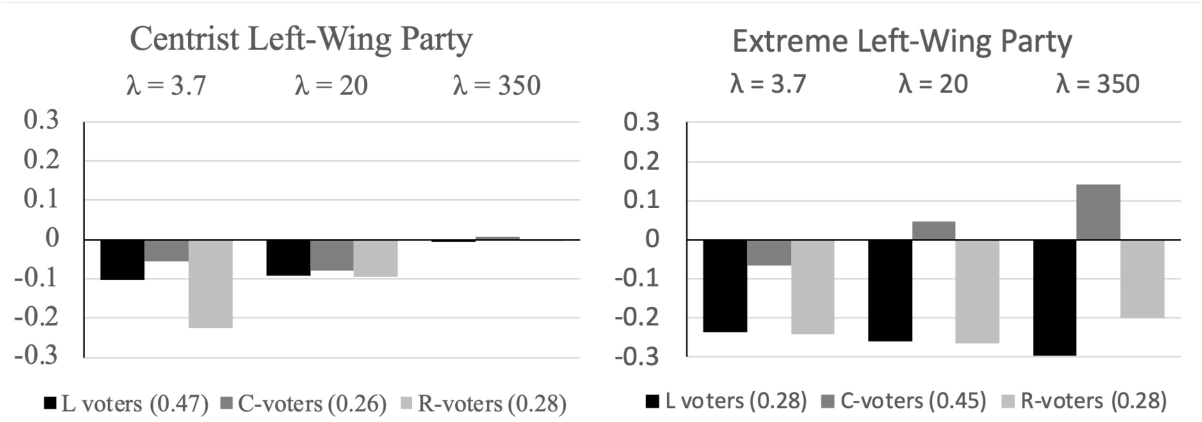
**Notes.** Cells show the vote share of the central party or the Dalton index in the QRE (with λ = 3.7) under mandatory and voluntary voting. The hypothetical voluntary row refers to the hypothetical situation where all voters cast a vote, but with the probabilities of party choice given by the equilibrium with endogenous turnout. The first and last rows of the table are the same as in Table C1.

The table reveals that in equilibrium, the two effects are of similar size. In our theoretical model, therefore, both the polarization effect and the extremism effect are predicted to hold. In Appendix D, we show that the occurrence of the polarization effect (Table D4) and the extremism effect (Figures D2 and D4) are robust to variations in λ. The only exception is that we do not observe the polarization effect in the Nash equilibrium of the environment with a muted left-wing party.

**Differences in strategic voting**

We argue in the main text that the cause of the polarization effect lies in lower strategic voting by supporters of the extreme parties when voting is voluntary. Figure C1 shows the difference in strategic voting, between the hypothetical voluntary environment defined above and the case where voting is mandatory.

**Figure C1: Effects on strategic voting**




**Notes**. Bars show the percentage point difference is strategic voting between the equilibrium for the hypothetical voluntary scenario described in the note to Table C2 and the equilibrium for mandatory voting. Negative numbers reflect more strategic voting under the mandatory rule. A vote is defined to be strategic if it is for a different party than the one closest to the voter on the policy space. L-/C-/R-supporters are defined as being closest to the L-/C-/R-parties. Fractions in the legend refer to the electorate composition across these three groups.

Consider first the QRE for λ = 3.7. For each of the three voter groups we observe less strategic behavior under (hypothetical) voluntary voting than when it is mandatory to cast a vote. The effect is stronger for voters with a sincere preference for L or R (which we dub 'L supporters' and 'R supporters') than for those with an ideal point closest to C ('C supporters'). Note that reduced strategic voting by L and R supporters increases polarization (because their strategic votes are overwhelmingly for C), while reduced strategic voting by C supporters reduces it. The observation that only 26% of the voters are C supporters, while the difference in strategic voting is lowest for them explains why we observe the polarization effect in the QRE. This holds for both positions of the left-wing party.

The same conclusion can be drawn for $\lambda = 20$. For $\lambda = 350$ (which is very close to the Nash equilibrium, as discussed in Appendix A), the conclusion is the same when the left-wing party has an extreme ideal point, but not in the muted case. In the latter environment there is little difference in strategic voting between the two cases.[2] This confirms the conclusion presented above (and illustrated in Table D4 of Appendix D) that the Nash equilibrium does not predict a polarization effect.

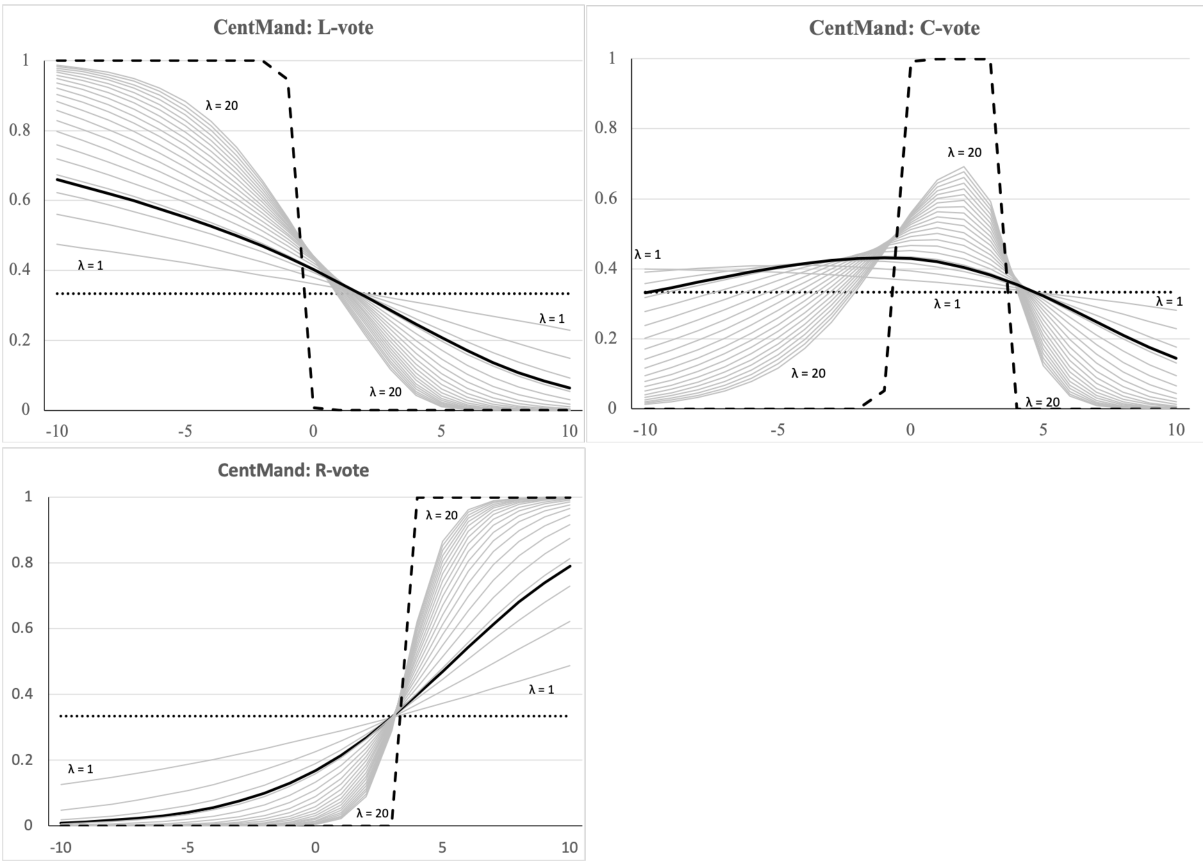
[2] This can be attributed to very low levels of strategic voting in the first place.

