# Public Persuasion in Elections: Single-Crossing Property and the Optimality of Censorship*

Junze Sun[1,†] Arthur Schram[1,2,‡] Randolph Sloof[2,§]

[1]Department of Economics, European University Institute, 50014 Fiesole, Italy
[2]Amsterdam School of Economics, University of Amsterdam, 1001 NJ Amsterdam, the Netherlands

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**Abstract**

We study public persuasion in elections, in which a monopoly designer or multiple competing designers attempt to influence the election outcome by manipulating public information about a payoff relevant state. We allow for a wide class of designer preferences, ranging from pursuing pure self-interest to maximizing any social welfare function expressed as weighted sum of voter payoffs (e.g., utilitarian). Our main result identifies a novel single-crossing property and shows that it guarantees the optimality of censorship policies – which reveal intermediate states while censor extreme states – in large elections under both monopolistic and competitive persuasion. The single-crossing property is (i) generically satisfied when designers are self-interested, or (ii) satisfied for generic designer preferences under a mild assumption on the distribution of voters' preferences. We also analyze how the structure of the equilibrium censorship policy varies with the designer's preference and voting rules. Finally, we apply our results to study the welfare impacts of media bias and competition and show that, contrary to common wisdom, increased media competition may in fact harm voter welfare by inducing excessive information disclosure.

**JEL Codes:** D72, D82, D83

**Keywords:** voting, single-crossing property, censorship, Bayesian persuasion, competition in persuasion, information design

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†Corresponding author. Email: Junze.Sun@eui.eu.

‡Email: A.J.H.C.Schram@uva.nl.

§Email: R.Sloof@uva.nl.

# 1 Introduction

In modern democracies, important choices are often made through collective decisions. For instance, presidents are selected via general elections and many important policies (like Brexit) are determined in referenda. In general, many different individuals and organizations have interests in the outcome of such collective decisions; think of (possibly foreign) governments, politicians, media, interest groups, representatives of industry or community leaders. Anyone with a stake in the outcome may try to influence the election outcome through manipulating public information, e.g., via public announcements or debate.

In this paper, we study such strategic manipulation of public information in elections with binary outcomes, such as referenda. From a positive perspective, we aim to understand the equilibrium behavior of actor(s) interested in manipulating public information to influence the election outcome. At the same time, we also consider such persuasion from a normative angle, by deriving the optimal information policy for a social planner whose objective is to maximize some social welfare function. This normative benchmark allows us to go one step further and study the welfare effects of varying the number of information providers.

We model the environment of interest as a public Bayesian persuasion problem (Kamenica and Gentzkow, 2011), in which information designers manipulate public information to maximize their own expected payoffs.[1] Compared to the previous literature, the framework of our paper has two important and distinguishing features. First and foremost, we allow for a wide class of utility functions for information designers that embed both the pursuit of self-interest and the maximization of utilitarian social welfare as special cases. Second, we characterize the equilibrium information transmission under both monopolistic persuasion with a single information designer and competitive persuasion with multiple designers, in a unified framework. Our main contribution is to identify a novel *single-crossing property* – which is widely held, easily verifiable and economically meaningful – and to show that it ensures the optimality of *censorship policies* for an information designer under both monopolistic and competitive persuasion in large elections.

We consider an election in which voters (she) collectively decide between adopting a reform and maintaining the status quo.[2] An ex-ante unknown *state* $k$ determines the quality of the reform relative to the status quo. Each voter's preference depends linearly on both the state realization $k$ and her *private type*, which can be interpreted as a 'threshold of

[1] Kamenica (2019) and Bergemann and Morris (2019) provide comprehensive overviews of this literature.
[2] Throughout this paper, we will refer to voters as feminine and information designers as masculine.

acceptance' for the reform; she prefers the reform to be adopted only if $k$ is larger than her private type, and prefers the status quo to be maintained otherwise. There is a finite set of information designers (he). Prior to knowing either the state realization or voters' types, each designer simultaneously chooses an *information policy*, which is a strategy about how to publicly reveal information about state $k$.

One class of information policies that will prove to be important is a so-called *censorship policy*, as illustrated in Figure 1. Specifically, a censorship policy can be characterized by a revelation interval $[a,b]$ such that all state realizations within $[a,b]$ are precisely communicated to voters, while other state realizations above $b$ or below $a$ are 'censored' separately under different pooling messages. This means that for all state realizations above threshold $b$, voters are only informed that $k > b$ and cannot further distinguish these state realizations. Similarly, if the state realization is below threshold $a$, then voters only learn $k < a$ without further details.

Figure 1: Censorship Policy


After observing their private types and the public information jointly released by all designer(s), voters simultaneously decide to vote for either the reform or the status quo. The reform will be adopted if and only if the fraction of votes it receives exceeds a cutoff that is determined by the voting rule. For example, under simple majority rule this cutoff is 50%. Since information transmission is public and voters' payoffs are linear and monotone in state $k$, they must share the same posterior expectation about state realization and the election outcome is determined by a *pivotal voter* under the cutoff voting rule. For instance, under the familiar simple majority rule the pivotal voter is the one whose type realization is the sample median (i.e., the median voter).

Any designer's utility function is a weighted average of on the one hand his private payoff and on the other hand a (rank-dependent) weighted average of voters' payoffs. Varying both kinds of weights allows for a broad spectrum of designer preferences. As mentioned above, voters' payoffs depend on their private types, which are unknown to the designer. The designer can, however, make inferences about voters' payoffs based on the pivotal voter's type realization. We show that such inference leads to a cutoff structure in a designer's preference; that is, conditional on the pivotal voter's type, the designer prefers reform to be adopted if and only if the realized state $k$ is above a certain threshold. In particular, this threshold is strictly increasing in the pivotal voter's type realization whenever the designer assigns a positive weight to voter welfare. This allows us to draw the indifference curves of both the designer and the pivotal voter in a same plane where the state is plotted on the vertical axis and the pivotal voter's type is plotted on the horizontal axis.

Our main result identifies a single-crossing property, which ensures the optimality of censorship policies in large elections under both monopolistic persuasion with a single information designer, and competitive persuasion with multiple designers. In our model, we say that the single-crossing property holds for a designer if for sufficiently large elections the designer's indifference curve crosses the pivotal voter's indifference curve at most once, and if so only from above, on the 'state vs. pivotal type' plane described above.[3] It implies that whenever the pivotal voter weakly prefers reform in state $k$, the designer must strictly prefer the reform in all higher states, in which the reform has a higher quality. Conversely, if the pivotal voter weakly prefers status quo in state $k$, then the designer must strictly prefer status quo in all lower states, namely when the quality of the reform is lower. We show that the single-crossing property is widely held; it is always satisfied if the designer is sufficiently self-interested, or satisfied for generic designer utility functions and voting rules under a mild assumption on the distribution of voters' preferences.

Consider first monopolistic persuasion by a single information designer for whom the single-crossing property holds. In this case we show that some censorship policy with revelation interval $[a,b]$ (as in Figure 1) must be uniquely optimal for this designer in sufficiently large elections. This result holds robustly for any prior distribution of state $k$ that admits a positive and continuous density function. The optimal choices of thresholds $a$ and $b$ are driven by the tradeoff between the capability of manipulating voters' beliefs in more states on the one hand, and the effectiveness of belief manipulation on the other hand. For instance, suppose the designer increases threshold $b$ to some $b+\Delta$ with $\Delta$ small. Then the designer losses the opportunity to manipulate voter's beliefs for state realizations $k \in [b, b+\Delta]$ because these states will be fully revealed. Nevertheless, this expansion of $b$ changes the pooling message from '$k > b$' to '$k > b+\Delta$', which signals a higher posterior expectation of state realization and is more capable of manipulating voters' beliefs upwards

[3] The fact that our single-crossing property can be interpreted in terms of indifference curves is reminiscent of the Spence-Mirrlees condition in the signaling and mechanism design literature. However, the interpretation and application of the Spence-Mirrlees condition is very different than ours. In the context of Bayesian persuasion, Mensch (2021) proposes a single-crossing condition that ensures optimality of monotone partitional signals. Again, both the context and meaning of single-crossing condition therein are very different from ours.

(so that the pivotal voter is more likely to be convinced to pass the reform). When $b$ is at the optimum the losses and gains of any such marginal expansion are precisely equal. The tradeoff that drives the choice of the lower cutoff $a$ is analogous.

We further explore how the two thresholds $a$ and $b$ of a designer's monopolistically optimal censorship policy vary with his preference and the voting rule. For a purely self-interested designer who does not care about voter welfare, we show that in large elections both thresholds $a$ and $b$ increase monotonically as the required vote share for passing the reform rises. This is entirely due to a *stringency* effect: such a change in voting rule makes it more difficult to convince the pivotal voter to pass the reform but easier to convince her to maintain the status quo (Alonso and Câmara, 2016*a*). In sharp contrast, however, if the designer cares about voter welfare, then an increase the required vote share for passing the reform may move thresholds $a$ and $b$ in opposite directions. This is because, apart from the stringency effect, in this case a change in voting rule also has a novel *designer-preference* effect: raising the required vote share for passing the reform makes a pro-social designer more leaning towards reform. As we explain in Section 6, the reason is that the election outcome is informative about the realized distribution of voters' payoffs, and the informativeness depends on the voting rule.

Next consider competitive persuasion in which multiple information designers simultaneously choose their public information policies. In this case we show that if the single-crossing property holds for a designer and the electorate size is sufficiently large, then it is *without loss of optimality* for this designer to restrict attention to censorship policies in the following sense: for any feasible pure strategy profile chosen by other designers (which need not be censorship policies), this designer can always find a censorship policy as his best response. Moreover, suppose that the single-crossing property holds for all designers and they all commit to use censorship policies only. Then there is a unique pure-strategy equilibrium that survives iterated deletions of weakly dominated strategies. In this equilibrium, the information jointly provided by all designers can be equivalently reproduced by a censorship policy whose revelation interval is precisely the convex hull of the revelation intervals of the optimal censorship polices of each designer under monopolistic persuasion alone. In fact, this equilibrium is the least informative one among all pure strategy equilibria and hence is Pareto optimal for all designers (Gentzkow and Kamenica, 2016*a*). This result implies under competition all designers may actually benefit from an exogenous restriction to use censorship policies only, because this avoids the risk of coordinating to inefficient equilibria that are excessively informative and hence make all designers worse off.

We also identify a sufficient condition – which is both easily verifiable and economically meaningful – that ensures full information disclosure as the unique equilibrium outcome under competitive persuasion. This condition requires that for each state realization there are designers who are biased towards different alternatives relative to the pivotal voter in that state. Contrary to many existing papers (Gentzkow and Kamenica, 2016*a*, 2017; Cui and Ravindran, 2020), our sufficient condition does not demand large conflicts of interests among information designers. For instance, we show that full disclosure can be the unique equilibrium outcome under competition in persuasion by two social planners who both care about voter welfare but differ in their weights.

Finally, we apply our results to study the welfare impacts of media bias and competition. We ask whether increased media competition necessarily improves voter welfare. We show that, contrary to common wisdom, full information disclosure is generically sub-optimal from the welfare perspective. Increased media competition may in fact harm voter welfare by inducing excessive information disclosure. Such harm can be severe if the ex-ante conflict of interests between the average voter and the pivotal voter is large. These results imply that it is important to account for institutional background, such as voting rules, when evaluating the welfare impacts of media bias and competition.

The rest of the paper is organized as follows. The next section discusses the related literature. Section 3 introduces our theoretical framework. Sections 4 and 5 present our main results. In Section 4 we introduce the single-crossing property and explain its economic implications. In Section 5 we relate the single-crossing property to the optimality of censorship policies under both monopolistic and competitive persuasion. Section 6 examines how, under the single-crossing property, the structure of a designer's optimal censorship policy responds to variations in his preference or the voting rule. Section 7 studies the welfare implication of media bias and competition. Section 8 concludes.

## 2 Related literature

This paper speaks to several strands of literature. First of all, our paper belongs to a strand of literature that studies information transmission in elections using the Bayesian persuasion or information design approach.[4] Aside from a few exceptions discussed below,

[4] Of course, strategic information transmission in elections has been extensively studied under various other communication protocols, such as cheap talk (Schnakenberg, 2015, 2017; Kartik and Van Weelden, 2019; Sun, Schram and Sloof, 2021) and verifiable disclosure (Liu, 2019), among others. One important feature that

most papers in this literature study monopolistic persuasion problems by a single designer whose goal is to sway the election outcome in favor of his preferred alternative (Wang, 2013; Alonso and Câmara, 2016*a*,*b*; Bardhi and Guo, 2018; Chan et al., 2019; Ginzburg, 2019; Kerman, Herings and Karos, 2020; Heese and Lauermann, 2021).[5] Our paper complements these works by either allowing for a wider class of designer preferences – ranging from pursuing self-interest to maximizing any social welfare function that can be expressed as a weighted average of voters' payoffs – or analyzing both monopolistic and competitive persuasion in a unified framework.

In terms of both modeling approaches and results, our paper is closely related to Alonso and Câmara (2016*b*) and Kolotilin, Mylovanov and Zapechelnyuk (2021). The models of both papers can be interpreted as a monopoly designer persuading a privately informed representative voter. In Alonso and Câmara (2016*b*), the information designer is an incumbent party leader whose aim is to maximize the re-election probability. They show that, under some regularity conditions, the optimal information policy takes the form of upper censorship – which reveals states below a certain cutoff while pool states above the cutoff – if the distribution of the representative voter's private type has a log-concave density.[6] Kolotilin, Mylovanov and Zapechelnyuk (2021) characterize sufficient and necessary conditions for the optimality of upper censorship for general linear persuasion problems. They show that the same log-concavity density assumption ensures the optimality of upper censorship for a wider class of designer preferences, ranging from pure persuasion motive to maximizing the welfare of the representative voter.

Our paper enriches and generalizes the results of both papers to an environment that allows for multiple designers and voters. Looking at a setup with multiple voters instead of

separates our paper from these papers is that our theory can answer the normative question: what is the optimal information policy for a social planner aiming at maximizing some weighted average of voters' payoffs? This question is not trivial because, as we show in Section 7, full information disclosure is generically sub-optimal from the welfare perspective In fact, the structure of the welfare-maximizing information policy depends subtly on many factors, such as the the distribution of voter preferences and voting rules.

[5] Two points are worth noticing. First, some of these papers (e.g., Heese and Lauermann (2021)) allow the designer's preferred alternative to be state-dependent. However, they do not allow the designer's payoff to depend on voters' private types and hence cannot capture a pro-social designer's preference. Second, all these papers except Alonso and Câmara (2016*a*,*b*) and Ginzburg (2019) study targeted persuasion in which the designer can privately communicate to voters (Bergemann and Morris, 2016; Taneva, 2019; Mathevet, Perego and Taneva, 2020). Our paper instead focuses on public persuasion whereby a designer must send the same message to all voters.

[6] Ginzburg (2019) studies a similar persuasion problem by an office-motivated designer. In his model, however, the designer is restricted to only use censorship policies (whose definition is broader than ours). He shows that upper censorship is optimal if the distribution of the representative voter's type is uni-model.

a single representative voter enables us to (i) model a wider class of social preferences for designers, and (ii) study the influence of voting rules on the information policies induced in equilibrium. We show that in large elections the optimality of censorship can be ensured under much weaker assumptions regarding the underlying distribution of voter types. Moreover, we show that the same conditions that ensure the optimality of censorship for a designer under monopolistic persuasion continue to do so under competitive persuasion with multiple designers.

Alonso and Câmara (2016*a*) study public persuasion in elections by a monopoly designer in a model similar to ours. A crucial difference between our paper and theirs is that we allow voters to have private types, whereas in their model the designer perfectly knows voters' preferences. This difference is important in two ways. First, the structures of the optimal information policies are very different depending on whether the designer knows voters' preferences.[7] Second, we show that when a designer cares about social welfare and is imperfectly informed about voters' preferences, changes in voting rule can affect his optimal information policy through a novel designer-preference effect. This effect is absent if the designer has perfect information. Van der Straeten and Yamashita (2020) and Ferguson (2020) study monopolistic persuasion problems in which the designer aims at maximizing voters' utilitarian welfare. Both papers show that full information disclosure is sub-optimal from the utilitarian perspective. Our paper extends this insight to general social welfare functions. Moreover, we also study welfare effects of increasing the number of information designers. Finally, Innocenti (2021) and Mylovanov and Zapechelnyuk (2021) study competition in Bayesian persuasion by two opposite-minded designers with pure persuasion motives. The former does so in a model where each voter can only hear from one designer. The latter, like ours, consider public persuasion a la Gentzkow and Kamenica (2016*a*). Our paper allows for a richer set of designer preferences compared to theirs.

Second, methodologically, our paper relates to a recent strand of literature that develops the duality approach to solve linear persuasion problems in which designers' utility functions depend only on the posterior expected state (Kolotilin, 2018; Dworczak and Martini, 2019; Dworczak and Kolotilin, 2019; Dizdar and Kováč, 2020; Kolotilin, Mylovanov and Zapechelnyuk, 2021).[8] All these papers formulate linear persuasion problems as linear

[7] Our model would reduce to a special case of Alonso and Câmara (2016*a*) if the designer perfectly knows voters' preferences. In this case, the optimal information policy would be a simple binary cutoff partition, which only reveals whether the realized state is above or below a certain cutoff.

[8] Several papers study linear persuasion problems using other methods. For instance, Gentzkow and Kamenica (2016*b*) and Kolotilin et al. (2017) characterize the set of implementable outcomes under public

programs. Dworczak and Martini (2019) and (Dizdar and Kováč, 2020) establish strong duality and provide convenient tools to verify a candidate solution to a linear persuasion problem. Moreover, Dworczak and Martini (2019) show that the problem of finding an equilibrium outcome under competitive persuasion can be converted to solving the monopolistic persuasion problems of each designer with modified utility functions. This allows us to treat monopolistic and competitive persuasion in a unified framework. The duality method is also powerful in that it establishes a strong link between the shape of a designer's utility function and the structure of his optimal information policy. For instance, Kolotilin, Mylovanov and Zapechelnyuk (2021) exploit the duality method to show that upper (lower) censorship policies are uniquely optimal for a monopoly designer independent of the prior distribution of the state if and only if his utility function is strictly S-shaped (inverse S-shaped).

Our paper exploits the duality approach and builds on insights above to establish our main results. In addition to that, we also deliver three novel results that apply to general linear persuasion problems. First, we identify a novel *increasing slope property* on a designer's utility function (cf. Observation 1 in Section 5) that ensures, for all continuous and full-support prior distribution of state and under both monopolistic and competitive persuasion, any equilibrium information policy must be more informative than a given censorship policy. Second, we generalize the analyses of Kolotilin, Mylovanov and Zapechelnyuk (2021) by showing that their conditions for utility functions ensure optimality of censorship policies not only under monopolistic persuasion, but also under competition; given any pure-strategy profile of other designers, it is always possible to find a censorship policy as a best response (cf. Observations 2 and 3). Third, we derive a sufficient condition – which is easy to verify and economically intuitive – for full information disclosure to be the unique equilibrium outcome under competitive persuasion (cf. Observation 4).

Our Observation 4 also relates to papers studying competition in Bayesian persuasion with multiple senders (Gentzkow and Kamenica, 2016*a*, 2017; Cui and Ravindran, 2020; Au and Kawai, 2020, 2021; Li and Norman, 2021; Mylovanov and Zapechelnyuk, 2021). An important theme of this literature is to identify conditions under which full information disclosure is the unique equilibrium outcome. Our Observation 4 contributes to this research agenda by providing such a sufficient condition for general linear persuasion games. In contrast to many existing findings, we show that strong conflicts of interests between

and private signals, respectively, using an implication of Blackwell's theorem. More recently, Arieli et al. (2020), Ivanov (2020) and Kleiner, Moldovanu and Strack (2021) develop methods based on extreme points and majorization to characterize structures of solutions to linear persuasion problems.

competing senders are not necessary to sustain full information disclosure as the unique equilibrium outcome.[9]

Finally, our application to evaluate the welfare impact of media bias and competition speaks to the strand of literature on the electoral impacts of media. An important theme of this literature is the debate over whether media competition can improve voter welfare. Most of the existing works, however, assume from the outset that more information is better and focus on whether media competition can improve information revelation.[10] On the one hand, media competition can benefit voters and improve political accountability by increasing the costs of media capture (Besley and Prat, 2006) that aims at suppressing disclosure of unfavorable information to the politician. Competition may also urge media to provide information that aligns better with the interests of their audiences (Gentzkow and Shapiro, 2006; Chan and Suen, 2008). These insights support the view that media competition is welfare-improving. On the other hand, the literature also identifies channels through which media competition can deteriorate voter welfare. For instance, competition can drive profit-maximizing media to invest fewer resources in the provision of political news or topics of common interests (Chen and Suen, 2018; Cagé, 2019; Perego and Yuksel, 2021).[11] In contrast to these works, we show that even if media competition can improve information revelation, this itself could harm welfare.

# 3 Framework

We consider an election in which $n+1$ voters collectively decide whether to adopt a *Reform* or to maintain the *Status quo*. The election outcome is determined by a cutoff rule with threshold $q \in (0,1)$: the reform is adopted if and only if it obtains strictly more than $nq$ votes. For instance, $q = 0.5$ corresponds to the familiar simple majority rule. Unless explicitly mentioned otherwise, we assume that $nq$ is an integer for ease of exposure.

An ex-ante unknown state $k$ is drawn from a commonly prior $F$, which admits a positive

[9] Mylovanov and Zapechelnyuk (2021) study competition in persuasion by two designers in a linear environment similar to ours. They also identify a sufficient condition for full disclosure to be the unique equilibrium and their condition does not demand a zero-sum game between designers. Our sufficient condition (Observation 4 in Section 5.3) includes theirs (Corollary 3 therein) as a special case.

[10] An exception is Carrillo and Castanheira (2008), who show that a profit-maximizing media outlet may acquire excessive information compared to social optimum. This is because intermediate precision of information may lead parties to choose polarized policies that deviate from the median voter's preference. The mechanism therein is completely different from ours.

[11] From a supply-side perspective, Innocenti (2021) shows that competition of two opposite-minded partisan media outlets may also decrease the quality of information when voters have limited attention.

and continuous density $f$ on $[-1,1]$. Without loss of generality, we normalize all players' payoffs to zero if the status quo is maintained. When the reform is adopted, each voter $i$'s payoff equals $k-v_i$, where $v_i$ is her private type. We assume that each $v_i$ is independently drawn from a commonly known distribution $G$, which admits a positive and twice continuously differentiable density $g$ on $[\underline{v},\overline{v}]$ with $\underline{v}<-1$ and $\overline{v}>1$. For any profile of type realization $v=(v_1,\cdots,v_{n+1})$, we let $v^{(1)}\leq v^{(2)}\leq\cdots\leq v^{(n+1)}$ be its ascending permutation. Since $k-v_i$ decreases in $v_i$, voters with lower type realizations receive higher ex-post payoffs when the reform is adopted.

There is a finite set $M$ of information designers indexed by $m$. For each designer $m\in M$, his payoff $u_m(k,v)$ under reform being adopted is given by

$$u_m(k,v)=\rho_m\sum_{j=1}^{n+1}w_{m,j}\left(k-v^{(j)}\right)+(1-\rho_m)(k-\chi_m) \tag{1}$$

where $\rho_m\in[0,1]$, $\chi_m\in\mathbb{R}$ and $(w_{m,1},\cdots,w_{m,n+1})$ is a non-negative vector of weights that sum up to 1. Parameter $\rho_m$ captures the extent to which designer $m$ cares about 'voter welfare' relative to his 'self-interest'. If $\rho_m=0$ then $u_m(k,v)=k-\chi_m$ so that designer $m$ prefers reform to be adopted if and only if $k\geq\chi_m$. In this case, designer $m$ is *self-interested* in the sense that his preference over alternatives is independent of voters' interests.[12]

Conversely, if $\rho_m=1$ then $u_m(k,v)=\sum_{j=1}^{n+1}w_{m,j}\left(k-v^{(j)}\right)$ is a weighted average of voters' realized payoffs when the reform is adopted. For each $j=1,\cdots,n+1$, $w_{m,j}$ is the *rank-dependent welfare weight* designer $m$ assigns to the voter whose payoff under reform is ranked the $j$-th highest under the realized type profile $v$. The vector $(w_{m,1},\cdots,w_{m,n+1})$ is generated by a *weighting function* $w_m(\cdot)$, which is non-decreasing, absolutely continuous on $[0,1]$ and satisfies $w_m(0)=0$ and $w_m(1)=1$. Hence, $w_m(\cdot)$ is the cumulative distribution function (cdf) of a random variable on $[0,1]$.[13] For any integer $n\geq 0$ and $j\in\{1,\cdots,n+1\}$, element $w_{m,j}$ is uniquely generated by

$$w_{m,j}=w_m\left(\frac{j}{n+1}\right)-w_m\left(\frac{j-1}{n+1}\right) \tag{2}$$

This setup captures a wide class of social welfare functions in a unified way. For instance,

[12] This captures transparent persuasion motives as limiting cases. For instance, the preference of a designer $m$ who aims at maximizing the winning probability of reform (resp. status quo) independent of state realizations can be captured by letting $\chi_m\to-\infty$ (resp. $\chi_m\to\infty$).

[13] $w_m(\cdot)$ is reminiscent of the probability weighting function in rank-dependent utility theory (Quiggin, 1982).

the familiar Utilitarian welfare function can be obtained by letting $\rho_m = 1$ and $w_m(x) = x$ for all $x \in [0,1]$. With this $w_m(\cdot)$, it follows from (2) that $w_{m,j} = \frac{1}{n+1}$ for each $j$ so that the welfare weights are indeed placed equally across voters. If $w_m(\cdot)$ is not the cdf of a uniform distribution on $[0,1]$, then it represents the preference of some non-Utilitarian social planner who may discriminate voters according to the ranking of their ex-post payoffs. We will discuss some examples in Section 5.

Each designer $m \in M$ can affect voters' information about $k$ by designing an *information policy*. Following the literature, we define an information policy $\pi_m$ by a pair $(S_m, \sigma_m)$, where $S_m$ is a sufficiently rich signal space[14] and $\sigma_m : [-1,1] \mapsto \Delta(S_m)$ maps each state realization $k$ to a probability distribution on $S_m$ (e.g., Kamenica and Gentzkow (2011) and Dworczak and Martini (2019)). Let $\Pi$ denote the set of all feasible information policies. Given any profile $\{\pi_m\}_{m\in M}$, we denote by $\pi := \langle \{\pi_m\}_{m\in M} \rangle$ the *joint information policy* induced by observing the signal realizations for all $\pi_m$'s in $\{\pi_m\}_{m\in M}$. Notice that $\pi \in \Pi$ because it is clearly feasible. In this way, our information environment is *Blackwell-connected*; given any strategy profile $\pi_{-m} := \times_{j\in M\setminus\{m\}} \pi_j$ of other designers, each designer $m$ can unilaterally deviate to any feasible joint information policy that is Blackwell more informative (Gentzkow and Kamenica, 2016*a*).

The timing of the game is as follows. First, prior to observing state realization $k$, all designers $m \in M$ simultaneously choose their information policies $\pi_m \in \Pi$, which together induce a joint policy $\pi := \langle \{\pi_m\}_{m\in M} \rangle$. Second, state $k$ is realized and a public signal is drawn according to $\pi$. Observing the realized public signal, voters simultaneously decide to vote for either the reform or the status quo. The reform is adopted if and only if its vote tally strictly exceeds $nq$. All players' payoffs then realize. Throughout, we focus on equilibria in pure and weakly undominated strategies.[15]

[14] It is sufficient to assume that the cardinality of each $S_m$ is as large as that of the state space $[-1,1]$.

[15] It is well known that the voting game at the second stage has a plethora of uninteresting equilibria in weakly dominated strategies. For example, whenever $n > 0$ it is an equilibrium for all voters to vote for reform regardless of their private types or the public information they obtain from designers, because no single vote can unilaterally change the election outcome. In this case, any strategy profile $\{\pi_m\}_{m\in M}$ of designers can hold in equilibrium as well because they have no influence on the election outcome anyway. By restricting to weakly undominated strategies we rule out such uninteresting equilibria. The restriction to pure strategies is without loss of generality for the voting game and for monopolistic persuasion with one designer. For competition in persuasion with multiple designers, however, this is a substantive restriction. As explained in Gentzkow and Kamenica (2017), when designers are allowed to use mixed strategies the information environment may no longer be Blackwell-connected, which is a key property we rely on to characterize equilibria under competition. Li and Norman (2018) show by examples that allowing for mixed strategies indeed changes the set of equilibria.

## 3.1 Voting behavior and election outcome

Because voters have a common prior $F$ and information transmission is public, they must share a common posterior about state realization after hearing from designers. Since voters' payoffs under reform is linear in state $k$, their expected payoffs depend only on their posterior expectation $\theta$ and they are given by $\theta - v_i$ for each voter $i$. It is then a weakly dominant strategy for each voter $i$ to vote for reform if and only if $\theta \geq v_i$.[16] Therefore, under the cutoff voting rule with threshold $q$, the election outcome is determined by the choice of the *pivotal* voter, whose type realization is $v^{(nq+1)}$. Note that $v^{(nq+1)}$ is a random variable and let $\hat{G}_n(\cdot;q)$ denote its cumulative distribution function. Since reform is adopted only if $v^{(nq+1)} \leq \theta$, $\hat{G}_n(\theta;q)$ equals the winning probability of reform when voters share a common posterior expectation $\theta$ about state realization. The expression and useful properties of $\hat{G}_n(\theta,q)$ are derived in Appendix A.1.

**Lemma 1.** *$\hat{G}_n(\cdot;q)$ is strictly increasing. $v^{(nq+1)}$ converges in probability to $v_q^* := G^{-1}(q)$.*

Lemma 1 says that the winning probability of reform strictly increases in $\theta$. Moreover, as $n \to \infty$ the reform will be adopted almost surely if $\theta > v_q^*$, while the status quo will be maintained almost surely if $\theta < v_q^*$.

# 4 Indifference curves and the single-crossing property

In this section we introduce the single-crossing property and discuss its implications for a designer's temptation to manipulate information. We also characterize sufficient conditions for the single-crossing property to hold for any designer. All omitted derivations and proofs are relegated to Appendix A.2.

## 4.1 Indifference curves and inference from pivotal voter's choice

For any designer $m \in M$ and any realized voter type profile $v$, it follows from (1) designer $m$ weakly prefers the reform to be adopted if and only if

$$k \geq \varphi_n^m(v) := \rho_m \sum_{j=1}^{n+1} w_{m,j} v^{(j)} + (1-\rho_m)\chi_m \ .$$

[16] The tie-breaking rule in event $\theta = v_i$ does not matter because it is a zero-probability event.

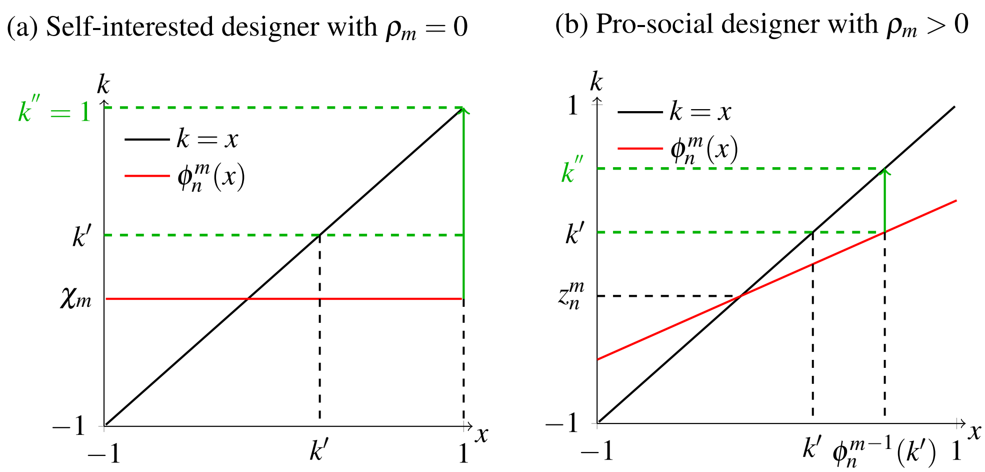
$\varphi_n^m(v)$ is designer $m$'s threshold of acceptance for the reform and it depends on voters' realized type profile $v$ whenever $\rho_m > 0$. Importantly however, at the time of choosing his information policy, any designer $m$ with $\rho_m > 0$ cannot precisely observe $\varphi_n^m(v)$ because realized types are voters' private information. Nevertheless, the election outcome, which is essentially the choice of the pivotal voter, is informative about the realization of $\varphi_n^m(v)$.

To make this point clear, it is instructive to draw the indifference curves of both the pivotal voter and the designer in a same plane as in Figure 2. In each panel, the horizontal axis $x$ equals the pivotal voter's type realization $v^{(nq+1)}$ and the vertical axis denotes the realized state $k$. The pivotal voter's indifference curve is simply the 45-degree line; she is indifferent between alternatives if and only if $k = x$. For any designer $m$, let

$$\phi_n^m(x) := \mathbb{E}\left[\varphi_n^m(v)\middle|v^{(nq+1)} = x\right] = \rho_m \sum_{j=1}^{n+1} w_{m,j}\mathbb{E}\left[v^{(j)}\middle|v^{(nq+1)} = x\right] + (1-\rho_m)\chi_m \qquad (3)$$

denote the expectation of $\varphi_n^m(v)$ *conditional on* event $v^{(nq+1)} = x$. Then, if the designer only knows that $v^{(nq+1)} = x$, he would be indifferent between alternatives if and only if $k = \phi_n^m(x)$. For this reason, we refer to $\phi_n^m(x)$ as the *indifference curve* of designer $m$.

Figure 2: Indifference Curves and the Single-Crossing Property




Note: In the horizontal axis $x$ denotes the pivotal voter's type realization $v^{(nq+1)}$, the black line $k = x$ denotes the pivotal voter's indifference curve, and the red line $\phi_n^m(x)$ denotes the designer's indifference curve.

Panel (a) of Figure 2 depicts the indifference curve of a self-interested designer with $\rho_m = 0$. In this case it is obvious from (3) that $\phi_n^m(x) = \chi_m$ for all $x$. The preference of a self-interested designer is thus independent of the pivotal voter's type realization.

Panel (b) of Figure 2 depicts the indifference curve of a pro-social designer with $\rho_m > 0$.

In this case, we show in Appendix A.2 (cf. Proposition A.3 therein) that $\phi_n^m(x)$ must be strictly increasing in $x$ for all $n \geq 0$ and weighting function $w_m(\cdot)$. This is because the pivotal voter's type realization $v^{(nq+1)}$ is positively associated with all other order statistics $v^{(j)}$ for $j = 1, \cdots, n+1$. Therefore, no matter how designer $m$ assigns his welfare weights, the pivotal voter's type realization is either *directly relevant* or *indirectly informative* about the designer's threshold of acceptance for the reform. It is precisely in this way that inference from pivotal voter's choice is relevant for any pro-social designer $m$ with $\rho_m > 0$.[17]

## 4.2 Single-crossing property and its economic implications

In this subsection we introduce our *single-crossing property*. To formally define it we need the following lemma, which characterizes the limiting properties of $\phi_n^m(\cdot)$ as $n \to \infty$.

**Lemma 2.** *For $x \in [\underline{v}, \overline{v}]$, define*

$$\phi^m(x) := \rho_m \left[ \int_0^q G^{-1}\left(\frac{y}{q}G(x)\right) dw_m(y) + \int_q^1 G^{-1}\left(\frac{y-q}{1-q} + \frac{1-y}{1-q}G(x)\right) dw_m(y) \right] + (1-\rho_m)\chi_m$$

*As $n \to \infty$, $\phi_n^m(x)$ and $\phi_n^{m\prime}(x)$ converge uniformly to $\phi^m(x)$ and $\phi^{m\prime}(x)$, respectively, on $[\underline{v}, \overline{v}]$. Moreover, $\varphi_n^m(v)$ converges almost surely to*

$$\phi_m^* := \rho_m \int_0^1 G^{-1}(y) dw_m(y) + (1-\rho_m)\chi_m$$

For any continuously differentiable function $h(\cdot)$, we say that $h(\cdot)$ is *single-crossing*on some interval $[l, r]$ if (i) $h(x)$ crosses zero at most once and if so from above on $[a, b]$, and (ii) $h'(x) < 0$ whenever $h(x) = 0$ and $x \in [l, r]$.

**Definition 1.** *(Single-crossing property) We say that single-crossing property holds for designer $m \in M$ if $x - \phi^m(x)$ is single-crossing on $[\underline{v}, \overline{v}]$.*[18]

By Lemma 2, $\phi_n^m(x)$ and $\phi_n^{m\prime}(x)$ converge uniformly to $\phi^m(x)$ and $\phi^{m\prime}(x)$, respectively, on $[\underline{v}, \overline{v}]$. Therefore, when single-crossing property holds for designer $m$, there exists a threshold $n_m$ such that for all $n > n_m$ function $\phi_n^m(x) - x$ is single-crossing on $[\underline{v}, \overline{v}]$; that is,

[17] The inference problem here is conceptually different from the inference about state conditional on the event of being pivotal, which is central to the literature on information aggregation in voting (e.g., Feddersen and Pesendorfer (1996, 1997)). In our model voters have no private information about state $k$, so the information aggregation issue is absent.

[18] We impose single-crossing on the entire $[\underline{v}, \overline{v}]$ rather than $[-1, 1]$ in order to simplify the proof. All our results hold under a weaker assumption that $x - \phi^m(x)$ is single-crossing on $[-1-\varepsilon, 1+\varepsilon]$ for some $\varepsilon > 0$.