**Appendix D: Robustness checks**

**D.1 Sensitivity to the noise parameter**

In this appendix, we investigate the sensitivity of our logit equilibria to variations in the QRE noise parameter λ. We start with the treatment where the left-wing party's position is muted and voting is mandatory. Figure D1 shows the equilibrium voting probabilities for discrete values of λ between 0 (only noise) and 20. It also includes the values λ = 3.7 (chosen for our predictions) and λ = 350 (as an approximation of the Nash equilibrium).[3]

**Figure D1: Quantal Response Equilibria, CentMand**

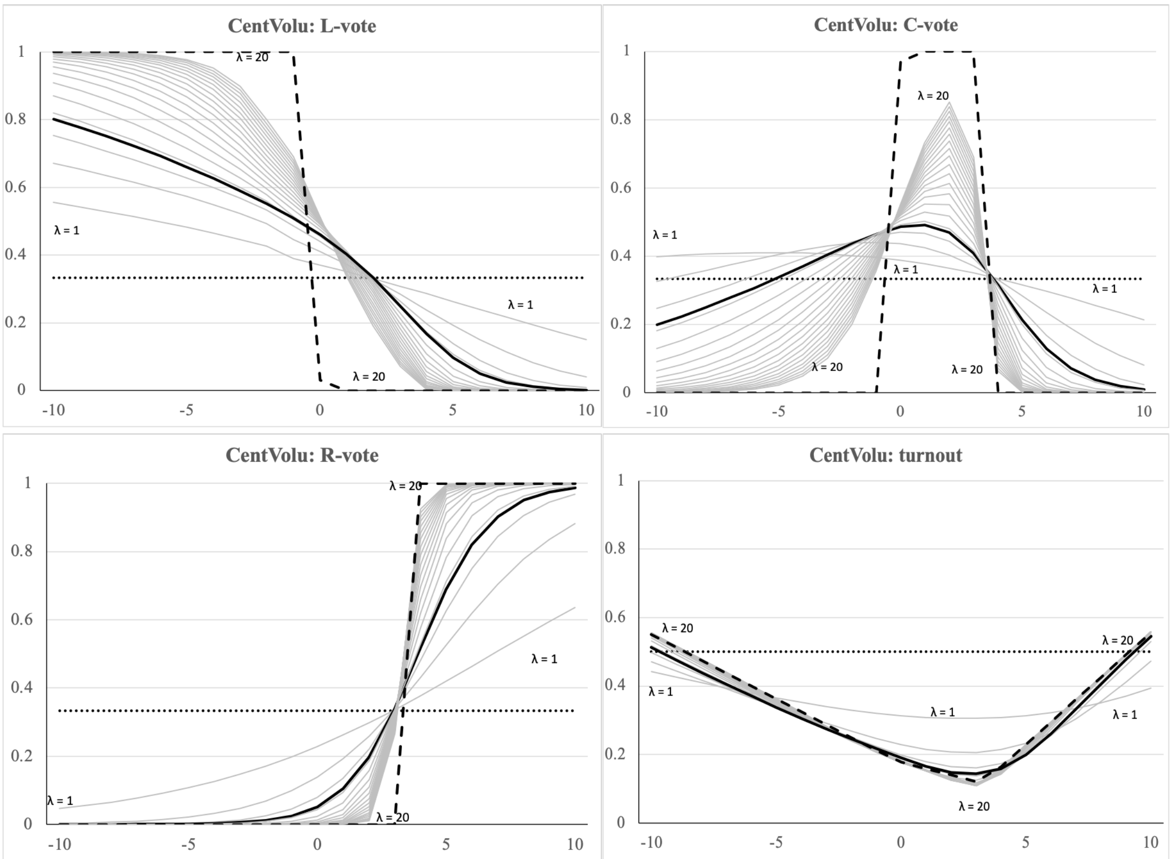



**Notes.** The graphs show for the treatment with mandatory voting and a muted left-wing party the QRE probabilities of voting for parties L (top left), C (top right) and R (bottom left). The QRE were calculated for all discrete positions between –10 and 10. The horizontal dotted line represents random voting (each party with probability 0.33), which corresponds to λ = 0. The dashed line corresponds to λ = 350, which approaches the Nash equilibrium, and the solid black line corresponds to λ = 3.7 as used for our QRE predictions. Grey lines represent QRE with λ varying between 1 and 20, in steps of 1.

The graphs show a smooth convergence from random behavior to the Nash equilibrium as λ increases. A similar convergence is observed in the other treatments (Figures D2-D4).

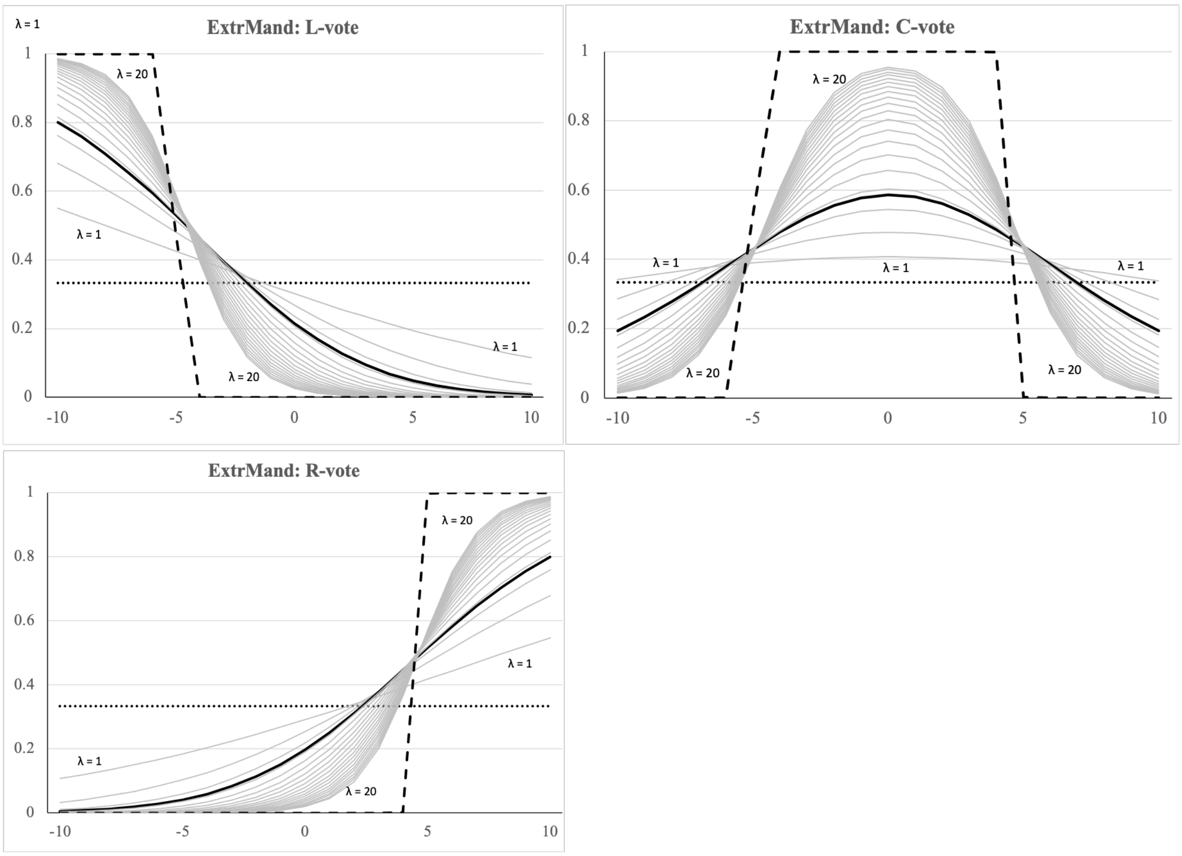
[3] For larger values than λ = 350, we ran into computational problems for some voter positions. We show in Appendix A that this approximation of the Nash equilibrium by using λ =350 is very accurate.

**Figure D2: Quantal Response Equilibria, CentVolu**



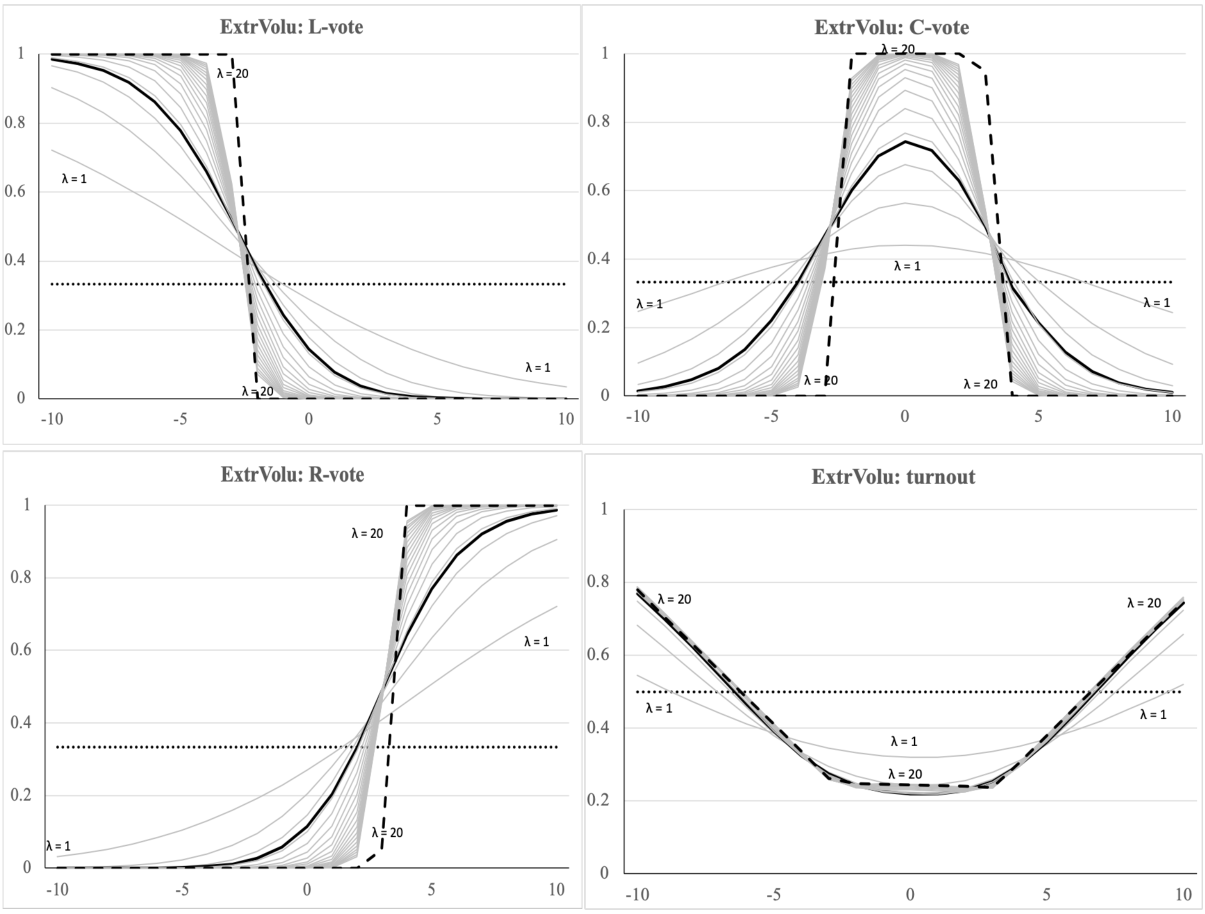

**Notes.** The graphs show for the treatment with voluntary voting and a muted left-wing party the QRE probabilities of voting for parties L (top left), C (top right) and R (bottom left), and the QRE probability of turning up to vote (bottom right). The QRE were calculated for all discrete positions between –10 and 10. The horizontal dotted line represents random voting (each party with probability 0.33; turnout with probability 0.5), which corresponds to λ = 0. The dashed line corresponds to λ = 350, which approaches the Nash equilibrium, and the solid black line corresponds to λ = 3.7 as used for our QRE predictions. Grey lines represent QRE with λ varying between 1 and 20, in steps of 1.

**Figure D3: Quantal Response Equilibria, ExtrMand**




**Notes.** The graphs show for the treatment with mandatory voting and an extreme left-wing party the QRE probabilities of voting for parties L (top left), C (top right) and R (bottom left). The QRE were calculated for all discrete positions between –10 and 10. The horizontal dotted line represents random voting (each party with probability 0.33), which corresponds to λ = 0. The dashed line corresponds to λ = 350, which approaches the Nash equilibrium, and the solid black line corresponds to λ = 3.7 as used for our QRE predictions. Grey lines represent QRE with λ varying between 1 and 20, in steps of 1.

**Figure D4: Quantal Response Equilibria, ExtrVolu**




**Notes.** The graphs show for the treatment with voluntary voting and an extreme left-wing party the QRE probabilities of voting for parties L (top left), C (top right) and R (bottom left), and the QRE probability of turning up to vote (bottom right). The QRE were calculated for all discrete positions between –10 and 10. The horizontal dotted line represents random voting (each party with probability 0.33; turnout with probabilty 0.5), which corresponds to λ = 0. The dashed line corresponds to λ = 350, which approaches the Nash equilibrium, and the solid black line corresponds to λ = 3.7 as used for our QRE predictions. Grey lines represent QRE with λ varying between 1 and 20, in steps of 1.

## D.2 Robustness of the comparative static effects

### Polarization effect

To investigate the robustness of the polarization effect, we repeat the analysis of Appendix C for λ = 20 and λ = 350. As explained in Appendix C, we measure the polarization effect in two ways, to wit the vote share of the central party and the Dalton index. We compare these measures between the cases of mandatory and voluntary voting, the latter under the hypothetical case that everyone votes (with the party choice probabilities determined by the equilibrium with endogenous turnout). Table D1 shows these measures of the polarization effect for the cases under consideration.

**Table D1: Polarization Effect**

| | | muted left-wing party | | | extreme left-wing party | | |
|---|---|---|---|---|---|---|---|
| | | λ = 3.7 | λ = 20 | λ = 350 | λ = 3.7 | λ = 20 | λ = 350 |
| Vote share C | mandatory | 0.368 | 0.278 | 0.261 | 0.451 | 0.546 | 0.585 |
| | polarization effect | –0.044 | –0.045 | –0.005 | –0.089 | –0.165 | –0.199 |
| | hypothetical voluntary | 0.324 | 0.233 | 0.256 | 0.362 | 0.381 | 0.386 |
| Dalton index | mandatory | 0.363 | 0.380 | 0.383 | 0.363 | 0.359 | 0.348 |
| | polarization effect | 0.010 | 0.010 | 0.000 | 0.021 | 0.060 | 0.068 |
| | hypothetical voluntary | 0.373 | 0.390 | 0.383 | 0.384 | 0.419 | 0.416 |

**Notes.** Cells show the vote share of the central party or the Dalton index under mandatory and hypothetical voluntary voting. The latter considers the hypothetical situation where all voters cast a vote, but with the probabilities of party choice given by the equilibrium with endogenous turnout.

The results show that our theoretical model predicts a polarization based on both measures and all λ values except λ = 350 in the treatment with a muted left-wing party. This is consistent with the observation that for this treatment no polarization effect is observed in the Nash equilibrium. Remarkably, we do observe a polarization effect for λ = 20, which already gives a very low noise level. We conclude that the polarization effect is broadly robust to such changes in our equilibrium concept, except for the limit case of a Nash equilibrium.

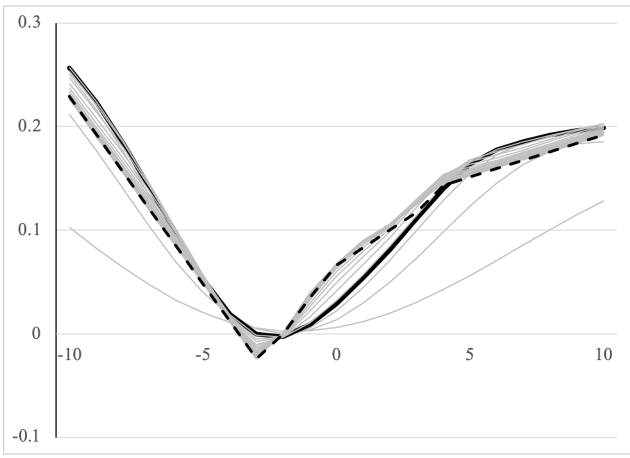
**Extremist effect**

It is clear from the fourth panels in Figures D2 and D4 that for all λ-values investigated and for both positions of the left-wing party, turnout is higher amongst voters towards the extremes of the policy space than for voters closer to the center. The same holds for the Nash equilibria depicted in Figure 3 of the main text. The effect that we observe for λ = 3.7 is therefore robust to variation in the equilibrium concept.

**Turnout effect**

To investigate the robustness of the turnout effect, we consider for each voter position the difference between equilibrium turnout with an extreme left-wing party and that with a muted left-wing party. Figure D5 shows this for the same parameter values as in Figures D1-D4.