$\phi_n^m(x)$ crosses the pivotal voter's indifference curve $k = x$ at most once and if so only from above. For such $\phi_n^m(x)$ we can pin down a unique *switching state* $z_n^m$ defined as follows

$$z_n^m := \begin{cases} -1 & \text{if } x > \phi_n^m(x) \text{ for all } x \in [-1,1] \\ x & \text{if } x = \phi_n^m(x) \text{ for some } x \in [-1,1] \\ 1 & \text{if } x < \phi_n^m(x) \text{ for all } x \in [-1,1] \end{cases}. \tag{4}$$

The definition of $z_n^m$ implies that $k > (<) \phi_n^m(k)$ for all $k > (<) z_n^m$. Therefore, in any state $k > z_n^m$ the designer is more biased towards the reform than the pivotal voter in the following sense: whenever the pivotal voter is indifferent between alternatives (i.e., in event $k = x$) the designer must strictly prefer the reform to be adopted because $k > \phi_n^m(k)$. Similarly, in any state $k < z_n^m$ the designer is more biased towards the status quo than the pivotal voter in that the designer must strictly prefer the status quo to be maintained whenever the pivotal voter is indifferent.

Depending on the value of $z_n^m$, there are three possible cases. First, $z_n^m \in (-1,1)$ so that $\phi_n^m(x)$ crosses $k = x$ at an interior state. In this case, the designer is more biased towards passing the reform (status quo) for state realizations above (below) the switching state $z_n^m$. Second, $z_n^m = -1$ so that $\phi_n^m(x) < x$ for all $x \in (-1,1)$. In this case, the designer is uniformly more biased towards the reform than the pivotal voter in all states. Third, $z_n^m = 1$ so that $\phi_n^m(x) > x$ for all $x \in (-1,1)$. In this case the designer is uniformly more biased towards the status quo than the pivotal voter in all states.

An important economic implication of the single-crossing property is that the designer is tempted to manipulate voters' beliefs upwards (downwards) for state realizations above (below) the switching state $z_n^m$. Figure 2 illustrates this. Consider any state realization $k'$ in $(z_n^m, 1)$. Under single-crossing property $k' > \phi_n^m(k')$ must hold. Let $k'' = \phi_n^{m-1}(k')$ if $k' \leq \phi_n^m(1)$ (left panel) or set $k'' = 1$ otherwise (right panel). As is evident in Figure 2, $k'' > k'$ must hold so that the designer and pivotal voter prefer different alternatives whenever $v^{(nq+1)} = x \in (k', k'')$. Since $x$ is the pivotal voter's private information, the designer is tempted to lie and let the pivotal voter believe that the realized state is $k''$, which is higher than the true state $k'$. It is in this sense that the designer is tempted to manipulate voters' beliefs about state realization upwards. Following the same logic, if the state realization $k'$ is below $z_n^m$ then the designer is tempted to manipulate voters' beliefs about the state realization downwards.

The following lemma provides two sufficient conditions, which are economically meaningful and easily verifiable, for single-crossing property to hold for any designer $m$.

**Lemma 3.** *Suppose $G$ has a strictly positive and twice continuously differentiable density $g$ on $[\underline{v}, \overline{v}]$. Then single-crossing property holds for designer $m$ if either*

*(i). $\rho_m$ is sufficiently close to 0 (i.e., designer $m$ is sufficiently self-interested), or*

*(ii). both $G$ and $1-G$ are strictly log-concave.*[19]

Notice that the constraint on $G$ in the second statement is in fact very mild; it is satisfied if the density function $g$ is strictly log-concave, which already includes a wide class of distributions (see Bagnoli and Bergstrom (2005) for examples) that are frequently assumed in applied theories. Once this mild assumption for $G$ is satisfied, the single-crossing property holds generically for all designer preferences and voting rules.

# 5 Single-crossing property and optimality of censorship

This section presents our main results, which relate the single-crossing property to the optimality of censorship policies under both monopolistic persuasion with a single designer (Section 5.1) and competitive persuasion with multiple designers (Section 5.2). Formal formulations of persuasion problems and proofs are collected in Section 5.3.

## 5.1 Optimal information policy under monopolistic persuasion

In this subsection we consider the monopolistic persuasion problem with only one designer. Since $|M| = 1$ we omit index $m$ and let $\phi_n(x)$ denote this monopoly designer's indifference curve. We assume single-crossing property holds so there exists some $\tilde{n} \geq 0$ such that for all $n > \tilde{n}$ function $\phi_n(x) - x$ is single-crossing on $[-1, 1]$ and the unique switching state $z_n$ is identified by (4) with $\phi_n^m(\cdot)$ therein replaced by $\phi_n$.

On the other hand, as explained in the Introduction, a censorship policy is characterized by a revelation interval $[a, b]$ with $-1 \leq a \leq b \leq 1$ such that (i) all intermediate state realizations $k \in [a, b]$ are precisely revealed, and (ii) extreme state realizations $k > b$ and $k < a$ are censored under different pooling messages as in Figure 1. Under such censorship

[19] Strict log-concavity of $1-G$ is equivalent to strictly increasing hazard rate $g(x)/(1-G(x))$. Strict log-concavity of $G$ is equivalent to strictly decreasing reversed hazard rate $g(x)/G(x)$. In fact, this condition is tight: suppose either $G$ or $1-G$ is strictly log-convex on some sub-interval within $[-1, 1]$, then one can construct a designer preference and voting rule under which the single-crossing property fails to hold.

policy voters' (common) posterior expectation equals $k$ for all state realization or state realizations $k \in [a,b]$ due to full revelation, and equals $\mathbb{E}_F[k|k > b]$ (resp. $\mathbb{E}_F[k|k < b]$) for all state realizations $k > b$ (resp. $k < a$). Observe that both *full disclosure* (with $a = -1$ and $b = 1$) and *no disclosure* (with $a = b \in \{-1, 1\}$) are special cases of censorship policies.

Our first main result, Theorem 1, relates the single-crossing property to the optimality of censorship policies in large elections under monopolistic persuasion.

**Theorem 1.** *Suppose $|M| = 1$ and the single-crossing property holds for this monopoly designer. Then there exists a $N \geq 0$ such that for all $n > N$ any monopolistically optimal information policy is outcome-equivalent to a censorship policy with revelation interval $[a_n, b_n]$ that satisfy $-1 \leq a_n \leq z_n \leq b_n \leq 1$.*[20] *Moreover, the following holds:*

1. *If $-1 < z_n < 1$ (i.e., the designer is more biased towards the reform (status quo) than the pivotal voter in state $k > (<) z_n$), then $a_n < z_n < b_n$ so that both sufficiently high and low states are likely to be censored.*

2. *If $z_n = -1$ (i.e., the designer is uniformly more biased towards the reform than the pivotal voter), then $a_n = -1$ so that only sufficiently high states can be censored.*

3. *If $z_n = 1$ (i.e., the designer is uniformly more biased towards the status quo than the pivotal voter), then $b_n = 1$ so that only sufficiently low states can be censored.*

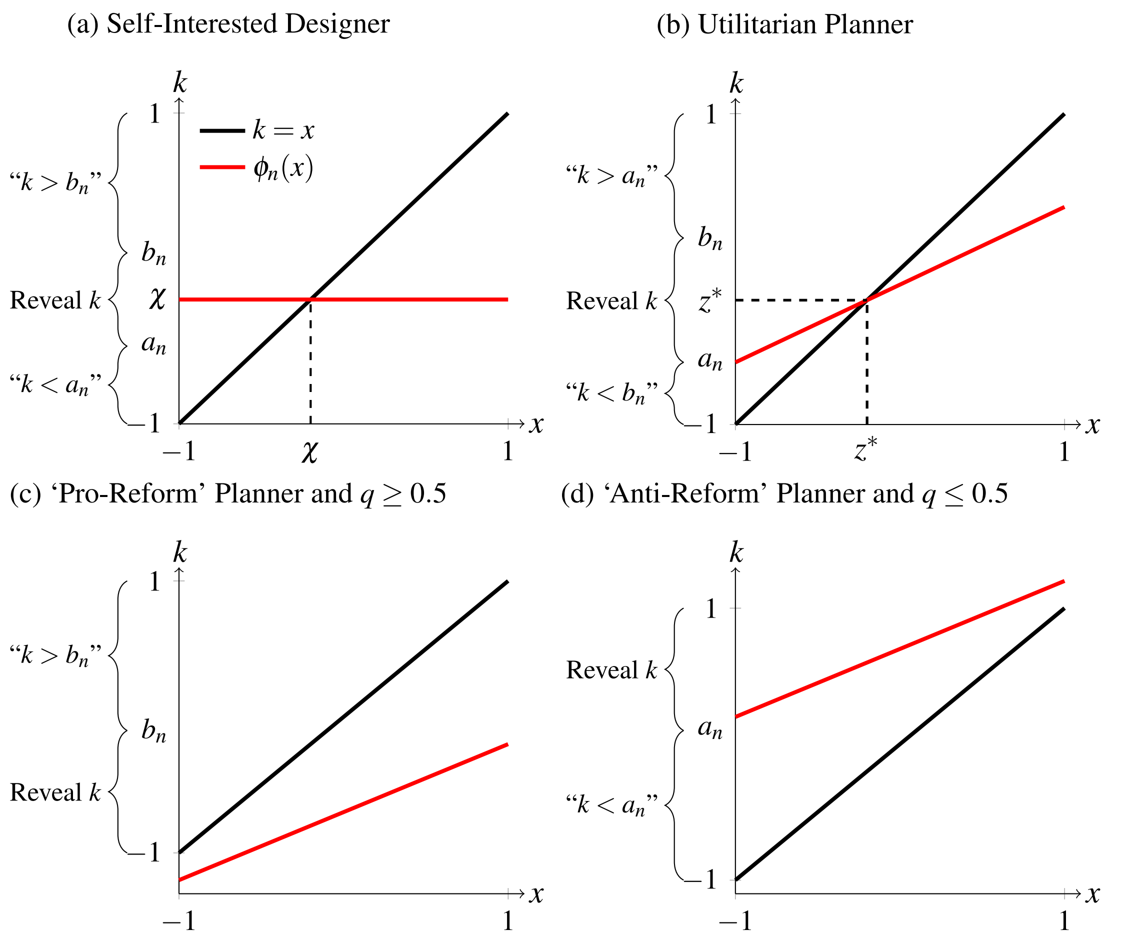
*In particular, if $g(\cdot)$ is log-concave and $\rho$ is sufficiently close to $0$, then $N = 0$ so that the three properties above hold for all $n > 0$.*

Theorem 1 establishes a one-to-one mapping between the three possible locations of the switching state $z_n$ and the structure of the optimal censorship policy. If $z_n \in (-1, 1)$ so that $\phi_n(x) - x$ crosses zero at some interior state, then the optimal policy has a feature of *two-sided censorship* in the sense that both very high and very low states can be censored. If instead $z_n = -1$, then $\phi_n(x) < x$ for all $x \in (-1, 1)$ and the designer is uniformly more biased towards reform than the pivotal voter. In this case the optimal policy takes the form of *upper censorship* in the sense that only sufficiently high states can be censored. Finally, if $z_n = 1$, then $\phi_n(x) > x$ for all $x \in (-1, 1)$ and the designer is uniformly more biased towards the status quo than the pivotal voter. In this case the optimal policy takes the form of *lower censorship* in the sense that only sufficiently low states can be censored.

[20] Two information policies are *outcome-equivalent* if their induced mappings from state realization $k$ to voters' posterior expected state are equal almost everywhere.

Before explaining the intuition of Theorem 1, we apply this theorem to characterize the structures of the monopolistically optimal censorship policies for the four examples of designer preferences. These are illustrated by the four panels of Figure 3.

Figure 3: The Monopolistically Optimal Censorship Policies for Four Examples




Note: In the horizontal axis $x$ denotes the pivotal voter's type realization $v^{(nq+1)}$, the black line $k = x$ denotes the pivotal voter's indifference curve, and the red line $\phi_n(x)$ denotes the designer's indifference curve.

**Example 1**: *Self-interested designer*. Panel (a) depicts the indifference curve and structure of the optimal censorship policy for a self-interested designer with $\rho = 0$. By (3), $\phi_n(x) = \chi$ for all $x \in [\underline{v}, \overline{v}]$. His switching state $z_n$ thus depends solely on $\chi$. If $\chi \in (-1, 1)$, then $z_n = \chi$ and he is more biased towards the reform (status quo) in state $k > (<)\chi$. By Theorem 1, some two-sided censorship policy with $a_n < \chi < b_n$ is optimal for this designer in large elections. If instead $\chi \leq -1$ (resp. $\chi \geq 1$), then he is uniformly more biased towards the reform (resp. status quo) than the pivotal voter in all states. For these cases, Theorem 1 implies the designer's optimal information policy must be either upper (if $\chi \leq -1$) or lower (if $\chi \geq 1$) censorship in large elections.

**Example 2**: *Utilitarian social planner*. Panel (b) depicts the indifference curve and structure of the optimal censorship policy for a Utilitarian planner who aims at maximizing voters' ex-post average payoffs. We assume that both $G$ and $1-G$ are strictly log-concave so that by Lemma 3 the single-crossing property holds. A Utilitarian planner's indifference curve $\phi_n(x)$ is given by[21]

$$\phi_n(x) = \frac{n}{n+1}\left(q\mathbb{E}_G\left[v_i|v_i \leq x\right] + (1-q)\,\mathbb{E}_G\left[v_i|v_i \geq x\right]\right) + \frac{1}{n+1}x \tag{5}$$

for $x \in [\underline{v}, \overline{v}]$. Therefore, $\phi_n(x) = x$ if and only if

$$q\mathbb{E}_G\left[v_i|v_i \leq x\right] + (1-q)\,\mathbb{E}_G\left[v_i|v_i \geq x\right] = x \tag{6}$$

When both $G$ and $1-G$ are strictly log-concave (6) admits a unique solution $z^*$ on $(\underline{v}, \overline{v})$. A Utilitarian planner's switching point $z_n$ thus depends only on $z^*$. If $z^* \in (-1,1)$, then $z_n = z^*$ and he is more biased towards the reform (status quo) in states $k > (<) z^*$. By Theorem 1, some two-sided censorship policy with $a_n < z^* < b_n$ is Utilitarian optimal in large elections. Interestingly, if $z^* \leq -1$ (resp. $z^* \geq 1$) then even a Utilitarian planner can be uniformly more biased towards reform (resp. status quo) than the pivotal voter. For these cases Theorem 1 implies that the Utilitarian optimal information policy is either upper (if $z^* \leq -1$) or lower (if $z^* \geq 1$) censorship in large elections.

**Example 3**: *'Pro-Reform' social planner*. In panel (c) we consider a non-Utilitarian social planner who aims at maximizing the average payoff of the subset of voters whose ex-post payoffs under reform are above the 50%-percentile.[22] Suppose $q \geq 0.5$ so that a strict majority of vote share is required in order to pass the reform. In this case, $\phi_n(x) < x$ must hold for all $x \in (-1,1)$; that is, the designer must be uniformly biased towards the reform than the pivotal voter in all states. This is because a designer with such a skewed social preference assigns positive weights only to voters whose realized types are below the pivotal voter's type $v^{(nq+1)}$. Recall that any voter's payoff under reform is decreasing in her type. It follows that all voters the designer cares about must prefer the the reform to status quo whenever the pivotal voter is indifferent. The designer thus must also prefer the

[21] To see why (5) is true, notice that for event $v^{(nq+1)} = x$ to hold, there must be one voter with type $v_i = x$, $nq$ other voters with $v_i \leq x$, and the remaining $n(1-q)$ voters with $v_i \geq x$. Since each voter's type is independently drawn from $G$, the conditional expectation of any voter with $v_i \leq x$ (resp. $v_i \geq x$) equals $\mathbb{E}_G\left[v_i|v_i \leq x\right]$ (resp. $\mathbb{E}_G\left[v_i|v_i \geq x\right]$). Taking average over the whole electorate size $n+1$ yields (5).

[22] The weighting function for such 'pro-Reform' planner is given by $w(x) = \min\{2x, 1\}$ for $x \in [0,1]$.

reform in such event. Therefore, by Theorem 1, in large elections some upper censorship policy must be optimal for such 'pro-Reform' planner whenever a strict majority of votes is required to pass the reform.

**Example 4**: *'Anti-Reform' social planner*. In panel (d) we consider a non-Utilitarian social planner who aims maximizing the average payoff of the subset of voters whose ex-post payoffs under form are below the 50% percentile.[23] Following the same logic, we can show that for all $q \leq 0.5$ (i.e., a strict majority vote share is required to main the status quo) $\phi_n(x) > x$ must hold for all $x \in (-1,1)$; that is, such a designer must be uniformly biased towards the status quo than the pivotal voter in all states. Theorem 1 thus implies that some upper censorship policy must be optimal for such 'anti-Reform' planner in large elections, whenever a strict majority of votes is required to maintain the status quo.

Now we explain the intuition of Theorem 1. Observe that when $\phi_n(x) - x$ crosses zero from above at an interior switching state $z_n \in (-1,1)$, the revelation interval $[a_n, b_n]$ of the optimal censorship policy must contain $z_n$ in its interior so that voters can always perfectly distinguish state realizations above and below $z_n$. Indeed, single-crossing property implies that the designer has no incentive to hide state realization $k = z_n$. This is because under $k = z_n$ the interests of the designer and the pivotal voter are aligned; whenever the pivotal voter strictly prefers either alternative, the designer always weakly prefers it. Moreover, it is always optimal for the designer to fully separate any pair of state realizations on different sides of $z_n$.[24] To see why, consider any $k_1$ and $k_2$ such that $k_1 < z_n < k_2$ and suppose they are not full separated. As explained above, the designer is tempted to manipulate voters' beliefs about state realizations upwards in state $k_2$ while downwards in $k_1$. By fully separating these two states, the induced posterior expectation about state realization will indeed be lower in $k_1$ while higher in $k_2$. The designer thus strictly benefits from such separation.

What drives the optimal choices of thresholds $b_n$ and $a_n$ then? To answer this question, consider $b_n$ first. The reasoning above implies that $b_n \geq z_n$ must hold in order perfectly separate state realizations above and below $z_n$. In the meanwhile, recall that the designer is tempted to manipulate voters' beliefs upwards for all states $k > z_n$. By lowering $b_n$, the designer expands the set of states $(b_n, 1]$ in which he can manipulate voters' beliefs. However, this is at the expense of decreased effectiveness of persuasion because voters' posterior expectation of state realization under the pooling message "$k > b_n$" decreases as

[23] The weighting function for such 'anti-Reform' planner is given by $w(x) = \max\{2x-1, 0\}$ for $x \in [0,1]$.

[24] Formally, this means that any optimal information policy $\pi$ will not induce any posterior belief $\gamma$ such that $\{k_1, k_2\} \subseteq \text{supp}(\gamma)$ for any $k_1 \leq z_n \leq k_2$ (with at least one inequality holds strictly).

$b_n$ becomes lower. The optimal choice of $b_n$ therefore balances the marginal benefit of being able to manipulate voters' beliefs in more states on the one hand, and the marginal cost of losing effectiveness of such manipulation on the other hand. The tradeoff governing the optimal choice of threshold $a_n$ is similar.

We end this subsection with two remarks. First, Theorem 1 is robust in the sense that it holds for all continuous prior $F$, and for all $G$ as long as single-crossing property holds. By Lemma 3, this implies that Theorem 1 holds for generic $G$ if the designer is sufficiently self-interested (i.e., $\rho$ is equal or close to 0)[25], and it holds for all designer preferences characterized by (1) if both $G$ and $1-G$ are strictly log-concave. Second, single-crossing property is in fact almost necessary for the optimality of censorship policies under monopolistic persuasion; suppose instead that $\phi(x)$ crosses the pivotal voter's indifference curve $k = x$ from below at some interior state, then there exists some prior $F$ under which the optimal information policy is not censorship.