**Figure D5: Robustness of the Turnout Effect**



**Notes.** The graph shows for each voter position the difference between the QRE turnout probabily when the left-wing party has position –7.0 and when it is at position –1.5. The QRE were calculated for all discrete positions between –10 and 10. The horizontal axis corresponds to $\lambda = 0$, where turnout is random in both treatments. The dashed line corresponds to $\lambda = 350$ –which approaches the difference between the Nash equilibria depicted in Figure 3 of the main text– and the solid black line corresponds to $\lambda = 3.7$ as used for our QRE predictions. Grey lines represent the cross-treatment differences in QRE with $\lambda$ varying between 1 and 20, in steps of 1.

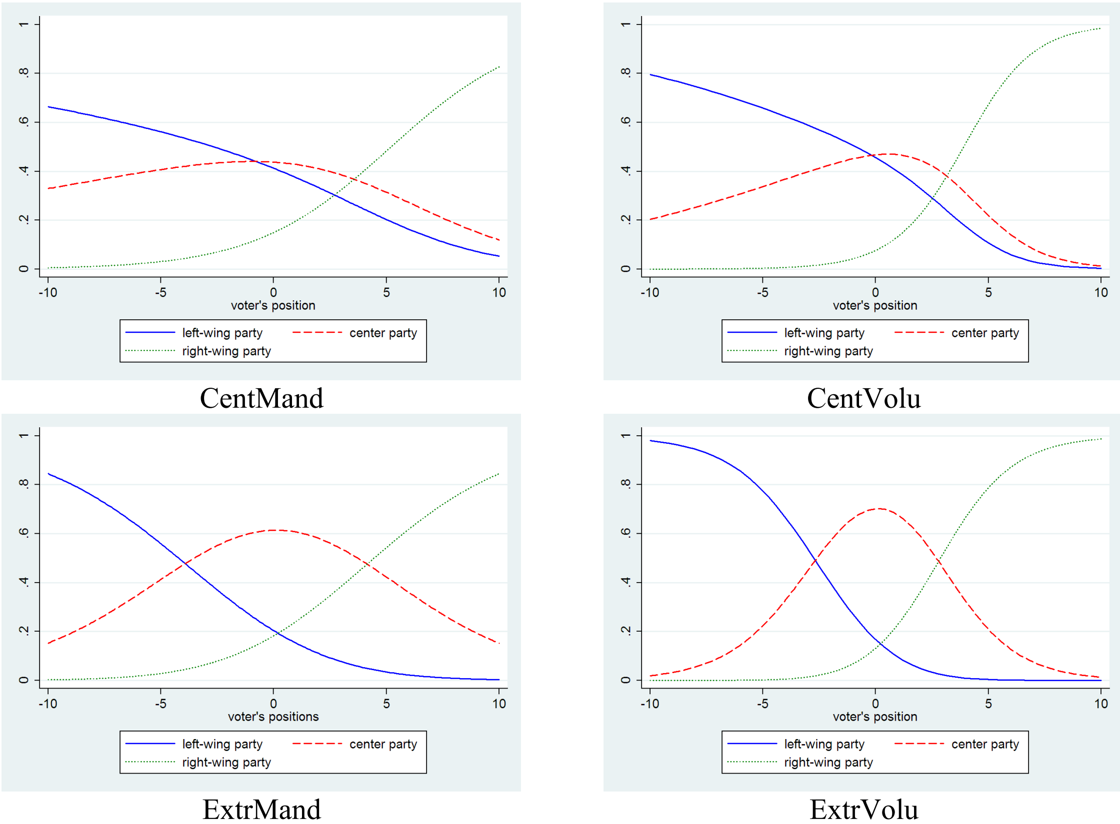
Positive values of the difference depicted in Figure D5 indicate that voters at that position turnout to vote at a lower rate with a muted left-wing party than when this party is more extreme. The figure shows that for all values of $\lambda$ and almost all voter positions, turnout is higher in the latter case. Note that this also holds for the Nash equilibrium (cf. Figure 3 in the main text). The marginally lower turnout for positions –2 and –3 is negligible in comparison to the much larger differences at other positions. This is evidence of the robustness of the Turnout Effect.

**Appendix E: Analysis for a uniform distribution of voters**

**Equilibria with a uniform specification**

As a robustness check we analyze the QRE model assuming that the voters' positions are distributed uniformly along the policy space.

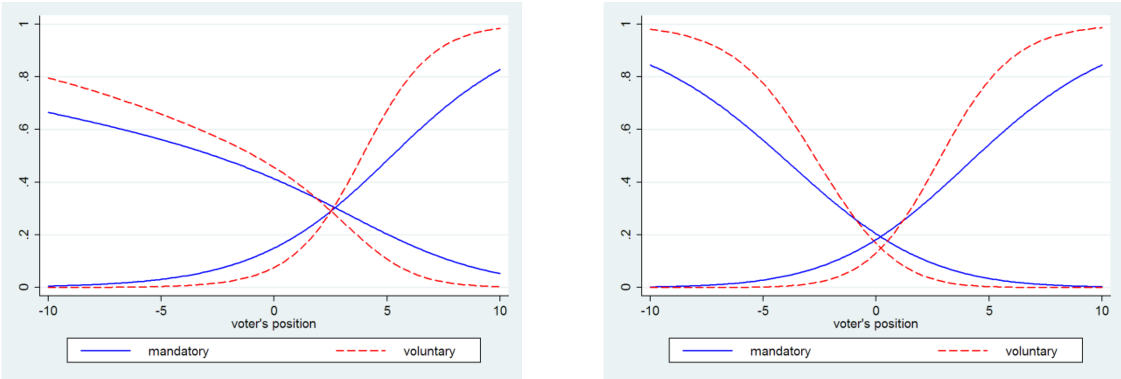
**Figure E1: Predicted party choice**




*Notes.* The figure shows the predicted probability of voting for each of the three parties as the voters position varies along the horizontal axis. The predictions are based on the QRE model with λ=3.7 and a uniform distribution of voter's positions.

Figure E1 shows the QRE predictions for party choice in this model. Note the close resemblance to the QRE predictions with a t-distribution of voter preferences (cf. Appendix B). A consequence of this resemblance is that both specifications predict the same interaction effects. For example, Figure E2 investigates the interaction between the turnout regime and party choice in the uniform distribution case by comparing the predictions of the treatment with mandatory voting to the predictions with voluntary voting. It shows that the probability of voting for an extreme party is higher when voting is voluntary than when it is mandatory. Therefore, the 'Polarization Effect' is also observed when we assume that voters' ideal points are uniformly distributed.

**Figure E2: Predicted party choice (voluntary versus mandatory voting)**

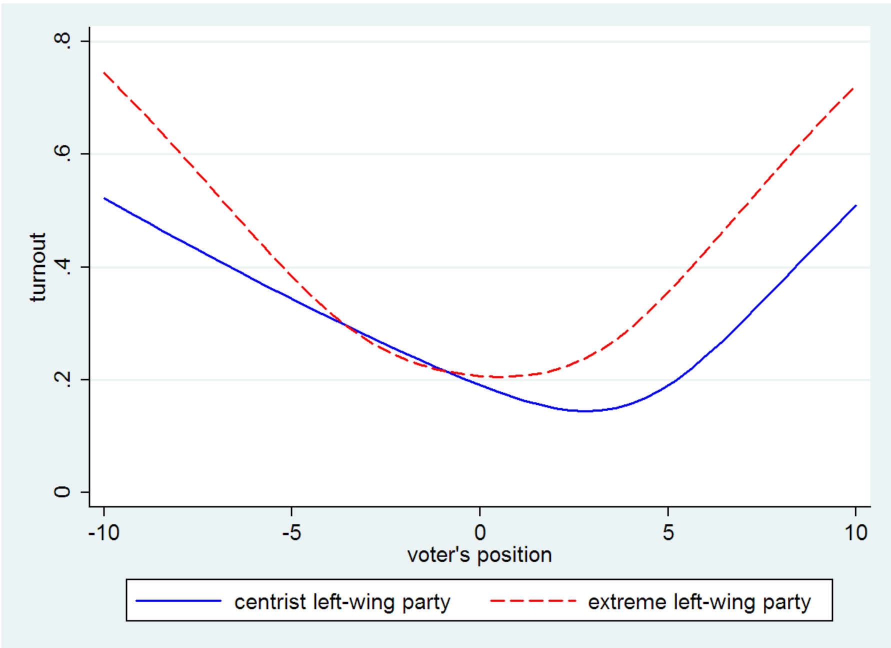



Muted left-wing party | Extreme left-wing party

*Notes.* The figure compares the predicted probability of voting for each of the extreme parties (conditional on voting) between compulsory and voluntary voting as the voter's position varies along the horizontal axis. The predictions are based on the QRE model with λ=3.7 and a uniform distribution of voter's positions.

To replicate the other effects, Figure E3 compares the equilibrium turnout for the two levels of party polarization used in the experiment. The horizontal axis shows the voter's positions and the vertical axis depicts the predicted turnout rates.

**Figure E3: Predicted turnout rates**




*Notes.* The figure shows the predicted turnout rates for CentVolu and ExtrVolu as the voter's position varies along the horizontal axis. The predictions are based on the QRE model with λ=3.7 and a uniform distribution of voter's positions.

This figure shows that the 'Turnout Effect' and the 'Extremist Effect' are also present when assuming a uniform distribution of policy positions.

**Appendix F: Instructions and screenshots of the experiment**

In this appendix, we provide the instructions that the subjects read on their monitors. We also give the summary of the instructions that was handed out to subjects after they had read these on-screen instructions. Finally, we provide screenshots of the user interface of the experiment.

**F.1 Instructions[4]**

Welcome to this experiment on decision-making. Please carefully read the following instructions. If you have any questions, please raise your hand, and we will come to your table to answer your question in private.

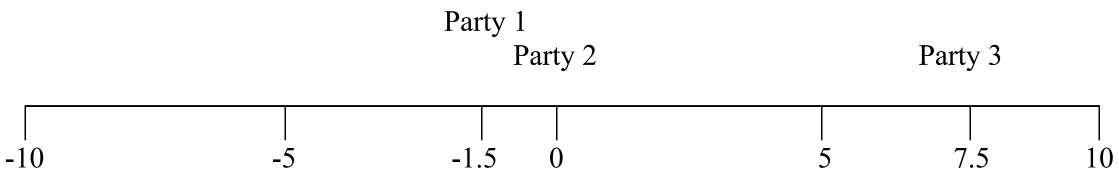
In this experiment you will earn points. At the end of the experiment, your earnings in points will be exchanged for money at the rate 1 eurocent for each point. This means that for each 100 points you earn, you will receive 1 euro. Additionally, you will receive a show-up fee of 7 euros. Your earnings will be privately paid to you in cash at the end of the experiment.

This experiment will consist of 30 elections. In each election you will be one of five voters in the electorate that is electing a new government. Your earnings will be based on the outcome of these elections. The rest of these instructions will explain exactly how the experiment works.

---

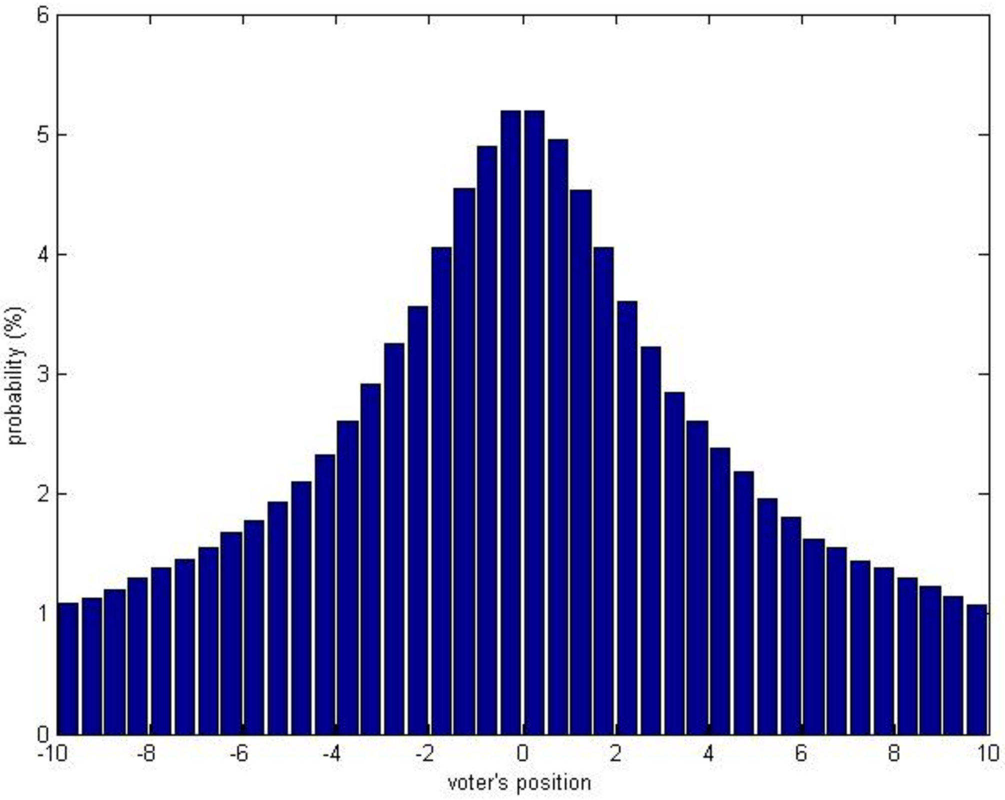
**Parties and Voters**

Three parties participate in the elections. Each party is described by a number between -10 and 10, which signifies their policy position. They will keep the same policy position throughout the experiment. You can see where they are located on the graph below (you will also find this graph on the handout) and on your monitor during the experiment.




Every voter is also described by a position on the line from -10 to 10. This position corresponds to the policy that the voter would prefer to see implemented (this is her or his favorite policy). The five voters are randomly distributed over the policy space according to the distribution shown in the graph below. The height of a bar signifies how likely it is that a value occurs. In the experiment we will round the voter positions to one decimal point.

[4] We provide here the instructions used for the treatment centrist-voluntary. The instructions for other treatments are analogous and available upon request.




As you can see a position of 0 is most likely. The probability that the position is very close to zero (between -0.5 and 0.5) is 10%. This means that in (about) 10 of 100 cases the position will be in this interval.

Furthermore, you can note that the distribution is symmetric around zero and therefore it is equally likely to be to the left and to the right of zero.

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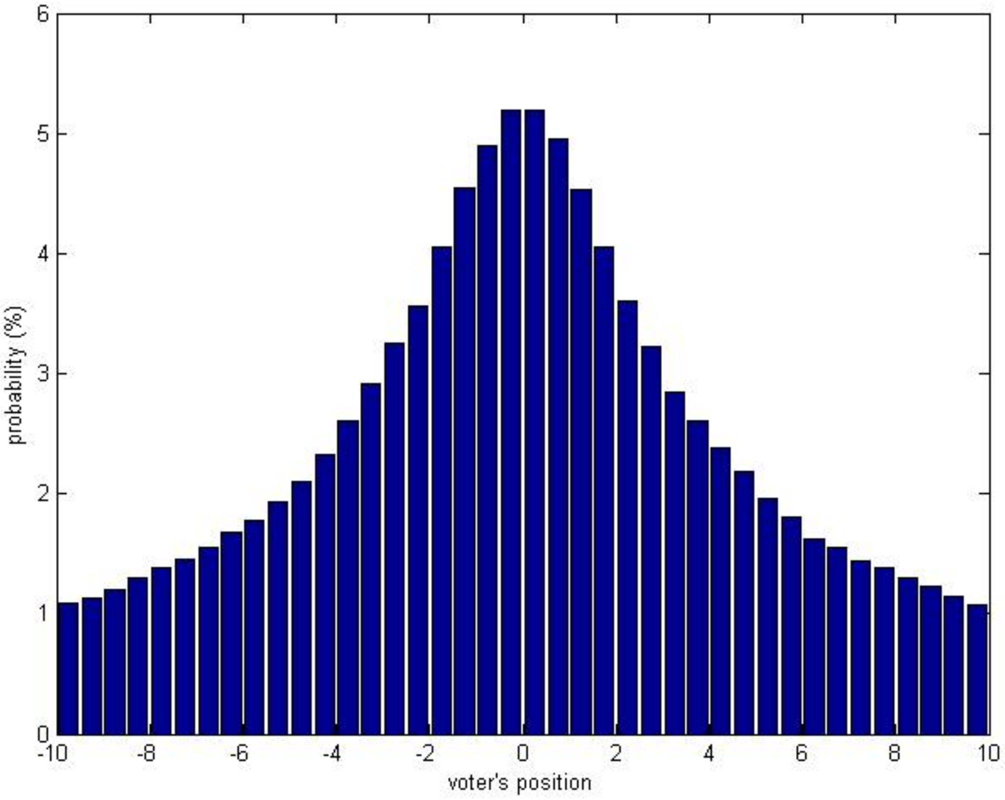



What follows are some further illustrating examples of the shape of the distribution

In about 45% of the cases (45 out of 100) a voter's position will be between -2.5 and 2.5.