## 5.2 Equilibrium outcomes under competitive persuasion

In this section we study information provision in equilibrium under competition in persuasion with $|M| \geq 2$ designers. We impose the following regularity assumption.

**Assumption 1.** *For all $m \in M$ and $n > 0$, $\phi_n^{m\prime}(x) < 2$ holds on $[-1,1]$.*

We show in Appendix A.2 that either condition (i) or (ii) in Lemma 3 is sufficient for this assumption to hold. Therefore, Assumption 1 is not very restrictive; it holds for generic designer preferences under the mild condition that both $G$ and $1-G$ are strictly log-concave.

We first derive an economically meaningful sufficient condition that ensures full disclosure as the unique outcome in equilibrium. We say state $k$ is a *disagreeing state* if there exists at least two designers $\mathrm{I}, \mathrm{II} \in M$ who are weakly biased towards different alternatives at state $k$ relative to the pivotal voter; formally, with $\phi_n^{\mathrm{I}}(k) \leq k \leq \phi_n^{\mathrm{II}}(k)$.

**Theorem 2.** *Suppose Assumption 1 holds. Then full disclosure is the unique equilibrium outcome if all states are disagreeing states.*

[25] The result would be sharply different in a setup with only one representative voter (i.e., $n = 0$). This case is studied by Kolotilin, Mylovanov and Zapechelnyuk (2021). They show that $G$ must be uni-model (i.e., $g$ is single-peaked) to ensure the optimality of censorship policy for a self-interested designer with $\rho = 0$. In our setup this is not required because, as we show in Appendix A.1, the density function of the pivotal voter's type distribution $\hat{g}_n(\cdot;q)$ will be single-peaked for sufficiently large $n$ for all $g$ that are positive and twice-continuously differentiable.

To see why Theorem 2 is true, observe that in any disagreeing state there at least two designers who are weakly biased towards different alternatives relative to the pivotal voter. These two designers are thus tempted to manipulate voters' beliefs in opposite directions. Therefore, while revealing some extra information about this state realization could hurt one designer, it may benefit another. In fact, we can show that indeed at least one designer must strictly prefer to unilaterally disclose more information whenever this state realization is not already fully revealed. This is because, as we show in the next subsection, in any disagreeing state $k$ there exists at least one designer $m \in M$ whose utility function is strictly convex in the posterior expected state in a neighborhood around $k$. This local convexity implies positive gains from revealing more information (because it induces a mean-preserving spread on the distribution of posterior expectations about state realization).

Theorem 2 immediately implies the following corollary.

**Corollary 1.** *Suppose Assumption 1 holds. If there exists $I, II \in M$ such that $\phi_n^I(k) < k < \phi_n^{II}(k)$ holds for all $k \in (-1,1)$, then full disclosure is the unique equilibrium outcome.*

This corollary says that full disclosure is the unique equilibrium outcome as long as there are two designers who are uniformly biased towards different alternatives than the pivotal voter. For instance, this is the case if the competition is between two self-interested designers uniformly biased towards different alternatives (e.g., $\chi_{\text{I}} \leq -1$ and $\chi_{\text{II}} \geq 1$), or between a 'pro-Reform' planner and an 'anti-Reform' planner under simple majority rule (cf. Examples 3 and 4 in the previous subsection). An interesting observation is thus that, contrary to many existing works (Gentzkow and Kamenica, 2017, 2016*a*; Cui and Ravindran, 2020), a strong conflict of interests between designers is not necessary to ensure full disclosure as the unique outcome in equilibrium.

Next we consider any designer $m \in M$ for whom the single-crossing property holds. In this case, Theorem 1 ensures the optimality of censorship policies for designer $m$ under monopolistic persuasion in large elections. Our next theorem extends this observation to competition in persuasion. We show that for sufficiently large $n$ it is *with loss of optimality* for this designer $m$ to restrict attention to censorship policies in the following sense; for any pure strategy profile of other designers (which need not be censorship policies), there always exists a best response in censorship policy.

**Theorem 3.** *Suppose single-crossing property holds for designer $m$ and let $N_m$ be the threshold identified in Theorem 1 that ensures unique optimality of censorship under monopolistic* *persuasion for designer m. Then for all $n > N_m$, given any pure strategy profile of designers other than m, there exists a censorship policy which is designer m's best response.*

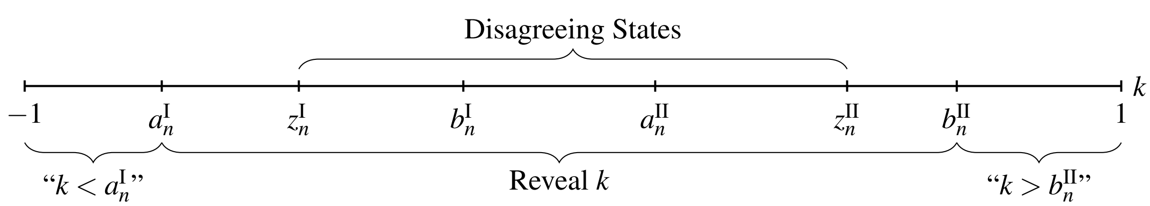
Now assume single-crossing property holds for all designers. By Theorem 1, for each $m \in M$ there exists a $N_m$ such that for all $n > N_m$ the monopolistically optimal information policy for designer $m$ is a censorship policy whose revelation interval $[a_n^m, b_n^m]$ contains the switching state $z_n^m$.

**Theorem 4.** *Suppose Assumption 1 and single-crossing property holds for all designers and let $N \geq \max_{m \in M} \{N_m\}$. Then for all $n > N$, the following holds:*

1. *In the minimally informative equilibrium[26], the joint information policy induced by all designers is outcome-equivalent to a censorship policy with revelation interval $\left[a_n^{\min}, b_n^{\max}\right]$, where $a_n^{\min} = \min_{m \in M} \{a_n^m\}$ and $b_n^{\max} = \max_{m \in M} \{b_n^m\}$.*

2. *If each designer is only restricted to use censorship policies, then the minimal informative equilibrium is the unique pure strategy equilibrium that survives (two rounds of) iterated elimination of weakly dominated strategies.*

Figure 4: Minimal Informative Equilibrium with Two Competing Designers




Note: $a_n^m$ and $b_n^m$ are cutoffs of the optimal censorship policy under monopolistic persuasion for $m \in \{\mathrm{I}, \mathrm{II}\}$.

Figure 4 illustrates Theorem 4 with two designers. This theorem implies that when single-crossing property holds for all designers, for sufficiently large elections the joint information policy induced in the minimally informative equilibrium is outcome-equivalent to a censorship policy whose revelation interval is the convex hull of the revelation intervals of all designers' monopolistically optimal censorship policies. Moreover, this equilibrium policy inherits the feature of fully revealing all disagreeing states; for any two designers $\mathrm{I}, \mathrm{II} \in M$ with $z_n^{\mathrm{I}} \leq z_n^{\mathrm{II}}$, it holds that all states $k \in \left[z_n^{\mathrm{I}}, z_n^{\mathrm{II}}\right]$ must be precisely disclosed.

[26] Following Gentzkow and Kamenica (2016*a*), we say an equilibrium is *minimally informative* if the joint information policy it induces is no more Blackwell informative than any information policy that can be induced by some other equilibrium. In fact, the joint information policy in the minimally informative equilibrium identified here is strictly less Blackwell informative than any other equilibria.

Under competition in public Bayesian persuasion it is well-known that there are multiple equilibria. The literature typically focus on the minimally informative equilibria because it is Pareto-optimal for all designers (Gentzkow and Kamenica, 2016*a*).[27] Building on Theorems 3 and 4, we provide an alternative justification for selecting the minimal informative equilibria outcome by restricting designers to use censorship policies only. When all designers are restricted to use censorship policies only, the minimally informative equilibrium is the unique pure strategy equilibrium that survives iterated deletion of weakly undominated strategies. Due to this favorable equilibrium selection, all designers would prefer these restrictions to censorship policies to be exogenously enforced (or by committing to use only censorship policies themselves). This is because these restrictions could partially avoid the risk of coordinating on equilibrium outcomes that are excessively informative and therefore make all designers strictly worse off. Such benefit of exogenous restriction is impossible under monopolistic persuasion.

## 5.3 Formal derivations and proofs of main results

In this subsection we formally present the persuasion problem and prove Theorems 1 to 3 and statement (1) of Theorem 4. The proof for statement (2) of Theorem 4 is more lengthy and we relegate it to Appendix C. Subsequent sections of this paper do not rely on results presented here, so readers uninterested in the formal arguments may skip this subsection.

Our proofs are based on a series of auxiliary results, which are labeled as "Observations" and presented in subsection 5.3.2, that apply to general linear persuasion problems. These observations are of independent interests and their proofs are relegated to Appendix B.

### 5.3.1 The persuasion problem and censorship policy

We start by formally stating the problems for both monopolistic persuasion and competitive persuasion. Let $\theta$ denote the common posterior expectation about state realization share by both voters and the designer(s). Given $\phi_n^m(\cdot)$, each designer $m$'s expected utility under any posterior expectation $\theta$ is given by

$$W_n^m(\theta) = \int_{\underline{v}}^{\theta} \left(\theta - \phi_n^m(x)\right) \hat{g}_n(x;q)dx \tag{7}$$

[27] An exception is Mylovanov and Zapechelnyuk (2021), who propose an equilibrium refinement based on a vanishing (entropy-based) cost of information disclosure. Unlike theirs, our refinement is based on iterated deletion of weakly dominated strategies.

where $\hat{g}_n(\cdot;q) = \hat{G}'_n(\cdot;q)$ is the density function of the pivotal voter's type realization.[28] Because each designer $m$'s expected payoff depends on voters' posterior expectation $\theta$ only, it is convenient to present any information policy $\pi$ by the distribution $H_\pi$ of posterior means it induces. We say that a distribution of posterior means $H$ is *feasible* if it can be induced by some information policy $\pi \in \Pi$ given prior $F$. It is well known that given prior $F$, a distribution of posterior means $H$ is feasible if and only if $F$ is a *mean-preserving spread* of $H$ (Gentzkow and Kamenica, 2016*b*; Kolotilin et al., 2017; Dworczak and Martini, 2019).[29] In the sequel we write $F \succeq_{MPS} H$ if $F$ is a mean-preserving spread of $H$.

*Monopolistic persuasion*. When $|M| = 1$, there is only one information designer. An information policy $\pi$ is optimal (hence holds in equilibrium) if and only if $H_\pi$ is a solution to the monopolistic persuasion problem

$$\max_{H \in \Delta([-1,1])} \int_{-1}^{1} W_n^m(\theta) dH(\theta), \quad \text{s.t. } F \succeq_{MPS} H \tag{MP}$$

*Competition in persuasion*. For the case $|M| \geq 2$, let $\pi = \langle \pi_m \rangle_{m \in M}$ be any joint information policy induced by all designers and $H_\pi$ denote the distribution of the posterior means induced by $\pi$. We say that $H_\pi$ is *unimprovable for designer $m \in M$* if he has no incentive to reveal more information. For each $m \in M$, let $\mathscr{H}_m$ denote the set of all distributions $H$ that are unimprovable for designer $m$. The set of distributions $H$ that are *unimprovable for all designers* is then $\mathscr{H} = \cap_{m \in M} \mathscr{H}_m$. By Proposition 2 of Gentzkow and Kamenica (2016*a*), $\pi$ can be sustained in equilibrium if and only if $H_\pi \in \mathscr{H}$.

*Censorship policy*. Observe that the distribution of posterior expectation induced by a censorship policy with revelation interval $[a,b]$ is given by

$$H_{\mathscr{P}(a,b)}(\theta) := \begin{cases} F(a) \cdot \mathbb{1}\{\theta \geq \mathbb{E}_F[k|k<a]\}, & \text{if } \theta \in [-1,a) \\ F(\theta), & \text{if } \theta \in [a,b) \\ F(b) + [1-F(b)] \cdot \mathbb{1}\{\theta \geq \mathbb{E}_F[k|k>b]\}, & \text{if } \theta \in [b,1] \end{cases} \tag{8}$$

where $\mathbb{1}\{\cdot\}$ is the indicator function. We say that an information policy $\pi \in \Pi$ is a *censorship policy* if $H_\pi$ coincides with (8) for some $-1 \leq a \leq b \leq 1$ almost everywhere. In the sequel

[28] To see why (7) holds, recall that $x$ denotes the type realization of the pivotal voter. By the discussion in Section 3.1, the reform is adopted if $\theta \geq x$ and in this case the designer gets an expected payoff $\theta - \phi_n^m(x)$, otherwise the status quo is maintained and the designer's payoff is zero.

[29] $F$ is a mean-preserving spread of $H$ if $\int_{-1}^{x} H(\theta)d\theta \leq \int_{-1}^{x} F(\theta)d\theta$ for all $x \in [-1,1]$, where equality holds for $x \in \{-1,1\}$. Another equivalent definition is that $\mathbb{E}_F[\omega(\cdot)] \geq \mathbb{E}_H[\omega(\cdot)]$ for any convex function $\omega(\cdot)$.

we slightly abuse notation and let $\mathscr{P}(a,b)$ denote both any censorship policy or the set of all censorship policies with revelation interval $[a,b]$, whenever it does not lead to confusion. For the special case $a=b$ we simply write $\mathscr{P}(a,b)$ as $\mathscr{P}(a)$ and refer to it as a *cutoff policy* because it only reveals whether the realize state is above, equal, or below cutoff $a$.

### 5.3.2 Properties of solutions for general persuasion problem

We present some useful observations about properties of solutions to a general linear persuasion problem as the following:

$$\max_{H\in\Delta([\underline{\kappa},\overline{\kappa}])}\int_{\underline{\kappa}}^{\overline{\kappa}} U(\theta)dF(\theta), \quad \text{s.t. } F\big|_{[\underline{\kappa},\overline{\kappa}]} \succeq_{MPS} H \qquad \text{(MP')}$$

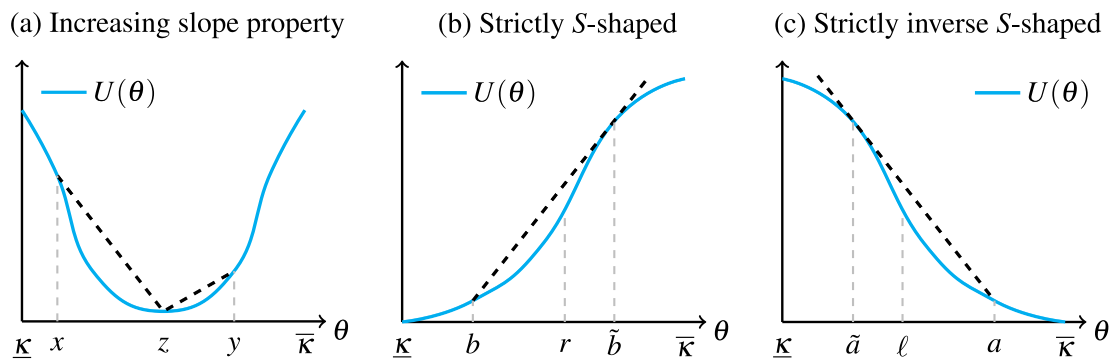
where $U(\cdot)$ is a designer's utility function defined on some closed interval $[\underline{\kappa},\overline{\kappa}]\subseteq[-1,1]$ and $F\big|_{[\underline{\kappa},\overline{\kappa}]}$ is the cdf of prior $F$ truncated on interval $[\underline{\kappa},\overline{\kappa}]$.[30] For all following results except Observation 1 we assume that $U(\cdot)$ is twice continuously differentiable.

We say that $U(\cdot)$ satisfies the *increasing-slope property at point z* if

$$\frac{U(x)-U(z)}{x-z}\leq\frac{U(y)-U(z)}{y-z}, \forall y>x$$

and strict inequality holds if $x<z<y$. Panel (a) of Figure 5 illustrates this property.[31]

Figure 5: Increasing Slope Property, S-shaped, and Inverse-S-shaped Utility Functions




Observation 1 shows that $U(\cdot)$ satisfying the single-crossing property at point $z$ ensures

[30] That is, $F\big|_{[\underline{\kappa},\overline{\kappa}]}(k)=\frac{F(k)-F(\underline{\kappa})}{F(\overline{\kappa})-F(\underline{\kappa})}$ for $k\in[\underline{\kappa},\overline{\kappa}]$ and it equals 1 (resp. 0) for $k>\overline{\kappa}$ (resp. $k<\underline{\kappa}$).

[31] Geometrically, $U(\cdot)$ satisfies the increasing-slope property at point $z$ only if for all $x\neq z$ the line segment connecting $(x,U(x))$ and $(z,U(z))$ lies above $U(\cdot)$. Notice that if $U(\cdot)$ is strictly convex on $[-1,1]$, then it satisfies the increasing-slope condition for all points $z\in[-1,1]$. The converse, however, is not true.

that any monopolistically optimal or unimprovable information policy for a designer must be Blackwell more informative than the cutoff policy $\mathscr{P}(z)$, independent of prior $F$.

**Observation 1.** *Suppose $U(\cdot)$ satisfies the increasing slope property at some point $z \in (\underline{\kappa}, \overline{\kappa})$. Then, for any continuous prior $F$ with full support on $[-1,1]$, $H \succeq_{MPS} H_{\mathscr{P}(z)}$ holds for all $H$ that either solves (MP') or is unimprovable for a designer with utility function $U(\cdot)$.*[32]

This observation can be easily extended to any general censorship policy with a non-degenerate revealing interval $[a,b]$: if $U(\cdot)$ satisfies increasing property for all points $z \in [a,b]$ with $a < b$, then for all continuous prior $F$ any monopolistically optimal or unimprovable information policy must be Blackwell more informative than censorship policy $\mathscr{P}(a,b)$.

Next we introduce two important classes of utility functions that are *strictly S-shaped* or *strictly inverse S-shaped*, respectively. These are illustrated in panels (b) and (c) of Figure 5.

**Definition 2.** *$U(\cdot)$ is* strictly S-shaped *on $[\underline{\kappa}, \overline{\kappa}]$ if there exists an inflection point $r \in [\underline{\kappa}, \overline{\kappa}]$ such that $U(\cdot)$ is strictly convex on $[\underline{\kappa}, r]$ and strictly concave on $[r, \overline{\kappa}]$. $U(\cdot)$ is* strictly inverse S-shaped *if there exists an inflection point $\ell \in [\underline{\kappa}, \overline{\kappa}]$ such that $U(\cdot)$ is strictly concave on $[\underline{\kappa}, \ell]$ and strictly concave on $[\ell, \overline{\kappa}]$.*[33]

The next two observations establish, for these two classes of utility functions, the optimality of censorship policies under both monopolistic and competitive persuasion.