In about 71% of the cases (71 out of 100) a voter's position will be between -5 and 5.

In about 96% of the cases (96 out of 100) a voter's position will be between -9 and 9.

In about 22% of the cases (22 out of 100) a voter's position will be between 2.5 and 7.5 and with the same probability s/he will be between -7.5 and -2.5.

In about 15% of the cases (15 out of 100) a voter's position will be between 5 and 10 and with the same probability s/he will be between -10 and -5.

If you want to know how likely it is that a voter's position is in a given interval you can use the tool below (you will also be able to use this tool during the experiment).

**position is between** [ ] **and** [ ] Submit

For each voter, a new position will be drawn after every period and your position in the next period is completely independent of your position in the current period. You will always know your own position before making a decision but not the position of the other voters in your electorate.

## Government formation

In each electorate (i.e. group of voters that form an election) there will be five voters. You will be one of them and the other four voters are some of the other subjects in the lab. The identity of the four other subjects will be randomly determined in each of the 30 periods. Hence, you are in a new electorate in each of the thirty rounds.

In each period you will have to decide whether you want to vote, and if so, for which of the three parties you want to cast your vote. If you decide to vote you have to incur costs of voting which in every period are an integer randomly drawn from the interval -15 and 200. Every integer in this interval is equally likely to be drawn. You know your own costs of voting before making your decision, but only the distribution of the costs of voting for the other voters in your electorate. Note that there is a small chance that your costs are negative in a round. If this occurs, you will receive extra points if you vote.

The votes by the members of your electorate determine which government will be formed. When forming a government the following rules will be applied:

1. If a party receives an absolute majority (more than half) of the votes this party will form a single party government.

2. If no party receives an absolute majority, the party with the most votes forms a coalition with one of the other two parties. Which coalition will be formed for the different possible configurations of votes can be seen in the table below (you can also find this table on the handout). Coalitions are determined by assuming that the party that is forming the coalition tries to end up with a policy that is as close as possible to its own policy position.

3. If in 2. there are multiple parties with the most votes it is randomly determine which party forms the coalition.

(p.t.o)

**Coalition Formation**

| Votes for party 1 | Votes for party 2 | Votes for party 3 | Formed coalition | Implemented policy |
|---|---|---|---|---|
| 2 | 2 | 1 | parties 1 and 2 | -0. 8 |
| 2 | 1 | 2 | parties 1 and 2<br>OR (determined by coin toss)<br>parties 2 and 3 | -1.0<br><br>5.0 |
| 1 | 2 | 2 | parties 1 and 2<br>OR (determined by coin toss)<br>parties 1 and 3 | -0.5<br><br>4.5 |
| 2 | 1 | 1 | parties 1 and 2 | -1.0 |
| 1 | 2 | 1 | parties 1 and 2 | -0.5 |
| 1 | 1 | 2 | parties 2 and 3 | 5.0 |
| 2 | 2 | 0 | parties 1 and 2 | -0.8 |
| 2 | 0 | 2 | parties 1 and 3 | 3.0 |
| 0 | 2 | 2 | parties 2 and 3 | 3.8 |
| 1 | 1 | 1 | parties 1 and 2<br>OR (determined by throwing a dice; if it shows a 5 or 6 parties 2 and 3 form the coalition)<br>parties 2 and 3 | -0.8<br><br>3.8 |
| 1 | 1 | 0 | parties 1 and 2 | -0.8 |
| 1 | 0 | 1 | parties 1 and 3 | 3.0 |
| 0 | 1 | 1 | parties 2 and 3 | 3.8 |
| | 0 | 0 | party 1<br>OR (all with equal probability)<br>party 2<br>OR (all with equal probability)<br>party 3 | -1.5<br><br>0<br><br>7.5 |

Based on the government that is formed, a policy will be implemented. If there is a single party government the implemented policy is equal to this party's policy position. If there is a coalition the implemented policy is the weighted (by votes) average of the positions of the parties in the coalition. If for instance party 3 receives two votes and forms a coalition with party 2 which received one vote than the policy position of party 3 (7.5) receives weight 2/3 and the policy position of party 2 (0) receives weight 1/3. The implemented policy is then 5.0 (=1⁄3*0+2⁄3*7.5). See also the table on the handout for the policy implemented by any possible coalition.

---

Your earnings in points in a period are computed using the following formula:

$$160 - 2 * (\text{implemented policy - favorite policy})^2 - \text{costs of voting}$$

As mentioned before, the costs of voting are an integer between -15 and 200. Every integer in this interval is equally likely to be drawn and you will have a new draw in every period. You only pay (or receive) the costs of voting if you decide to vote in a period. The second term in the formula shows that your earnings are decreasing in the squared difference between your favorite policy (i.e. position) and the implemented policy. As a consequence, your earnings are higher the smaller is the distance between the implemented policy and your favorite policy. Below you can test what your earnings are for different configurations of your own position, your costs of voting and the government elected.

Assume that the following government
Forms

Position | costs of voting

At the end of the experiment the earnings from all periods will be added up and per 100 points, you will receive 1 euro. These earnings will be paid to you privately and confidentially.

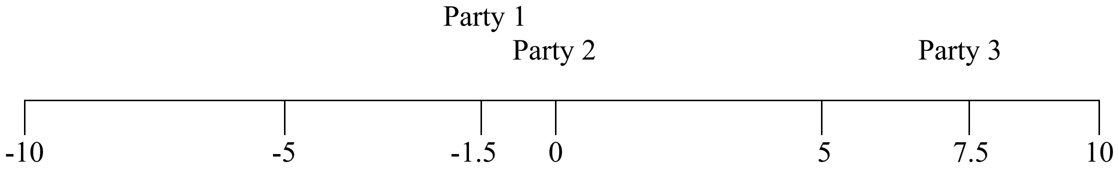
On the next screen you will be requested to answer some control questions to make sure that you have understood these instructions.[5] Please answer these questions now.

[5] The control questions are presented in Appendix E4.

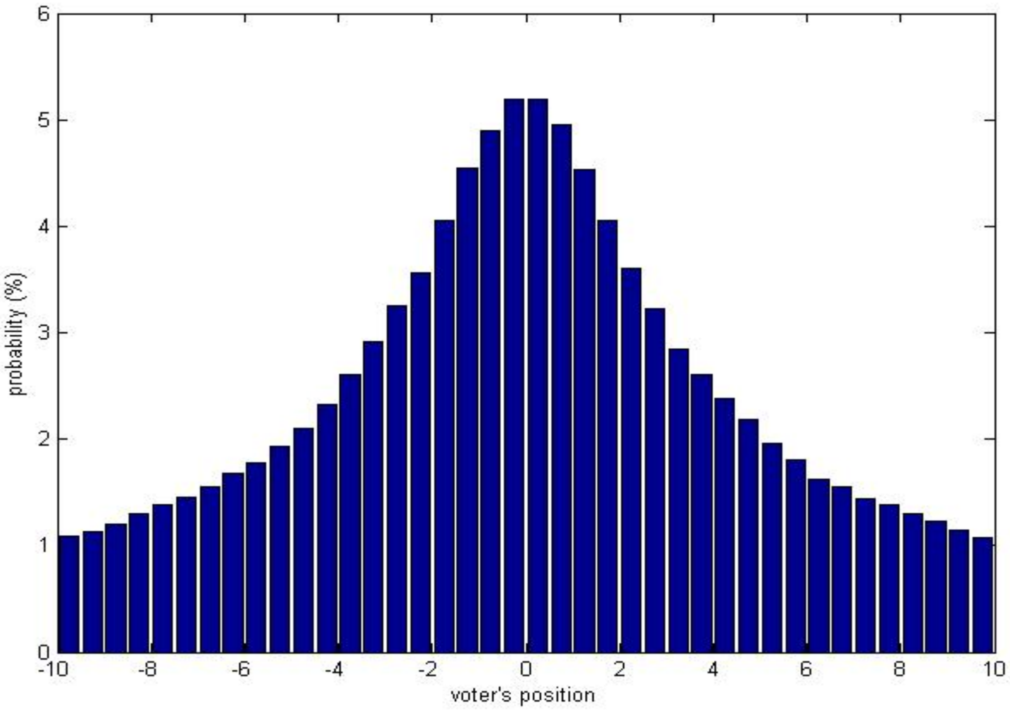
**F.2 Printed summary of instructions**

**Summary Instructions**

- Each electorate consists of five voters
- In each period you will be randomly rematched
- In each of the 30 periods you have to decide whether you want to vote and, if yes, for which party
- The three parties are described by policy positions as shown below




- Every voter is described by a position on the line from -10 to 10. The voters are randomly distributed over the policy space according to the distribution shown in the graph below.