**Observation 2.** *Let $U(\cdot)$ be designer $m$'s utility function and suppose that $U(\cdot)$ is strictly S-shaped on $[\underline{\kappa}, \overline{\kappa}]$ with inflection point $r$. The following properties hold:*

1. *Suppose $H$ solves problem (MP'), then $H$ is uniquely induced by an upper censorship policy $\mathscr{P}(\underline{\kappa}, b)$ with $b \geq \underline{\kappa}$ satisfying the complementary slackness condition:*

$$\left(\tilde{b} - b\right) U'(\tilde{b}) \leq U(\tilde{b}) - U(b) \tag{FOC: b}$$

   *where $\tilde{b} = \mathbb{E}_F[k | k \in [b, \overline{\kappa}]]$ and (FOC: b) is binding whenever $b > \underline{\kappa}$ (cf. panel (b) of Figure 5). Moreover, $b$ and $\tilde{b}$ satisfy $b < r < \tilde{b}$ for $r < \overline{\kappa}$, and $b = r$ if $r = \overline{\kappa}$.*

2. *Let $\mathscr{H}_m$ denote the set of unimprovable outcomes for designer $m$, then (i) $H \succeq_{MPS} H_{\mathscr{P}(\underline{\kappa}, b)}$ for all $H \in \mathscr{H}_m$, and (ii) $H_{\mathscr{P}(\underline{\kappa}, d)} \in \mathscr{H}_m$ for all $d \in [b, \overline{\kappa}]$.*

[32] Formally, $H$ is unimprovable for a designer with utility function $U(\cdot)$ if $\int_{\underline{\kappa}}^{\overline{\kappa}} U(\theta) dH(\theta) \geq \int_{\underline{\kappa}}^{\overline{\kappa}} U(\theta) d\widetilde{H}(\theta)$ holds for all $\widetilde{H} \in \Delta([\underline{\kappa}, \overline{\kappa}])$ that satisfies $F \succeq_{MPS} \widetilde{H} \succeq_{MPS} H$.

[33] Both definitions include strict convex and concave functions as special cases.

3. *Given any pure strategy profile of other designers $\pi_{-m}$, there exists some $d \in [b, r]$ such that the upper censorship policy $\mathscr{P}(\underline{\kappa}, d)$ is designer m's best response to $\pi_{-m}$.*

**Observation 3.** *Let $U(\cdot)$ be designer m's utility function and suppose that $U(\cdot)$ is strictly inverse S-shaped on $[\underline{\kappa}, \overline{\kappa}]$ with inflection point $\ell$. The following properties hold:*

1. *Suppose H solves problem (MP'), then H is uniquely induced by an lower censorship policy $\mathscr{P}(a, \overline{\kappa})$ with $a \leq \overline{\kappa}$ satisfying the complementary slackness condition:*

$$(a - \tilde{a})U'(\tilde{a}) \leq U(a) - U(\tilde{a}) \tag{FOC: a}$$

*where $\tilde{a} = \mathbb{E}_F[k|k \in [\underline{\kappa}, a]]$ and (FOC: a) is binding whenever $a < \overline{\kappa}$ (cf. panel (c) of Figure 5). Moreover, a and $\tilde{a}$ satisfy $a > \ell > \tilde{a}$ for $\ell > \underline{\kappa}$, and $a = \ell$ if $\ell = \underline{\kappa}$.*

2. *Let $\mathscr{H}_m$ denote the set of unimprovable outcomes for designer m, then (i) $H \succeq_{MPS} H_{\mathscr{P}(a,\overline{\kappa})}$ for all $H \in \mathscr{H}_m$, and (ii) $H_{\mathscr{P}(c,\overline{\kappa})} \in \mathscr{H}_m$ for all $c \in [\underline{\kappa}, a]$.*

3. *Given any pure strategy profile of other designers $\pi_{-m}$, there exists some $c \in [a, \ell]$ such that the lower censorship policy $\mathscr{P}(c, \overline{\kappa})$ is designer m's best response to $\pi_{-m}$.*

The first statement of the two observations above suggest that under monopolistic persuasion *upper* (resp. *lower*) censorship policies are uniquely optimal for a designer whose utility function $U(\cdot)$ is strictly S-shaped (resp. inverse S-shaped). These are proved by Kolotilin, Mylovanov and Zapechelnyuk (2021). The remaining statements of these observations extend this insight to competition in persuasion. For a designer whose whose utility function is either strictly S-shaped or inverse S-shaped, any censorship policy that is no less informative than the monopolistic optimal one is unimprovable for him. Moreover, it is without loss of optimality for him to restrict attention to a proper subset of censorship policies in the following sense: given any pure strategy profile of other designers, he can always find a best response from this subset of censorship policies. Notice that all policies in this subset are no less informative than his monopolistically optimal one.

Finally, we turn to competition in persuasion with $|M| \geq 2$ designers. For each designer $m \in M$ let $U_m(\cdot)$ denote his utility function. Our last observation states an easily verifiable sufficient condition for full disclosure to be the unique equilibrium outcome. This condition facilitates a simple proof for Theorem 2. Given $\{U_m(\cdot)\}_{m\in M}$, we say that *strictly convex finite open cover property* holds on interval $[x, y]$ if there exists a finite collection of open intervals $\{I_j\}_{j=1}^{J}$ such that (i) $[x, y] \subset \cup_{j=1}^{J} I_j$, and (ii) on each $I_j$ there exists some $m \in M$ such that $U_m(\cdot)$ is strictly convex.

**Observation 4.** *Let $\{U_m(\cdot)\}_{m\in M}$ be a profile of utility functions defined on $[\underline{\kappa},\overline{\kappa}]$. If strictly convex finite open cover property holds on $[\underline{\kappa},\overline{\kappa}]$, then full disclosure is the unique equilibrium outcome.*[34]

The following corollary, which is an immediate consequence of Observations 1 and 4, is useful to make robust predictions regarding whether under competition any joint information policy induced in equilibrium must be Blackwell more informative than some given censorship policy $\mathscr{P}(a,b)$, independent of the prior $F$.

**Corollary 2.** *Consider any $\underline{\kappa} < a \leq b < \overline{\kappa}$. Then $H \succeq_{MPS} H_{\mathscr{P}(a,b)}$ holds for all $H \in \mathscr{H} = \cap_{m\in M}\mathscr{H}_m$ under the following conditions:*

1. *Strictly convex finite open cover property holds on $[a,b]$.*
2. *There exists $i \in M$ such that $U_i(\cdot)$ satisfies increasing slope property at $a$ if $a > \underline{\kappa}$.*
3. *There exists $j \in M$ such that $U_j(\cdot)$ satisfies increasing slope property at $b$ if $b < \overline{\kappa}$.*

### 5.3.3 Proofs of main results

Now we come back to our model and consider any designer $m \in M$ for whom the single-crossing property holds. As explained in Section 4, there exists an $n_m \geq 0$ such that $\phi_n^m(x) - x$ is single-crossing on $[-1,1]$ and a unique switching state $z_n^m$ as defined by (4) for all $n > n_m$. The following two lemmas, whose proofs are in Appendix C, summarize important curvature properties about $W_n^m(\cdot)$ when single-crossing property holds.

**Lemma 4.** *Suppose $\phi_n^m(x) - x$ crosses zero from above at an interior point $z_n^m \in (-1,1)$. Then $W_n^m(\cdot)$ satisfies increasing slope property at point $z_n^m$.*

[34] In fact, this condition is close to necessary; if there exists a non-trivial interval $I \subseteq [\underline{\kappa},\overline{\kappa}]$ on which all designers' utility functions are linear or concave, then there exists at least one unimprovable outcome $\pi$ such that $H_\pi \in \mathscr{H}$ and $\pi$ reveals no information for state realizations $k \in I$. Mylovanov and Zapechelnyuk (2021) provide such a counter example in a setup with two designers. Moreover, in this statement "open cover" cannot be replaced by "closed cover". Otherwise a counter example, inspired by Hu and Zhou (2020), can be constructed as follows. Let all designers have a common utility function $U(\cdot)$ which has a kink $c \in (\underline{\kappa},\overline{\kappa})$ such that $U(\cdot)$ is strictly convex on $[\underline{\kappa},c]$ and $[c,\overline{\kappa}]$, but $U'(\theta)$ drops discontinuously at $c$ so that $U(\cdot)$ is not globally convex. Because utility functions are common, the monopolistically optimal information policy is also an equilibrium outcome. In this setup, Hu and Zhou (2020) show that the monopolisticallly optimal information policy is generically not full disclosure.

**Lemma 5.** *Suppose single-crossing property holds for designer $m \in M$. Then there exists an $N_m \geq 0$ such that for all $n > N_m$ there are $\ell_n^m$ and $r_n^m$ with $-1 \leq \ell_n^m \leq z_n^m \leq r_n^m \leq 1$ such that the following two properties hold:*[35]

1. *$W_n^m(\cdot)$ is strictly S-shaped on $[z_n^m, 1]$ with inflection point $r_n^m \in [z_n^m, 1]$.*

2. *$W_n^m(\cdot)$ is strictly inverse-S-shaped on $[-1, z_n^m]$ with inflection point $\ell_n^m \in [-1, z_n^m]$.*

*In particular, if $g(\cdot)$ is log-concave and $\rho_m$ is sufficiently close to 0, then $N_m = 0$ so the above curvature properties hold for all $n > 0$.*

With these lemmas we are ready to prove our main results. In all proofs below we directly use quantities $N_m$, $\ell_n^m$ and $r_n^m$ identified in Lemma 5. We first prove Theorems 1 and 3 because the arguments are closely related.

*Proof of Theorem 1.* Depending on the value of $z_n^m$, we distinguish between three cases.

*Case 1*: $z_n^m = -1$ so that $\phi_n^m(x) < x$ for all $x \in (-1,1)$. By Lemma 5, $\ell_n^m = -1$ and $W_n^m(\cdot)$ is strictly S-shaped on $[-1,1]$ with inflection point $r_n^m \in [-1,1]$ for all $n > N_m$. Therefore, by statement (1) of Observation 2, the monopolistically optimal information policy is uniquely given by an upper-censorship policy $\mathscr{P}(-1, b_n^m)$. Threshold $b_n^m$ satisfies condition (FOC: b) with $[\underline{\kappa}, \overline{\kappa}] = [-1,1]$ and $U(\cdot) = W_n^m(\cdot)$. This proves statement (2) of Theorem 1.

*Case 2*: $z_n^m = 1$ so that $\phi_n^m(x) > x$ for all $x \in (-1,1)$. By Lemma 5, $r_n^m = 1$ and $W_n^m(\cdot)$ is strictly inverse S-shaped on $[-1,1]$ with inflection point $\ell_n^m \in [-1,1]$ for all $n > N_m$. Hence, by statement (1) of Observation 3, the monopolistically optimal information policy is uniquely given by a lower-censorship policy $\mathscr{P}(a_n^m, 1)$. Threshold $a_n^m$ satisfies condition (FOC: a) with $[\underline{\kappa}, \overline{\kappa}] = [-1,1]$ and $U(\cdot) = W_n^m(\cdot)$. This proves statement (3) of Theorem 1.

*Case 3*: $\phi_n^m(x) - x$ crosses zero from above at a unique interior point $z_n^m \in (-1,1)$. By Lemma 4, $W_n^m(\cdot)$ satisfies increasing slope property at point $z_n^m$ and hence by Observation 1 any solution $H$ to problem (MP) must satisfy $H \succeq_{MPS} H_{\mathscr{P}(z_n^m)}$. Therefore, the monopolistic persuasion problem can be decomposed into two auxiliary problems on intervals $[-1, z_n^m]$ and $[z_n^m, 1]$, respectively. Recall that for all $n > N_m$, $W_n^m(\cdot)$ is strictly S-shaped on $[z_n^m, 1]$ with inflection point $r_n^m$ and strictly inverse S-shaped on $[-1, z_n^m]$ with inflection point

[35] In fact, single-crossing property is almost necessary; suppose instead that $\phi^m(x) - x$ crosses zero from below at some point, then this lemma no longer holds and for sufficiently large $n$ there exists some interval $[x,y] \subset (-1,1)$ and $\varepsilon > 0$ such that $W_n^m(\cdot)$ is strictly concave on $[x,y]$ but is strictly convex on $[x-\varepsilon, x]$ and $[y, y+\varepsilon]$, respectively. In this case, it follows from duality arguments in Dworczak and Martini (2019) and Kolotilin, Mylovanov and Zapechelnyuk (2021) that there exists some prior $F$ under which the optimal information policy is not censorship.

$\ell_n^m$. Observations 2 and 3 together imply that the solutions to these auxiliary problems are $\mathscr{P}(a_n^m, z_n^m)$ and $\mathscr{P}(z_n^m, b_n^m)$, with $a_n^m$ and $b_n^m$ characterized by conditions (FOC: a) and (FOC: b), respectively (again with $[\underline{\kappa}, \overline{\kappa}] = [-1, 1]$ and $U(\cdot) = W_n^m(\cdot)$). These together imply that the optimal solution is uniquely given by a censorship policy $\mathscr{P}(a_n^m, b_n^m)$. Now we show that $a_n^m < z_n^m < b_n^m$ must hold. By (7) we have

$$\begin{aligned} W_n^{m\prime\prime}(z_n^m) &= \hat{g}_n(z_n^m; q)\left(2 - \phi_n^{m\prime}(z_n^m)\right) + \left(z_n^m - \phi_n^m(z_n^m)\right)\hat{g}_n'(z_n^m; q) \\ &= \hat{g}_n(z_n^m; q)\left(2 - \phi_n^{m\prime}(z_n^m)\right) > \hat{g}_n(z_n^m; q) > 0 \end{aligned}$$

The second step holds because $z_n^m - \phi_n^m(z_n^m) = 0$ by definition of $z_n^m$, and the third step holds because single-crossing property requires $\phi_n^{m\prime}(z_n^m) < 1$ whenever $z_n^m - \phi_n^m(z_n^m) = 0$. Therefore, $W_n^m(\theta)$ is strictly convex in a neighborhood around $z_n^m$ and thus $r_n^m > z_n^m$. This implies that

$$W_n^{m\prime}(z_n^m) < \frac{W_n^m(\theta) - W_n^m(z_n^m)}{\theta - z_n^m} \tag{9}$$

holds for all $\theta \in [z_n^m, r_n^m]$. Moreover, since $W_n^m(\cdot)$ satisfies increasing slope property at point $z_n^m$, the right-hand side of (9) is increasing in $\theta$. Therefore, (9) must hold for all $\theta > z_n^m$ and the optimality condition (FOC: b) cannot be satisfied at $b_n^m = z_n^m$. This implies that $b_n^m > z_n^m$ must hold. $a_n^m < z_n^m$ can be proved analogously. This proves statement (1) of Theorem 1. □

*Proof of Theorem 3.* For all $n > N_m$, define

$$\mathscr{P}_n^m := \{\mathscr{P}(c, d) : \; [a_n^m, b_n^m] \subseteq [c, d] \subseteq [\ell_n^m, r_n^m]\} \tag{10}$$

where $a_n^m$ and $b_n^m$ are the thresholds of the monopolistically optimal censorship policy. Clearly, $\mathscr{P}_n^m$ is a subset of censorship policies. We show below that for any pure strategy profile $\pi_{-m}$ of other designers, there exists a $\pi_m \in \mathscr{P}_n^m$ such that $\pi_m$ is designer $m$'s best response to $\pi_{-m}$. Again, we distinguish between three cases depending on the value of $z_n^m$.

If $z_n^m = -1$, then $\ell_n^m = a_n^m = 0$ and $W_n^m(\cdot)$ is strictly S-shaped on $[-1, 1]$ with inflection point $r_n^m \geq b_n^m$. By statement (3) of Observation 2, for any $\pi_{-m}$ there exists $d \in [b_n^m, r_n^m]$ such that $\pi_m = \mathscr{P}(-1, d)$ is designer $m$'s best response to $\pi_{-m}$. Similarly, if $z_n^m = 1$ then $r_n^m = b_n^m = 1$ and $W_n^m(\cdot)$ is strictly inverse S-shaped on $[-1, 1]$ with inflection point $\ell_n^m \leq a_n^m$. Statement (3) of Observation 3 implies that for any $\pi_{-m}$ there exists $c \in [\ell_n^m, a_n^m]$ such that $\pi_m = \mathscr{P}(c, 1)$ is designer $m$'s best response to $\pi_{-m}$. In both cases $\pi_m \in \mathscr{P}_n^m$ holds.

Now we consider the case $z_n^m \in (-1, 1)$ and let $\pi_m$ be any best response to $\pi_{-m}$ for designer $m$. Because the information environment is Blackwell-connected, the induced joint information policy $\langle \pi_m, \pi_{-m} \rangle$ must be unimprovable for designer $m$. Recall that $W_n^m(\cdot)$ satisfies increasing slope property at point $z_n^m$ (cf. Lemma 4), it follows from Observation 1 that $\langle \pi_m, \pi_{-m} \rangle$ must be Blackwell more informative than the cutoff policy $\mathscr{P}(z_n^m)$. This implies that there always exists a best response $\pi_m$ that is Blackwell more informative than $\mathscr{P}(z_n^m)$ (i.e., $H_{\pi_m} \succeq_{MPS} H_{\mathscr{P}(z_n^m)}$).[36] For such $\pi_m$, it must be a best response to $\pi_{-m}$ on both $[-1, z_n^m]$ and $[z_n^m, 1]$ separately. Recall that $W_n^m(\cdot)$ is strictly inverse S-shaped on $[-1, z_n^m]$ with inflection point $\ell_n^m < z_n^m$ and strictly S-shaped on $[z_n^m, 1]$ with inflection point $r_n^m > z_n^m$. It follows from the previous argument that there exists $c \in [\ell_n^m, a_n^m]$ and $d \in [b_n^m, r_n^m]$ such that $\mathscr{P}(c, z_n^m)$ is a best response to $\pi_{-m}$ on $[-1, z_n^m]$ and $\mathscr{P}(z_n^m, d)$ is a best response on $[z_n^m, 1]$. These together produce a censorship policy $\pi_m = \mathscr{P}(c, d)$, which belongs to $\mathscr{P}_n^m$, that is a best response to $\pi_{-m}$. $\square$

Next we prove Theorem 2.

*Proof of Theorem 2.* By Observation 4, we only need to establish that strictly finite open cover property holds on $[-1, 1]$ to complete the proof. By (7) we have

$$W_n^{m\prime\prime}(k) = \left(2 - \phi_n^{m\prime}(k)\right) \hat{g}_n(k;q) + \left(k - \phi_n^m(k)\right) \hat{g}_n'(k;q)$$

The first term is strictly positive for all $m \in M$ because $\phi_n^{m\prime}(k) < 2$ for all $k \in [-1, 1]$ by Assumption 1. Moreover, since all states are disagreeing states, for any $k \in [-1, 1]$ there exist $\mathrm{I}, \mathrm{II} \in M$ such that

$$\phi_n^{\mathrm{I}}(k) \leq k \leq \phi_n^{\mathrm{II}}(k)$$

holds. So, no matter what the sign of $\hat{g}_n'(k;q)$ is, $\left(k - \phi_n^m(k)\right) \hat{g}_n'(k;q)$ must be non-negative for at least one $m \in \{\mathrm{I}, \mathrm{II}\}$. Hence, for any $k \in [-1, 1]$, there exists some $m \in M$ for whom $W_n^{m\prime\prime}(k) > 0$ holds. By continuity of $W_n^{m\prime\prime}(\cdot)$, $W_n^m(\cdot)$ must be strictly convex on an open interval $I_k$ that contains $k$. $\{I_k\}_{k \in [-1,1]}$ is then an collection of open intervals that covers $[-1, 1]$ and by Heine-Borel Theorem there exists a finite subcover. This implies strictly convex finite open cover property on $[-1, 1]$. $\square$

Combining Observations 1 to 3 with the curvature properties of $W_n^m(\cdot)$ summarized in Lemmas 4 and 5, we obtain the following corollary.

[36] To see why, let $\pi = \langle \pi_m, \pi_{-m} \rangle$ and $\pi' = \langle \pi_m, \pi_{-m}, \mathscr{P}(z_n^m) \rangle$. The best response property of $\pi_m$ ensures that $H_\pi \succeq_{MPS} H_{\mathscr{P}(z_n^m)}$. Therefore, $H_\pi = H_{\pi'}$. Consider $\pi_m' = \langle \pi_m, \mathscr{P}(z_n^m) \rangle$ (which is always feasible) and observe that $H_{\pi_m'} \succeq_{MPS} H_{\mathscr{P}(z_n^m)}$ and $\pi' = \langle \pi_m', \pi_{-m} \rangle$. Then $\pi_m'$ must also be a best response to $\pi_{-m}$ because $H_\pi = H_{\pi'}$.

**Corollary 3.** *Suppose single-crossing property holds for each designer $m \in M$. Let $n > N_m$ so that each designer $m$'s monopolistically optimal censorship policy is given by $\mathscr{P}(a_n^m, b_n^m)$. Then the following properties hold:*

1. *$H \succeq_{MPS} H_{\mathscr{P}(a_n^m, b_n^m)}$ holds for all $H \in \mathscr{H}_m$.*
2. *$\mathscr{P}(a,b) \in \mathscr{H}_m$ for all $a \in [-1, a_n^m]$ and $b \in [b_n^m, 1]$.*

Corollary 3 says that under single-crossing property and sufficiently large $n$ all unimprovable outcomes for designer $m$ must be no less informative than his monopolistically optimal censorship policy. Moreover, all censorship policies that are more informative than the monopolistically optimal one are unimprovable for designer $m$. With these we can prove statement (1) of Theorem 4.

*Proof of Statement (1) of Theorem 4.* Consider any $N \geq \max_{m \in M} N_m$ and $n > N$. Let $z_n^{\max} := \max_{m \in M} \{z_n^m\}$ and $z_n^{\min} := \max_{m \in M} \{z_n^m\}$. Then Lemma 4 and Corollary 2 together imply that $H \succeq_{MPS} H_{\mathscr{P}(z_n^{\min}, z_n^{\max})}$ must hold for all $H \in \mathscr{H} = \cap_{m \in M} \mathscr{H}_m$. By Corollary 3, $H \succeq_{MPS} H_{\mathscr{P}(a_n^m, b_n^m)}$ must hold for all $H \in \mathscr{H}_m$. Moreover, for each $m \in M$, it holds that $H_{\mathscr{P}(c,d)} \in \mathscr{H}_m$ for all $c \in [-1, a_n^m]$ and $d \in [b_n^m, 1]$. Therefore, $\mathscr{P}(a_n^{\min}, b_n^{\max})$ is unimprovable for all designers and hence $H_{\mathscr{P}(a_n^{\min}, b_n^{\max})} \in \mathscr{H}$. To conclude the proof we show that any $H \in \mathscr{H}$ must be weakly more informative than $\mathscr{P}(a_n^{\min}, b_n^{\max})$, that is $H \succeq_{MPS} H_{\mathscr{P}(a_n^{\min}, b_n^{\max})}$. Let $\tilde{i}$ (resp. $\tilde{j}$) denote the identity of the designer with $a_n^{\tilde{i}} = a_n^{\min}$ (resp. $b_n^{\tilde{j}} = b_n^{\max}$). Recall that any $H \in \mathscr{H}$ must satisfy $H \succeq_{MPS} H_{\mathscr{P}(a_n^m, b_n^m)}$ with $a_n^m \leq z_n^m \leq b_n^m$ for all $m \in M$. The choices of $\tilde{i}$ and $\tilde{j}$ imply that $\left[a_n^{\tilde{i}}, b_n^{\tilde{i}}\right]$, $\left[z_n^{\min}, z_n^{\max}\right]$ and $\left[a_n^{\tilde{j}}, b_n^{\tilde{j}}\right]$ are overlapping and $\left[a_n^{\tilde{i}}, b_n^{\tilde{i}}\right] \cup \left[z_n^{\min}, z_n^{\max}\right] \cup \left[a_n^{\tilde{j}}, b_n^{\tilde{j}}\right] = \left[a_n^{\min}, b_n^{\max}\right]$. Therefore, $H \succeq_{MPS} H_{\mathscr{P}(a_n^{\min}, b_n^{\max})}$ must hold for all $H \in \mathscr{H}$ and this completes the proof. □

# 6 Comparative statics

Theorem 1 in the previous section shows that, if single-crossing property holds for a designer, then for sufficiently large $n$ some censorship policy with revealing interval $[a_n, b_n]$ is uniquely optimal for this designer under monopolistic persuasion. It is then natural to ask how do the two thresholds $a_n$ and $b_n$ vary with the designer's preference and the voting rule. In this section we answer this question. We assume that there is a monopoly designer for whom the single-crossing property holds and drop index $m$ for convenience of exposure. Omitted proofs and materials are relegated to Appendix D.

## 6.1 Effects of designer's preference shifts

We first examine how $a_n$ and $b_n$ are affected by the designer's preference. To do so, we hold $\rho$ fixed and consider a shift of the designer's preference towards the reform. Such preference shift, for instance, can occur if $\rho < 1$ and $\chi$ decreases so that the designer's own threshold of acceptance for reform becomes lower. In this case the designer's personal payoff from reform increases. Proposition 1 explains how such a preference shift towards the reform affects the designer's optimal censorship policy.

**Proposition 1.** *Suppose $\rho < 1$ and either condition (i) or (ii) in Lemma 3 holds. Consider any $\chi_I > \chi_{II}$. Then, for sufficiently large n, as $\chi$ decreases from $\chi_I$ to $\chi_{II}$ the following holds:*

1. *$a_n$ weakly decreases, strictly so if $a_n \in (-1, 1)$ under $\chi = \chi_I$.*
2. *$b_n$ weakly decreases, strictly so if $b_n \in (-1, 1)$ under $\chi = \chi_I$.*

*In words, as the designer's personal payoff from reform decreases, his optimal censorship policy will censor fewer states upwards but more states downwards.*

The intuition of Proposition 1 is as follows. On the one hand, as the designer's personal payoff from reform increases, he becomes more tempted to persuade voters to pass the reform. Therefore, $b_n$ decreases because the designer is now tempted to manipulate voters' beliefs upwards in more states which he would previously be willing to reveal truthfully. On the other hand, such preference shift makes the designer less tempted to persuade voters to maintain the status quo. Consequently, $a_n$ also decreases because the designer is now willing to truthfully reveal more states in which he would previously lie to manipulate voters' beliefs downwards.

For a pro-social designer with $\rho > 0$, a similar preference shift towards the reform could also occur if his weighting function $w(\cdot)$ decreases in the sense of first order stochastic dominance. In this way the designer systematically puts more weights on voters whose ex-post type realizations are lower and hence receive higher payoffs under reform. Such a change also makes the designer favors reform more and thus more tempted to persuade voters to pass the reform. Following the intuition discussed above, one may expect that the result in Proposition 1 continues to hold in this case. In Appendix D we show that, under some mild conditions, this is indeed true: for sufficiently large $n$ both $a_n$ and $b_n$ decrease as the designer's weighting function $w(\cdot)$ shifts from $w_{\mathrm{I}}(\cdot)$ to $w_{\mathrm{II}}(\cdot)$, where $w_{\mathrm{I}}(\cdot)$ and $w_{\mathrm{II}}(\cdot)$ are absolutely continuous cdfs on $[0,1]$ and $w_{\mathrm{I}}(\cdot)$ first order stochastically dominates $w_{\mathrm{II}}(\cdot)$.

## 6.2 Effects of varying the voting rule

Next we turn to the effects of changing voting rule $q$, the required vote share to pass the reform, on thresholds $a_n$ and $b_n$. The results depend critically on whether the designer is purely self-interested ($\rho = 0$) or pro-social ($\rho > 0$).

**Proposition 2.** *Suppose $\rho = 0$ and consider any $q_I, q_{II} \in (0,1)$ with $q_{II} > q_I$. Then, for sufficiently large n, as q rises from $q_I$ to $q_{II}$ the following holds:*

1. *$a_n$ weakly increases, strictly so if $a_n \in (-1,1)$ under $q = q_I$.*
2. *$b_n$ weakly increases, strictly so if $b_n \in (-1,1)$ under $q = q_I$.*

*In words, if the designer is purely self-interested, then increasing the required vote share to pass the reform makes him censor more states upwards but fewer states downwards.*

Proposition 2 is driven by a *stringency effect*: as $q$ increases it becomes harder to persuade the pivotal voter to pass the reform while easier to persuader her to maintain the status quo (Alonso and Câmara, 2016*a*). This is because the pivotal voter's threshold of acceptance $v^{(nq+1)}$ increases in $q$. For a self-interested designer who does not care about voter welfare, his best response would be to shift up both thresholds $a_n$ and $b_n$. By raising $b_n$ the designer makes the upward pooling message '$k > b_n$' more effective in persuading the pivotal voter – who is now harder to convince – to pass the reform. In the meanwhile, the demand for the effectiveness of the downward pooling message '$k < a_n$' is lower because it is now easier to convince the pivotal voter to maintain the status quo. Therefore, by increasing $a_n$ the designer can expand the set of states in which he can successfully persuade the pivotal voter to maintain the status quo at minor costs of reduced effectiveness.

An important implication of Proposition 2 is that, under the stringency effect alone, both $b_n$ and $a_n$ increase monotonically in $q$ for sufficiently large $n$. In particular, if $-1 < a_n < b_n < 1$ holds under some $q$, then both $a_n$ and $b_n$ must strictly increase as $q$ rises. In this case, the informativeness of the optimal censorship policies before and after the change in voting rule are not Blackwell comparable. Proposition 3 shows that these properties immediately break down once the designer cares about voter welfare.

**Proposition 3.** *Suppose $\rho > 0$, both G and $1 - G$ are strictly log-concave, n is sufficiently large, and $-1 < a_n < b_n < 1$ holds under $q = q_I$. Then there exists $q_I, q_{II} \in (0,1)$ with $q_{II} > q_I$ such that, as q increases from $q_I$ to $q_{II}$, one of the following may happen:*

1. *$a_n$ strictly decreases and $b_n$ strictly increases;*

2. *$a_n$ strictly increases and $b_n$ strictly decreases.*

*In words, if the designer is pro-social, then raising the required vote share to pass the reform may make him reveal more states both upwards and downwards, or make him censor more states in both directions.*

Proposition 3 suggests that an increase in the required vote share to pass the reform is no longer guaranteed to increase both thresholds $a_n$ and $b_n$ of a pro-social designer's optimal censorship policy. In fact, following such a change in voting rule, thresholds $a_n$ and $b_n$ could move in opposite directions. The designer's optimal censorship policy thus becomes either strictly Blackwell more or less informative, depending on whether the revelation interval $[a_n, b_n]$ strictly expands or shrinks on both sides. As explained above, neither case is possible under the stringency effect or a shift of designer's preference towards some alternative alone.

Why could $a_n$ and $b_n$ move in opposite directions as $q$ increases? It turns out that this is because, aside from the stringency effect above, an increase in $q$ also affect thresholds $a_n$ and $b_n$ through a novel *designer-preference effect*: increasing the required vote share to pass the reform can induce a shift of a pro-social designer's preference towards the reform. Therefore, through the intuition discussed in the previous subsection, the designer-preference effect *per se* drives both $a_n$ and $b_n$ downwards as $q$ increases.

The designer-preference effect here is driven by the fact that an increase in $q$ systematically shifts the designer's indifference curve $\phi_n(\cdot)$ downwards whenever $\rho > 0$.[37] To see why, let $q$ increase from $q'$ to $q''$. The pivotal voter's type thus shifts from $v^{(nq'+1)}$ to $v^{(nq''+1)}$. Under cutoff $q''$ the pivotal event $v^{(nq''+1)} = x$ necessarily implies $v^{(nq'+1)} \leq x$ (that is, the pivotal voter's type must be lower than $x$ were the cutoff $q'$). Therefore, for any fixed $x$, the event that the pivotal voter's type equals $x$ implies that the entire realized type profile is systematically lower (and thus voters' ex-post payoffs from reform becomes systematically higher) as $q$ increases. This makes any pro-social designer more leaning towards reform.

Therefore, for a pro-social designer, the net effect of an increase in $q$ on thresholds $a_n$ and $b_n$ depend on the strengths of both the stringency effect and the designer preference effect. For instance, if the designer-preference effect dominates in driving $a_n$ while the stringency effect dominates in driving $b_n$, then the net effect would be a strict expansion of

[37] This claim is proved for general pro-social preferences in Proposition A.3 in Appendix A.2. For a Utilitarian planner whose $\phi_n(\cdot)$ is given by (5), this can be straightforwardly shown by taking partial derivative with respect to $q$, which yields $\frac{\partial \phi_n(x)}{\partial q} = \frac{n}{n+1}\left(\mathbb{E}_G[v_i|v_i \leq x] - \mathbb{E}_G[v_i|v_i \geq x]\right) < 0$ for all $x \in (\underline{v}, \overline{v})$.

the revelation interval $[a_n, b_n]$ on both sides. Conversely, if the designer-preference effect dominates in driving $b_n$ while the stringency effect dominates in driving $a_n$, then the net effect would be a strict shrink of revelation interval $[a_n, b_n]$ on both sides.

# 7 An application to media and elections

In this section we turn to the normative angle and characterize a monopoly designer's asymptotic payoff as the electorate size $n \to \infty$ (Section 7.1). We then apply these asymptotic results to study the welfare impact of media bias and competition in elections (Section 7.2). Results presented in this section do not rely on the single-crossing property. Omitted proofs are relegated to Appendix E.

## 7.1 Asymptotic payoffs and election outcomes

We continue to assume that there is only one designer and omit index $m$. Let

$$v_q^* := G^{-1}(q) \quad \text{and} \quad \phi^* := \rho \int_0^1 G^{-1}(y) dw(y) + (1-\rho)\chi \ .$$

Recall that $v^{(nq+1)} \xrightarrow{p} v_q^*$ (cf. Lemma 1) and $\varphi_n(v) \xrightarrow{p} \phi^*$ (cf. Lemma 2). Therefore, as $n \to \infty$, the pivotal voter prefers reform (status quo) almost surely if $k > (<) v_q^*$, while the designer prefers reform (status quo) almost surely if $k > (<) \phi^*$. We focus on the interesting case $v_q^* \in (-1, 1)$ in which the pivotal voter's preference is state-dependent as $n \to \infty$.

Let $W_n$ denote the expected payoff of a monopoly designer under his optimal information policy given electorate size $n$.[38] Theorem 5 characterizes the designer's asymptotic payoff $W^* := \lim_{n\to\infty} W_n$ and the election outcome as $n \to \infty$.

**Theorem 5.** *Suppose $v_q^* \in (-1, 1)$ and $\phi^* \in [-1, 1]$.*[39] *Let*

$$\overline{\phi}(v_q^*) := \sup \left\{ y \in \mathbb{R} : \mathbb{E}_F \left[ k | k \le y \right] \le v_q^* \right\} \tag{11}$$

$$\underline{\phi}(v_q^*) := \inf \left\{ y \in \mathbb{R} : \mathbb{E}_F \left[ k | k \ge y \right] \ge v_q^* \right\} \tag{12}$$

*Then $W^*$ and the asymptotic election outcome are characterized as follows:*

[38] That is, $W_n$ is the value of the monopolistic persuasion problem (MP) with electorate size $n$.

[39] The restriction $\phi^* \in [-1, 1]$ simplifies exposure and is without loss of generality. For this theorem, any $\phi^* < -1$ (resp. $\phi^* > 1$) is equivalent to the case $\phi^* = -1$ (resp. $\phi^* = 1$).

1. *If $\underline{\phi}(v_q^*) \leq \phi^* \leq \overline{\phi}(v_q^*)$, then $W^* = \int_{\phi^*}^1 (k-\phi^*)\,dF(k)$ and the reform is adopted with probability 1 (0) as $n \to \infty$ when $k > (<) \phi^*$.*

2. *If $\phi^* < \underline{\phi}(v_q^*)$, then $W^* = \int_{\underline{\phi}(v_q^*)}^1 (k-\phi^*)\,dF(k)$ and the reform is adopted with probability 1 (0) as $n \to \infty$ when $k > (<) \underline{\phi}(v_q^*)$.*

3. *If $\phi^* > \overline{\phi}(v_q^*)$, then $W^* = \int_{\overline{\phi}(v_q^*)}^1 (k-\phi^*)\,dF(k)$ and the reform is adopted with probability 1 (0) as $n \to \infty$ when $k > (<) \overline{\phi}(v_q^*)$.*

The intuition of Theorem 5 is as follows. For the case $\underline{\phi}(v_q^*) \leq \phi^* \leq \overline{\phi}(v_q^*)$, it follows from (11) and (12) that $\mathbb{E}_F[k|k < \phi^*] \leq \phi^* \leq \mathbb{E}_F[k|k > \phi^*]$ holds. Therefore, the designer can ensure his preferred alternative being elected with probability one as $n \to \infty$ by using a simple cutoff information policy that reveals whether the realized state is above or below $\phi^*$. Under this simple cutoff policy the reform is passed with probability one (zero) as $n \to \infty$ for $k > (<) \phi^*$, and the designer's ex-ante expected payoff converges in probability to $\int_{-1}^1 \max\{0, k-\phi^*\} dF(k) = \int_{\phi^*}^1 (k-\phi^*) dF(k)$. Clearly, the designer cannot improve upon this because his preferred outcome is already elected with probability one as $n \to \infty$. Finally, notice that $\underline{\phi}(v_q^*) \leq \phi^* \leq \overline{\phi}(v_q^*)$ holds if (i) $\phi^*$ is close to $v_q^*$ so that the ex-ante conflict of interests between the designer and the pivotal voter is low, or if (ii) $v_q^*$ is close to $\mathbb{E}_F[k]$ so that the pivotal voter is almost indifferent between the reform and the status quo a priori (in this case even very weak evidence can manage to persuade the pivotal voter).

Suppose instead $\phi^* < \underline{\phi}(v_q^*)$, then $\mathbb{E}_F[k|k \geq \phi^*]$ and $\mathbb{E}_F[k|k \leq \phi^*]$ are both strictly lower than $v_q^*$. A simple cutoff policy that only reveals whether realized state $k$ is above or below $\phi^*$ is therefore no longer able to convince the pivotal voter to always choose the designer's favored outcome. In this case, the designer must truthfully reveal whether the realized state $k$ is above or below cutoff $\underline{\phi}(v_q^*)$ in order to convince the pivotal voter to choose reform. The designer's preferred alternative will therefore almost surely not be elected when $k \in (\phi^*, \underline{\phi}(v_q^*))$. The larger this interval is, the more losses the designer suffers. The situation is similar when $\phi^* > \overline{\phi}(v_q^*)$.