- In about 10% of the cases (10 out of 100) a voter's position will be between -0.5 and 0.5
- In about 45% of the cases (45 out of 100) a voter's position will be between -2.5 and 2.5
- In about 71% of the cases (71 out of 100) a voter's position will be between -5 and 5
- In about 96% of the cases (96 out of 100) a voter's position will be between -9 and 9

---

- Based on the votes a government will be formed
  - If a party receives an absolute majority of the votes this party will form a single party government.
  - Otherwise a coalition will be formed according to the table below

- The implemented policy is the position of the party in government; if there is a coalition it is the vote weighted average of the positions of the members of this coalition

| Votes for party 1 | Votes for party 2 | Votes for party 3 | Formed coalition | Implemented policy |
|---|---|---|---|---|
| 2 | 2 | 1 | parties 1 and 2 | -0. 8 |
| 2 | 1 | 2 | parties 1 and 2<br>OR (determined by coin toss)<br>parties 2 and 3 | -1.0<br><br>5.0 |
| 1 | 2 | 2 | parties 1 and 2<br>OR (determined by coin toss)<br>parties 1 and 3 | -0.5<br><br>4.5 |
| 2 | 1 | 1 | parties 1 and 2 | -1.0 |
| 1 | 2 | 1 | parties 1 and 2 | -0.5 |
| 1 | 1 | 2 | parties 2 and 3 | 5.0 |
| 2 | 2 | 0 | parties 1 and 2 | -0.8 |
| 2 | 0 | 2 | parties 1 and 3 | 3.0 |
| 0 | 2 | 2 | parties 2 and 3 | 3.8 |
| 1 | 1 | 1 | parties 1 and 2<br>OR (determined by throwing a dice; if it shows a 5 or 6 parties 2 and 3 form the coalition)<br>parties 2 and 3 | -0.8<br><br>3.8 |
| 1 | 1 | 0 | parties 1 and 2 | -0.8 |
| 1 | 0 | 1 | parties 1 and 3 | 3.0 |
| 0 | 1 | 1 | parties 2 and 3 | 3.8 |
| 0 | 0 | 0 | party 1<br>OR (all with equal probability)<br>party 2<br>OR (all with equal probability)<br>party 3 | -1.5<br><br>0<br><br>7.5 |

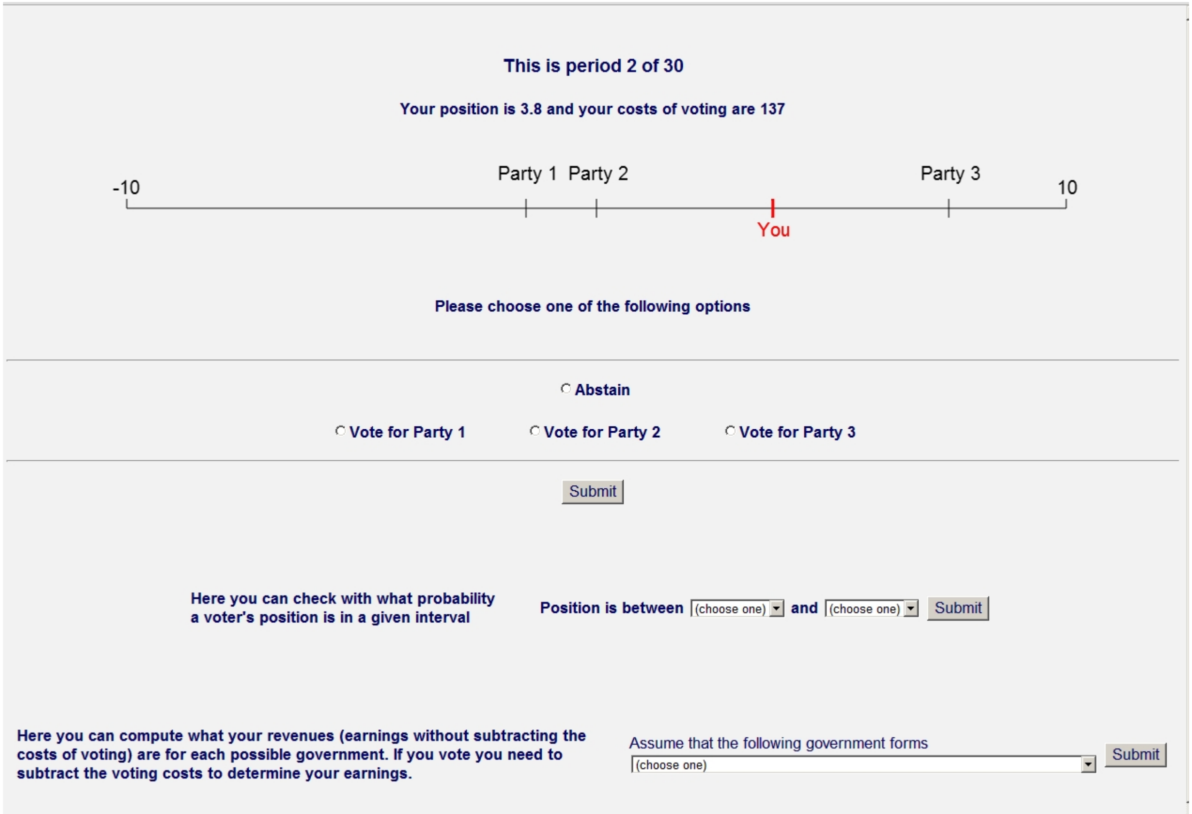
- Your payoff per round is

**160 – 2*(implemented policy - favorite policy)$^2$ - costs of voting**

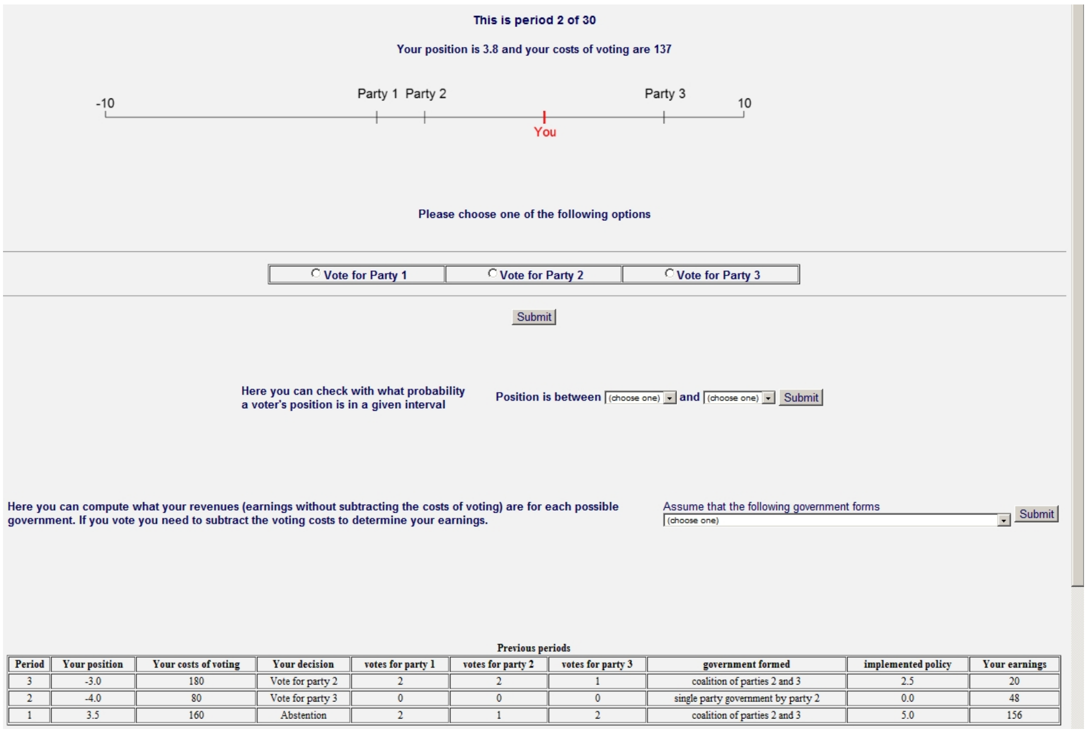
  - The cost of voting are an integer number in the interval between -15 and 200. Every integer in this interval is equally likely to be drawn.
  - You only have to pay the costs of voting in periods where you decide to vote

Your final payoff is 1 Euro for every 100 points plus a show-up fee of 7 Euros.