Next we compare $W^*$ with two important benchmarks. Let $\overline{W}$ be the designer's payoff under his *omniscient* control – i.e., he directly observes state $k$ and voters' type profile $v$ and dictates the election outcome – as $n \to \infty$. If the designer is a social planner (i.e., $\rho = 1$) who maximizes some voter welfare function, then $\overline{W}$ corresponds to the asymptotic voter welfare under the first best scenario. On the other hand, let $W^{\text{Full}}$ be the designer's payoff under full information disclosure as $n \to \infty$. Since $\varphi_n(v) \xrightarrow{p} \phi^*$ and $v^{(nq+1)} \xrightarrow{p} v_q^*$, we have

$$\overline{W} = \int_{\phi^*}^{1} (k - \phi^*)\, dF(k) \tag{13}$$

$$W^{\text{Full}} = \int_{v_q^*}^{1} (k - \phi^*)\, dF(k) \tag{14}$$

**Theorem 6.** *Suppose $v_q^* \in (-1,1)$. Then $\overline{W}$, $W^*$ and $W^{Full}$ are ranked as follows:*

1. *If $v_q^* = \phi^*$, then $\overline{W} = W^* = W^{Full}$.*
2. *If $v_q^* \neq \phi^*$ and $\underline{\phi}(v_q^*) \leq \phi^* < \overline{\phi}(v_q^*)$, then $\overline{W} = W^* > W^{Full}$.*
3. *If $\phi^* < \underline{\phi}(v_q^*)$ or $\phi^* > \overline{\phi}(v_q^*)$, then $\overline{W} > W^* > W^{Full}$.*

*Proof.* Let $\gamma(x) := \int_x^1 (k - \phi^*)\, dF(k)$ for $x \in [-1,1]$. By (13) and (14) we have $\overline{W} = \gamma(\phi^*)$ and $W^{\text{Full}} = \gamma(v_q^*)$. Note that $\gamma'(x) = (\phi^* - x) f(x) > (<) 0$ for $x < (>) \phi^*$. This suggests that $\gamma(x)$ is strictly increasing on $[-1, \phi^*]$ and strictly decreasing on $[\phi^*, 1]$. Therefore, $\overline{W} \geq W^{\text{Full}}$ and equality holds if and only if $\phi^* = v_q^*$ whenever $v_q^* \in (-1,1)$. Moreover, by Theorem 5, $W^* = \gamma(\phi^*) = \overline{W}$ if $\underline{\phi}(v_q^*) \leq \phi^* \leq \overline{\phi}(v_q^*)$. These together establish statements (1) and (2). To show statement (3), consider $\phi^* > \overline{\phi}(v_q^*)$ first. In this case, it follows from Theorem 5 that $W^* = \gamma(\overline{\phi}(v_q^*))$ with $\overline{\phi}(v_q^*) \in (\phi^*, v_q^*)$. Since $\gamma(x)$ is strictly decreasing for $x \geq \phi^*$, it holds that $\gamma(\phi^*) > \gamma(\overline{\phi}(v_q^*)) > \gamma(v_q^*)$, or equivalently $\overline{W} > W^* > W^{\text{Full}}$. The proof for the case $\phi^* < \underline{\phi}(v_q^*)$ is analogous. □

To explain the intuition of Theorem 6 we focus on the case $\phi^* \leq v_q^*$, which is without loss of generality. Observe that in any state in interval $\left(\phi^*, v_q^*\right)$ the preferences of the designer and the pivotal voter disagree almost surely as $n \to \infty$. Any optimal information policy should thus avoid disclosing states in this interval as much as possible. When the conflict of interests is small (i.e., $\underline{\phi}(v_q^*) < \phi^* < v_q^*$), this interval is small so that it is possible for the optimal information policy to avoid disclosing states therein and achieves the designer's omniscient payoff asymptotically (i.e., $W^* = \overline{W}$). As the conflict of interests increases, this interval expands and becomes harder to avoid. When the conflict is large enough (i.e., $\phi^* < \underline{\phi}(v_q^*) < v_q^*$), it becomes eventually unavoidable for any optimal information policy to disclose some states in this interval. In this case, the designer can no longer ensure his preferred outcome be elected with certainty as $n \to \infty$. Therefore, the designer cannot approach his omniscient payoff asymptotically through persuasion alone (i.e., $W^* < \overline{W}$).

Finally, full disclosure is generically sub-optimal compared to the other two benchmarks whenever $v_q^* \neq \phi^*$, i.e., the ex-ante conflicts of interests exists. $\overline{W} > W^{\text{Full}}$ holds precisely because full disclosure does not avoid disclosing information in the interval of conflicting states. The designer's payoff loss under full disclosure increases as the interval of conflicting states expands. In addition, $W^* > W^{\text{Full}}$ whenever $v_q^* \neq \phi^*$ reflects that full disclosure always reveals too much information compared to the optimal information policy.

## 7.2 The welfare impact of media bias and competition

In this subsection we apply the asymptotic results above to study the welfare impacts of media bias and competition. In particular, we reexamine the common wisdom that increased media competition necessarily enhances voter welfare. Contrary to this common wisdom, we show that media competition may harm voter welfare if the interests of the average and pivotal voters are poorly aligned under the voting rule in place. For ease of exposure, we impose the following assumption (our main results do not rely on this).

**Assumption 2.** *Both $F$ and $G$ have zero expectations and $v_q^* = G^{-1}(q) \in [0,1)$.*

The zero expectations imply that a priori, the two alternatives have equal quality and the average voter favors the quality-superior alternative. Moreover, ex-ante, the pivotal voter is either unbiased or biased towards the status quo under this assumption.

Note that as $n \to \infty$, a Utilitarian planner who maximizes the average voter's payoff is unbiased (that is, $\phi^* = 0$).[40] As a benchmark, let $W^*$ be a Utilitarian designer's asymptotic payoff (i.e., voter welfare) under the optimal information policy. By Theorem 5 we have

$$W^* = \int_{\max\{0, \underline{\phi}(v_q^*)\}}^{1} k dF(k) > 0 \,. \tag{15}$$

Let there be two media outlets I and II, which we model as self-interested designers ($\rho_{\text{I}} = \rho_{\text{II}} = 0$) so that $\phi_n^m(v) = \chi_m$ for $m \in \{\text{I}, \text{II}\}$. Moreover, we let $\chi_{\text{I}} = -1$ and $\chi_{\text{II}} = 1$ so that both outlets are *partisan* in the sense that their preferred alternatives are state-independent. We consider two cases: monopolistic persuasion by only one media outlet $m \in \{\text{I}, \text{II}\}$, or competitive persuasion by both outlets. For the latter case we know from Corollary 1 that full disclosure is the unique equilibrium outcome. Therefore, by (14), voter

[40] To see why $\phi^* = 0$, recall that $\rho = 1$ and $w(x) = x$ for $x \in [0,1]$ for a Utilitarian planner. Together with the zero-expectation assumption for $G$, this implies $\phi^* = \int_0^1 G^{-1}(y) dw(y) = \int_0^1 G^{-1}(y) dy = \int_{\underline{v}}^{\overline{v}} x dG(x) = 0$.

welfare under competition is given by

$$W_{\text{comp}} = W^{\text{Full}} = \int_{v_q^*}^{1} k dF(k) > 0 \,. \tag{16}$$

Next, we derive voters' Utilitarian welfare under monopolistic persuasion by either outlet I or II. We call outlet I *opposite-minded* because it favors the reform ex-ante while the pivotal voter favors the status quo. We call outlet II *like-minded* because it favors the same alternative as the pivotal voter ex-ante. The welfare under monopolistic persuasion by a single partisan media outlet is characterized below.

**Lemma 6.** *Suppose Assumption 2 holds. Then*

$$W_{mono}^{like} = 0 \quad and \quad W_{mono}^{oppo} = \int_{\underline{\phi}(v_q^*)}^{1} k dF(k) \,. \tag{17}$$

*Moreover, $W_{mono}^{oppo} \geq 0$ and equality holds if and only if $v_q^* = 0$.*

*Proof.* Under Assumption 2 we have $v_q^* \geq \mathbb{E}_F[k] = 0$. Therefore, under prior $F$, the pivotal voter selects the status quo almost surely as $n \to \infty$. Consider monopolistic persuasion by the like-minded outlet II. By Theorem 5, the status quo is maintained with probability one in large elections for all $k < \chi_{\text{II}} = 1$ (note that $\overline{\phi}(v_q^*) = \infty$ for all $v_q^* \geq 0 = \mathbb{E}_F[k]$ according to (11)). We therefore have $W_{\text{mono}}^{\text{like}} = 0$, the normalized payoff under the status quo.

Consider instead monopolistic persuasion by the opposite-minded outlet I. By Theorem 5, the reform is adopted almost surely in large elections if and only if $k > \underline{\phi}(v_q^*)$. Therefore, for any state $k > \underline{\phi}(v_q^*)$ the Utilitarian designer's payoff (i.e., voter welfare) converges almost surely to $k - \phi^* = k$ because $\varphi_n(v) \xrightarrow{p} \phi^*$ and $\phi^* = 0$. Conversely, for $k < \underline{\phi}(v_q^*)$, the status quo is maintained almost surely and voter welfare equals 0. These together imply (17). Finally, notice that for $v_q^* = \mathbb{E}_F[k] = 0$ we have $\underline{\phi}(v_q^*) = -\infty$ so that $W_{\text{mono}}^{\text{oppo}} = \int_{-1}^{1} k dF(k) = \mathbb{E}_F[k] = 0$. If instead $v_q^* > 0$, then we have $-1 < \underline{\phi}(v_q^*) < 0$ and $W_{\text{mono}}^{\text{oppo}} = \mathbb{E}_F[k] - \int_{-1}^{\underline{\phi}(v_q^*)} k dF(k) = -\int_{-1}^{\underline{\phi}(v_q^*)} k dF(k) > 0$. □

It is clear from the above that $W_{\text{mono}}^{\text{like}} = 0$ is the worst outcome amongst the options under consideration; that is, voter welfare is lowest under monopolistic persuasion by a like-minded partisan media outlet. The following proposition characterizes how the optimal voter welfare ($W^*$) relates to the welfare under outlet competition ($W_{\text{comp}}$) and under monopolistic persuasion by an opposite-minded outlet ($W_{\text{mono}}^{\text{oppo}}$). The ranking of these welfares depends crucially on the pivotal voter's type in the limit ($v_q^*$).

**Proposition 4.** *Under Assumption 2, there exists two thresholds $0 < \underline{\eta} < \overline{\eta} < 1$, such that the following holds:*

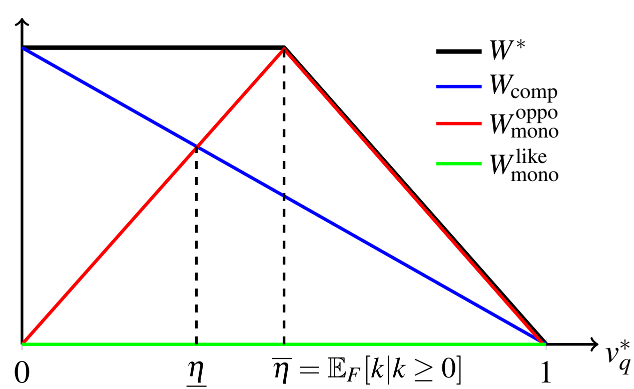
1. *If $v_q^* = 0$ then $W^* = W_{comp} > W_{mono}^{oppo}$.*
2. *If $0 < v_q^* < \underline{\eta}$ then $W^* > W_{comp} > W_{mono}^{oppo}$.*
3. *If $\underline{\eta} < v_q^* < \overline{\eta}$ then $W^* > W_{mono}^{oppo} > W_{comp}$.*
4. *If $v_q^* > \overline{\eta}$ then $W^* = W_{mono}^{oppo} > W_{comp}$.*

*Proof.* Since $W_{\text{comp}} = W^{\text{Full}}$, the ranking results between $W^*$ and $W_{\text{comp}}$ follow from Theorem 6. Below we compare $W_{\text{mono}}^{\text{oppo}}$ with $W^*$ and $W^{\text{Full}}$. By (15) and (17),

$$W_{\text{mono}}^{\text{oppo}} - W^* = \int_{\underline{\phi}(v_q^*)}^{\max\{0,\underline{\phi}(v_q^*)\}} k dF(k) \leq 0$$

and equality holds if and only if $\underline{\phi}(v_q^*) \geq 0$. Since $\mathbb{E}_F[k] = 0$, $\underline{\phi}(v_q^*)$ is strictly increasing on $(0,1)$ and $\underline{\phi}(v_q^*) \geq 0$ if and only if $v_q^* \geq \mathbb{E}_F[k|k \geq 0] = \overline{\eta}$. Therefore, $W^* > W_{\text{mono}}^{\text{oppo}}$ for $v_q^* < \overline{\eta}$ and $W^* = W_{\text{mono}}^{\text{oppo}}$ for $v_q^* \geq \overline{\eta}$. Regarding the comparison between $W_{\text{comp}}$ and $W_{\text{mono}}^{\text{oppo}}$, observe that $W_{\text{comp}}$ is continuously decreasing in $v_q^*$ (cf. (16)). In contrast, for $v_q^* \leq \overline{\eta}$, $W_{\text{mono}}^{\text{oppo}}$ is continuously increasing in $v_q^*$. Moreover, $W_{\text{mono}}^{\text{oppo}} = 0 < W^* = W_{\text{comp}}$ for $v_q^* = 0$ and $W_{\text{mono}}^{\text{oppo}} = W^* > W_{\text{comp}}$ for all $v_q^* \geq \overline{\eta}$. By intermediate value theorem there exists a unique $\underline{\eta} \in (0, \overline{\eta})$ such that $W_{\text{mono}}^{\text{oppo}} > (<) W_{\text{comp}}$ if and only if $v_q^* > (<) \underline{\eta}$. These together imply the ranking results in this proposition. □

Figure 6: The Welfare Impacts of Media Bias and Competition




Proposition 4 is illustrated by Figure 6. This shows that the welfare impact of media bias and competition depends crucially on the voting rule $q$, which determines $v_q^*$. First of all, observe that media competition maximizes asymptotic voter welfare (i.e., $W^* = W_{\text{comp}}$) if and only if $v_q^* = 0$, i.e., when there is no ex-ante conflict of interest between the mean and pivotal voters. As $v_q^*$ increases from 0 the conflict of interest between the two voters increases. Media competition then fails to maximize voter welfare because it induces excessive information revelation in the conflicting states. The harm of such excessive information revelation increases with the set of conflicting states $(0, v_q^*)$ expands. In particular, if $v_q^*$ is sufficiently large (i.e., $v_q^* > \underline{\eta}$), the harm of full information revelation is so large that welfare under competition is even lower than under monopolistic persuasion of an opposite-minded partisan outlet. Perhaps surprisingly, if the voting rule $q$ is very biased towards the status quo ex-ante, then asymptotically Utilitarian outcomes can be achieved with monopolistic persuasion by an opposite-minded partisan media outlet that is uniformly biased towards adopting the reform. Taken together, these results show that it is crucial to take institutional factors such as voting rules into account when evaluating the welfare consequences of media bias and competition in elections.

# 8 Conclusion

In this paper we study public persuasion in elections using a general framework with two distinguishing features. First, we allow for a wide class of designer preferences that contains both pursuit of self-interests and maximizing any social welfare function – any rank-dependent weighted average of voters' payoffs – as special cases. Second, we characterize information transmission in equilibrium under both monopolistic persuasion with one designer or competition in persuasion with multiple designers, in a unified framework.

We identify a novel single-crossing property regarding the relationship between the preferences of a designer and the pivotal voter, and relate it to the optimality of censorship policies. Under monopolistic persuasion by a single designer, we show that censorship policy is uniquely optimal in large elections if single-crossing property holds for this monopoly designer. Moreover, even under competition with other opponent designers, single-crossing property ensures that it is without loss of optimality for a designer to restrict attention to censorship policies. When single-crossing property holds for all designers, the equilibrium outcome that is Pareto optimal for all designers can be reproduced by a censorship policy, whose structure can be easily deduced from the monopolistically optimal censorship policies of each designer. This outcome turns out to be the unique pure-strategy equilibrium that survives iterated deletion of weakly dominated strategies if all designers commit to use censorship policies only. Methodologically, we exploit the duality approach for Bayesian persuasion to develop our main results. Along the way, we also obtain several novel results that shed lights on structures of solutions to general linear persuasion problems, especially when there is competition between designers.

We present two applications of our main results. First, we study, under the single-crossing property, how does the structure a monopoly designer's optimal censorship policy responds to a shift in his preference or a change in voting rule. We obtain an interesting result: if a designer is pro-social and imperfectly informed about voters' preferences, then increasing the required vote share for passing an alternative can affect his optimal censorship policy through both a *stringency effect* – by making it more difficult to persuade voters to pass that alternative – and a novel *designer-preference effect* – by inducing a shift of the designer's preference towards that alternative. Second, we study the welfare impact of media bias and competition. Contrary to the concern of insufficient information provision, we show that media competition can reduce voter welfare by inducing excessive information disclosure. Such harm can be particularly severe if the conflicts of interests between the average voter and the pivotal is large, under the voting rule in place. It is therefore important to take electoral characteristics, such as voting rules and the distribution of voter preferences, into account when evaluation the welfare impacts of media bias and competition.

# Appendix A Important Properties for $\hat{G}_n(\cdot;q)$ and $\phi_n^m(\cdot)$

In this appendix we derive the expression for $\hat{G}_n(\cdot;q)$ and establish relevant propositions for $\hat{G}_n(\cdot;q)$ and $\phi_n^m(\cdot)$ that imply unproved lemmas stated in Sections 3 and 4. To do so we need to introduce an auxiliary function. For all $y,q \in [0,1]$, define

$$\tau_n(y;q) := \frac{(n+1)!}{\lfloor x \rfloor ! \cdot \lceil n(1-q) \rceil !} y^{\lfloor nq \rfloor} (1-y)^{\lceil n(1-q) \rceil} \tag{A.1}$$

where $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ denote, respectively, the floor and ceiling functions.[41] In fact, $\tau_n(\cdot;q)$ is the density function of a Beta distribution $B(\alpha,\beta)$ with parameters $\alpha = \lfloor nq \rfloor + 1$ and $\beta = \lceil n(1-q) \rceil + 1$. The following properties about $\tau_n(y;q)$ are useful.

**Remark A.1.** *Suppose nq is an integer. Then the following properties hold:*
*(a)* $\tau_n'(y;q) = \tau_n(y;q)\frac{n(q-y)}{y(1-y)}$.
*(b)* $\tau_n(y;q)$ *is increasing on* $[0,q]$ *and decreasing on* $(q,1]$.
*(c)* $\lim\limits_{n\to\infty} \tau_n(y;q) = \infty$ *if* $y = q$ *and* $\lim\limits_{n\to\infty} \tau_n(y;q) = 0$ *if* $y \neq q$.