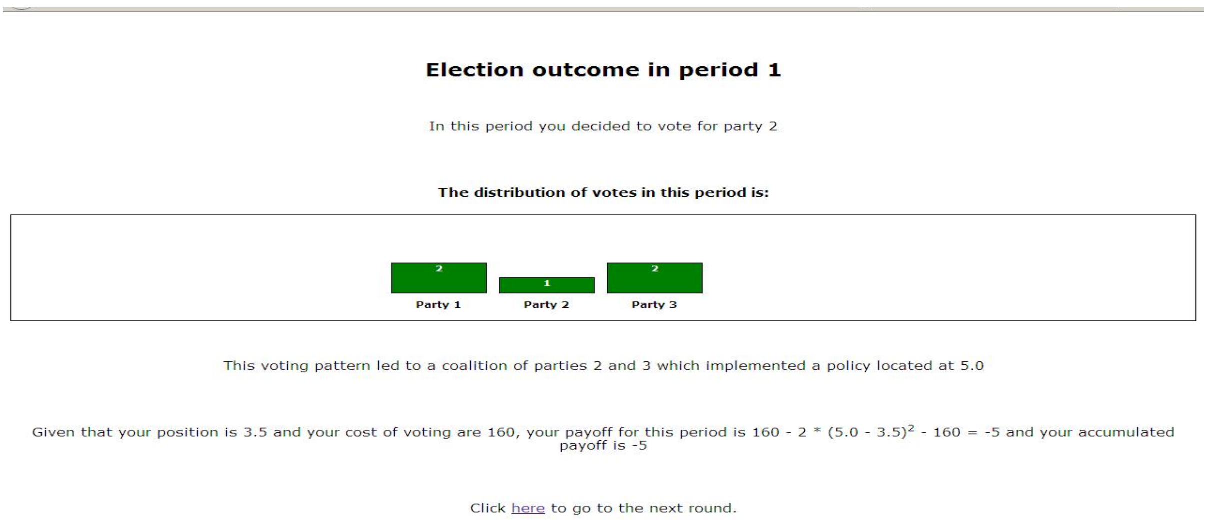
## F.3 Screenshots of the interface




*Notes.* The screen subjects saw when making a decision in the muted-voluntary treatment (in the mandatory treatment the button "abstain" is missing).




*Notes.* The screen subjects saw when making a decision in the mandatory treatment; the table at the bottom of the screen shows an example of the history box.




*Notes.* The screen subjects saw after an election was over.

### F.4 Comprehension Quiz

Participants need to answer all of the following questions correctly, before proceeding to the experiment.

**Control questions**

Before the experiment starts, please answer some questions on this page.

1. Is the following statement correct?

My position does not give me any information about the position of the other voters in the electorate.
- o Yes
- o No

2. How likely is it (in %) that a voter is located between -2.5 and 2.5?
Hint: Have a look at the handout

3. Assume that parties 1 and 2 receive two votes each and party 3 receives one vote. Which of the options below is a government that might form as a result?
- o A coalition of parties 1 and 3
- o A coalition of parties 1 and 2
- o A single party government by party 1
- o A grand coalition of all three parties

4. Assume that parties 1 and 3 receive two votes each and party 2 receives one vote. Which of the options below is not a government that might form as a result?
- o A coalition of parties 1 and 3
- o A coalition of parties 1 and 2
- o A coalition of parties 2 and 3
- o All options are possible

5. Assume as in the previous question that parties 1 and 3 receive two votes each and party 2 receives one vote. Furthermore, assume that parties 2 and 3 form a coalition. What policy (as an integer) will this government implement?

6. Assume that in the current period you are in the same electorate as subjects 3, 7, 9 and 10. Will you be in the same electorate as subject 7 in the next round?
- o Yes
- o No
- o Impossible to say

7. Assume that your position is 4.0 and your costs of voting are 50. Furthermore, assume that the other subjects in your electorate gave one vote to party 1, one vote to party 2 and two votes to party 3. Which policy (to one decimal point) will be implemented if you vote for party 3?

8. Staying with the situation of the previous question: What would your earnings be (to one decimal point) if you vote for party 3?

9. Staying with the situation of the previous question: If you voted for party 2, you would on average (since you do not know which coalition would be formed) earn 89.5. Given this information, would you earn more if you voted for party 2 or for party 3?
- o Party 2
- o Party 3

10. Staying with the situation in the previous three questions: What would you earn if you did not vote at all?

11. Assume that your position is -3.4 and your costs of voting are 100. Furthermore, assume that the other subjects in your electorate gave two votes to party 1, one vote to party 2 and one to party 3. What would your earnings be (to one decimal point) if you vote for party 1?

12. If you would vote for party 2, you would receive earnings of 46.0. Given this information, would you expect to earn more if you voted for party 1 or party 2?

- Party 1
- Party 2

**Appendix G: Multinomial logit estimates**

Below we report the estimation results underlying the logit choice functions for the party choice as depicted in Figure 6 of the main text. The variable "Voter's Position" measures a voter's position in the policy space. Standard errors are clustered at the matching group level.

**Multinomial Logit Results, CentMand**

| Constant and Independent Variables | Coefficients | | |
|---|---|---|---|
| | Vote for left-wing party | Vote for center party | Vote for right-wing party |
| Constant | –0.48*** | Base outcome | –2.17*** |
| | (0.131) | | (0.541) |
| Voter's position | –0.64*** | | 0.57*** |
| | (0.183) | | (0.139) |

*Notes.* Multinomial logit estimates for the party choice decision in treatment CentMand. Standard errors are clustered at the matching group level. *(**; ***) indicates significance at the 10% (5%; 1%) level.

**Multinomial Logit Results, ExtrMand**

| Constant and Independent Variables | Coefficients | | |
|---|---|---|---|
| | Vote for left-wing party | Vote for center party | Vote for right-wing party |
| Constant | –2.07*** | Base outcome | –1.95*** |
| | (0.519) | | (0.610) |
| Voter's position | –0.53*** | | 0.45*** |
| | (0.158) | | (0.181) |

*Notes.* Multinomial logit estimates for the party choice decision in treatment ExtrMand. Standard errors are clustered at the matching group level. *(**; ***) indicates significance at the 10% (5%; 1%) level.

**Multinomial Logit Results, CentVolu**

| Constant and Independent Variables | Coefficients | | |
|---|---|---|---|
| | Vote for left-wing party | Vote for center party | Vote for right-wing party |
| Constant | –0.45* | Base outcome | –2.40*** |
| | (0.249) | | (0.743) |
| Voter's position | –0.72*** | | 0.70*** |
| | (0.243) | | (0.195) |

*Notes.* Multinomial logit estimates for the party choice decision in treatment CentVolu. Standard errors are clustered at the matching group level. *(**; ***) indicates significance at the 10% (5%; 1%) level.

**Multinomial Logit Results, ExtrVolu**

| Constant and Independent Variables | Coefficients | | |
|---|---|---|---|
| | Vote for left-wing party | Vote for center party | Vote for right-wing party |
| Constant | –2.69*** | Base outcome | –2.63*** |
| | (0.610) | | (0.689) |
| Voter's position | –0.85*** | | 0.73*** |
| | (0.223) | | (0.183) |

*Notes.* Multinomial logit estimates for the party choice decision in treatment ExtrVolu. Standard errors are clustered at the matching group level. *(**; ***) indicates significance at the 10% (5%; 1%) level.

## References

Dalton, R. J. (2021). Modeling ideological polarization in democratic party systems. *Electoral Studies*, *72*, 102346.