*Proof of Remark A.1.* Since $nq$ is an integer, we can get rid of the floor and ceiling functions in (A.1). Taking natural logarithm of $\tau_n(y;q)$ and computing its derivative then yields

$$\frac{\tau_n'(y;q)}{\tau_n(y;q)} = \frac{n(q-y)}{y(1-y)}$$

Hence, $\tau_n'(y;q) > (<) 0$ for $y < (>) q$. This proves (a) and (b). To show (c), we use Stirling's formula to approximate $n!$ for all positive integer $n$: $n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$.[42] With this approximation, we obtain

$$\tau_n(y;q) \approx \sqrt{\frac{n}{2\pi q(1-q)}} \left(\frac{y}{q}\right)^{nq} \left(\frac{1-y}{1-q}\right)^{n(1-q)} \tag{A.2}$$

If $y = q$, then $\tau_n(y;q) \approx \sqrt{\frac{n}{2\pi q(1-q)}} \to \infty$. If $y \neq q$, we take the natural logarithm of (A.2) and get

$$\ln \tau_n(y;q) \approx \frac{1}{2}\ln n + n\psi(y;q) - \frac{1}{2}\ln 2\pi q(1-q) \tag{A.3}$$

[41] That is, $\lfloor x \rfloor$ (resp. $\lceil x \rceil$) gives the highest (resp. lowest) integer smaller (resp. greater) than $x$.
[42] The expression $l_n \approx r_n$ denotes $\lim\limits_{n\to\infty} \frac{l_n}{r_n} = 0$, where $l_n$ and $r_n$ are real number sequences.

where

$$\psi(y;q) := q \ln \frac{y}{q} + (1-q) \ln \frac{1-y}{1-q} \tag{A.4}$$

It holds that (i) $\psi(q;q) = 0$, and (ii) $\psi'(y;q) > (<) 0$ for $y < (>) q$. Therefore, if $y \neq q$, then $\psi(y;q) < 0$ and the right hand side of (A.3) converges to $-\infty$ as $n \to \infty$. This implies $\lim_{n\to\infty} \tau_n(y;q) = 0$ for $y \neq q$ and thus completes the proof for part (c). □

Observe that Remark A.1 easily extends to other values of $q$ in which $nq$ is not an integer. In this case, we can just replace $q$ by $\hat{q} := \frac{\lfloor nq \rfloor}{n}$. In this way, (a) and (b) of Remark A.1 hold with $\hat{q}$. Part (c) of Remark A.1 also holds for $q$ because $\hat{q}$ converges to $q$ as $n \to \infty$.

In the remainder of this appendix we assume $nq$ to be an integer for ease of exposure, with the understanding that this is without loss of generality.

## A.1 Derivation and relevant properties of $\hat{G}_n(\cdot;q)$

We start by deriving $\hat{G}_n(\cdot;q)$, the distribution of the pivotal voter's type $v^{(nq+1)}$. Let $\hat{g}_n(\cdot;q)$ denote the density function. Consider $x \in [\underline{v}, \overline{v}]$. For $v^{(nq+1)} = x$ to hold, there must be exactly $nq$ voters with $v_i \leq x$ and $n(1-q)$ others with $v_i \geq x$, while the remaining pivotal voter with exactly $v_i = x$. Because voters' types are independently drawn from $G$, we have

$$\hat{g}_n(x;q) = \frac{(n+1)!}{(nq)![n(1-q)]!} (G(x))^{nq} (1-G(x))^{n(1-q)} g(x) = \tau_n(G(x);q)\, g(x) \tag{A.5}$$

and

$$\hat{G}_n(x;q) = \int_{\underline{v}}^{x} \tau_n(G(x);q)\, g(x) dx = \int_0^{G(x)} \tau_n(y;q) dy \tag{A.6}$$

Next, we prove the following proposition about $\hat{G}_n(x;q)$.

**Proposition A.1.** *Let $v_q^* := G^{-1}(q)$. The following properties hold:*

*1. $\hat{G}_n(\cdot;q)$ is strictly increasing and $v^{(nq+1)}$ converges in probability to $v_q^*$.*

*2. $\hat{g}_n(\cdot;q)$ is single-peaked[43] for all $q \in (0,1)$ when $n$ is sufficiently large. In particular, if $g(\cdot)$ is log-concave, then $\hat{g}_n(\cdot;q)$ is strictly log-concave for all $n > 0$ and $q$.*

Notice that statement (1) of this proposition implies Lemma 1 in Section 3. Statement (2) says that regardless the shape of $G$ and voting rule $q$, for sufficiently large electorate the distribution of the pivotal voter will have a regular single-peaked shape. In particular,

[43] We say that $\hat{g}_n(\cdot;q)$ is single-peaked if there exists a value $\hat{x}$ such that $\hat{g}_n(x;q)$ is strictly increasing for $x < \hat{x}$ and strictly decreasing for $x > \hat{x}$.

large $n$ is not needed if $g$ is already log-concave. As is clear in subsequent appendices, this property plays a key role in establishing curvature properties of $W_n^m(\cdot)$, the utility function of any designer $m \in M$.

*Proof of Proposition A.1.* We first show part (1). The fact that $\hat{G}_n(\cdot;q)$ is strictly increasing follows immediately from $\hat{g}_n(x;q) = \tau_n(G(x);q)g(x) > 0$. To show that $v^{(nq+1)}$ converges in probability to $v_q^*$, it suffices to establish

$$\lim_{n\to\infty} \hat{G}_n(x;q) \to \begin{cases} 0, & \text{if } x < v_q^* \\ 1/2, & \text{if } x = v_q^* \\ 1, & \text{if } x > v_q^* \end{cases} \tag{A.7}$$

For $x < v_q^*$ we have $G(x) < q$ and

$$\hat{G}_n(x;q) = \int_0^{G(x)} \tau_n(y;q)dy < G(x)\tau_n(G(x);q) \to 0$$

The second and third steps follow from (b) and (c) of Remark A.1, respectively. If instead $x > v_q^*$, then $G(x) > q$ and $\int_{G(x)}^1 \tau_n(y)dy < (1-G(x))\tau_n(G(x);q) \to 0$. Therefore, $\hat{G}_n(x) = 1 - \int_{G(x)}^1 \tau_n(y)dy \to 1$. Finally, if $x = v_q^*$, then $G(x) = G(v_q^*) = q$ and $\hat{G}_n(x) = \int_0^q \tau_n(y;q)dy$. Below we show $\lim_{n\to\infty} \int_0^q \tau_n(y;q)dy = 1/2$. Recall that $\tau_n(y;q)$ is the density function of a random variable $Y$ following Beta distribution $B(\alpha,\beta)$ with parameters $\alpha = nq+1$ and $\beta = n(1-q)+1$. Let $q_n$ denote the median of $Y$; that is, $\int_0^{q_n} \tau_n(y;q)dy = 1/2$. We show that the sequence of medians $q_n$ converges to $q$ and thus $\lim_{n\to\infty} \int_0^q \tau_n(y;q)dy = \lim_{n\to\infty} \int_0^{q_n} \tau_n(y;q)dy = 1/2$. For a Beta-distributed random variable $Y \sim Beta(\alpha,\beta)$, Groeneveld and Meeden (1977) show that its median $q_n$ must be bounded between its mean $\mu_n$ and mode $m_n$. For a Beta distribution, it is well known that $\mu_n = \frac{\alpha}{\alpha+\beta}$ and $m_n = \frac{\alpha-1}{\alpha+\beta-2}$. Since $\alpha = nq+1$ and $\beta = n(1-q)+1$, both $\mu_n$ and $m_n$ converge to $q$ as $n \to \infty$. This implies that the median $q_n$ must converge to $q$ as well. This establishes part (1) of this proposition.

Below we prove part (2). By (A.5) and part (a) of Remark A.1, we have

$$\begin{aligned} \hat{g}_n'(x;q) &= \tau_n'(G(x);q)g^2(x) + \tau_n(G(x);q)g'(x) \\ &= \hat{g}_n(x;q)\left(n\frac{g(x)}{G(x)}\frac{q-G(x)}{1-G(x)} + \frac{g'(x)}{g(x)}\right) \end{aligned} \tag{A.8}$$

Therefore,

$$\frac{\hat{g}_n'(x;q)}{\hat{g}_n(x;q)} = n\frac{g(x)}{G(x)}\frac{q-G(x)}{1-G(x)} + \frac{g'(x)}{g(x)} \tag{A.9}$$

Suppose $g$ is log-concave and thus $\frac{g'(\cdot)}{g(\cdot)}$ is decreasing. By Theorem 1 of Bagnoli and Bergstrom (2005), $G$ inherits log-concavity and $\frac{g(\cdot)}{G(\cdot)}$ is decreasing. Since $\frac{q-G(x)}{1-G(x)}$ is strictly decreasing, it follows from (A.9) that $\frac{\hat{g}_n'(x;q)}{\hat{g}_n(x;q)}$ is strictly decreasing and thus $\hat{g}_n(\cdot;q)$ is strictly log-concave for all $n>0$.

Now we drop the log-concavity assumption of $g$ and show that $\hat{g}_n(x;q)$ is single-peaked for sufficiently large $n$. By (A.9),

$$\hat{g}_n'(x;q) > 0 \Longleftrightarrow \lambda_n(x) := q - G(x) + \frac{1}{n}\frac{G(x)\,(1-G(x))}{g(x)}\frac{g'(x)}{g(x)} > 0 \tag{A.10}$$

Recall that $g$ is strictly positive and twice-continuously differentiable on $[\underline{v},\overline{v}]$. These imply that (i) there exists some $\varepsilon>0$ such that $g(x)>\varepsilon$ for all $x$, and (ii) both $\frac{G(x)(1-G(x))}{g(x)}\frac{g'(x)}{g(x)}$ and its first order derivative are uniformly bounded. Therefore, as $n\to\infty$, $\lambda_n(x)$ and $\lambda_n'(x)$ converge uniformly to $q-G(x)$ and $-g(x)$, respectively. Hence, for sufficiently large $n$, $\lambda_n(x)$ must be strictly decreasing and its root $\hat{x}_n$ must converge to $v_q^*$. This implies that $\hat{g}_n(x;q)$ is single-peaked for sufficiently large $n$ and thus completes the proof. □

## A.2 Derivation and relevant properties of $\phi_n^m(\cdot)$ and its limit

In this subsection we establish some important properties for $\phi_n^m(x)$ – the indifference curve of any designer $m\in M$ – and its limit as $n\to\infty$. We also prove Lemma 2 and Lemma 3 in the main text. For ease of exposure we omit index $m$ throughout this part.

For each $j\in\{1,\cdots,n+1\}$ and $x\in[\underline{v},\overline{v}]$, let

$$\varphi_j(x;q,n) := \mathbb{E}\left[v^{(j)}\middle|v^{(nq+1)}=x;q,n\right]$$

denote the expectation of $v^{(j)}$ conditional on event $v^{(nq+1)}=x$. By (3) we have

$$\phi_n(x) := \mathbb{E}\left[\varphi_n(v)\middle|v^{(nq+1)}=x\right] = \rho\sum_{j=1}^{n+1} w_j\varphi_j(x;q,n) + (1-\rho)\chi \tag{A.11}$$

If $\rho=0$, it is obvious that $\phi_n(x)=\chi$ is a constant. If $\rho>0$, the properties of $\phi_n(x)$ depend closely on $\varphi_j(x;q,n)$. Proposition A.2 collects important properties of $\varphi_j(x;q,n)$.

**Proposition A.2.** *Let $j \in \{1,\cdots,n+1\}$. $\varphi_j(x;q,n)$ satisfies the following properties:*

1. *$\varphi_j(x;q,n)$ is strictly increasing in index $j$ and $\varphi_j(x;q,n) = x$ for $j = nq+1$;*
2. *$\varphi_j(x;q,n)$ is strictly increasing in $x$ and decreasing in $q$ for all $j$;*
3. *If $G$ is strictly log-concave, then $\varphi_j'(x;q,n) < 1$ for all $j < nq+1$;*
4. *If $1-G$ is strictly log-concave, then $\varphi_j'(x;q,n) < 1$ for all $j > nq+1$.*

### A.2.1 Proof of Proposition A.2

For any $j \neq nq+1$, let $\widetilde{g}_j(\cdot|x;q,n)$ denote the probability density function for the distribution of $v^{(j)}$ conditional on $v^{(nq+1)} = x$ given parameters $q$ and $n$. We show that

$$\widetilde{g}_j(y|x;q,n) = \begin{cases} \tau_{nq-1}\left(\frac{G(y)}{G(x)};\frac{j-1}{nq}\right)\frac{g(y)}{G(x)}, & \text{if} j < nq+1 \\ \tau_{n(1-q)-1}\left(\frac{G(y)-G(x)}{1-G(x)};\frac{j-nq-2}{n(1-q)}\right)\frac{g(y)}{1-G(x)}, & \text{if } j > nq+1 \end{cases}. \tag{A.12}$$

To see why, first consider $j < nq+1$. Conditional on $v^{(nq+1)} = x$, $v^{(j)}$ is the $j$-th lowest order statistic from $nq$ independent random draws from a truncated distribution with cdf $\frac{G(y)}{G(x)}$ for $y \in [\underline{v}, x]$. (A.12) for $j < nq+1$ thus follows from (A.5). Now consider $j > nq+1$. Conditional on $v^{(nq+1)} = x$, $v^{(j)}$ is the $(j-nq-1)$-th lowest order statistic from $n(1-q)$ independent random draws from a truncated distribution with cdf $\frac{G(y)-G(x)}{1-G(x)}$ for $y \in [x, \overline{v}]$. This implies (A.12) for $j < nq+1$ through (A.5). Claim A.1 explicitly characterizes $\varphi_j(x;q,n)$.

**Claim A.1.** *For all $x \in [\underline{v}, \overline{v}]$,*

$$\varphi_j(x;q,n) = \begin{cases} \int_0^1 \underline{t}(x,y)\tau_{nq-1}\big(y;\frac{j-1}{nq}\big)dy, & \text{if } j < nq+1 \\ x, & \text{if } j = nq+1 \\ \int_0^1 \overline{t}(x,y)\tau_{n(1-q)-1}\big(y;\frac{j-nq-2}{n(1-q)}\big)dy, & \text{if } j > nq+1 \end{cases} \tag{A.13}$$

*where*

$$\underline{t}(x,y) := G^{-1}\big(yG(x)\big) \tag{A.14}$$

$$\overline{t}(x,y) := G^{-1}\big(y + (1-y)G(x)\big) \tag{A.15}$$

*for all $x \in [\underline{v}, \overline{v}]$ and $y \in [0,1]$.*

*Proof of Claim A.1.* $\varphi_j(x;q,n) = x$ for $j = nq+1$ follows immediately from its definition.

For $j < nq+1$, it follows from (A.12) that

$$\begin{aligned}\varphi_j(x;q,n) &= \int_{\underline{v}}^{x} y\tilde{g}_j(y|x;q,n)dy = \int_{\underline{v}}^{x} y\tau_{nq}\left(\frac{G(y)}{G(x)};\frac{j-1}{nq-1}\right)\frac{dG(y)}{G(x)} \\ &= \int_0^1 G^{-1}\big(yG(x)\big)\tau_{nq-1}\left(y;\frac{j-1}{nq}\right)dy = \int_0^1 \underline{t}(x,y)\tau_{nq-1}\left(y;\frac{j-1}{nq}\right)dy\end{aligned}$$

Finally, for all $j > nq+1$ it follows from (A.12) that

$$\begin{aligned}\varphi_j(x;q,n) &= \int_x^{\overline{v}} y\tilde{g}_j(y|x;q,n)dy = \int_x^{\overline{v}} y\tau_{n(1-q)-1}\left(\frac{G(y)-G(x)}{1-G(x)};\frac{j-nq-2}{n(1-q)}\right)\frac{dG(y)}{1-G(x)} \\ &= \int_0^1 G^{-1}\big(y+(1-y)G(x)\big)\tau_{n(1-q)-1}\left(y;\frac{j-nq-2}{n(1-q)}\right)dy \\ &= \int_0^1 \overline{t}(x,y)\tau_{n(1-q)-1}\left(y;\frac{j-nq-2}{n(1-q)}\right)dy\end{aligned}$$

This completes the proof. □

Claim A.2 summarizes useful properties about $\underline{t}(x,y)$ and $\overline{t}(x,y)$ defined above.

**Claim A.2.** *$\underline{t}(x,y)$ and $\overline{t}(x,y)$ are three times continuously differentiable and satisfy the following properties:*

1. *$\underline{t}(x,y) < x < \overline{t}(x,y)$ for all $y \in (0,1)$.*
2. *Both $\underline{t}(x,y)$ and $\overline{t}(x,y)$ are strictly increasing in $x$ and $y$.*
3. *If $G$ is strictly log-concave, then $\underline{t}_x(x,y) < 1$ for all $y \in (0,1)$.*
4. *If $1-G$ is strictly log-concave, then $\overline{t}_x(x,y) < 1$ for all $y \in (0,1)$.*

*Proof of Claim A.2.* The fact that both $\underline{t}(x,y)$ and $\overline{t}(x,y)$ are three times continuously differentiable for all $(x,y) \in [\underline{v},\overline{v}] \times [0,1]$ follows from our assumption that $G$ is twice continuously differentiable on $[\underline{v},\overline{v}]$. Parts (1) and (2) of this claim follows immediately from the definitions of $\underline{t}(x,y)$ and $\overline{t}(x,y)$. To show part (3), note from (A.14) that

$$G\big(\underline{t}(x,y)\big) = yG(x)$$

Taking first order derivative with respect to $x$ on both sides and rearranging terms yields

$$g\big(\underline{t}(x,y)\big)\underline{t}_x(x,y) = yg(y) \Longleftrightarrow \underline{t}_x(x,y) = y\frac{g(x)}{g\big(\underline{t}(x,y)\big)} = \frac{g(x)}{G(x)}\Big/\frac{g\big(\underline{t}(x,y)\big)}{G\big(\underline{t}(x,y)\big)} \tag{A.16}$$

If $G$ is strictly log-concave, then $\frac{g(\cdot)}{G(\cdot)}$ is strictly decreasing. Since $\underline{t}_x(x,y) < x$ for $y \in (0,1)$, it follows from (A.16) that $\underline{t}_x(x,y) < 1$. To show part (4), note from (A.15) that

$$G\big(\bar{t}(x,y)\big) = y + \big(1-y\big)\big(1-G(x)\big)$$

holds for all $x$ and $y$. Simple algebra reveals that

$$\bar{t}_x(x,y) = \big(1-y\big)\frac{g(x)}{g\big(\bar{t}(x,y)\big)} = \frac{g(x)}{1-G(x)} \Big/ \frac{g(\bar{t}(x,y))}{1-G(\bar{t}(x,y))} \tag{A.17}$$

If $1-G$ is strictly log-concave, then $\frac{g(\cdot)}{1-G(\cdot)}$ is strictly increasing. Since $\bar{t}(x,y) > x$ for $y \in (0,1)$, it follows from (A.17) that $\bar{t}_x(x,y) < 1$. □

Notice that parameters $j$ and $q$ affect $\varphi_j(x;q,n)$ only through their impacts on $\widetilde{g}_j(\cdot|x;q,n)$. The next claim shows that $\widetilde{g}_j(\cdot|x;q,n)$ can be ranked in strict monotone likelihood-ratio dominance as $j$ increases and $q$ decreases. For two probability density functions $l(\cdot)$ and $r(\cdot)$, we write $l(\cdot) \succ_{LR} r(\cdot)$ if the likelihood ratio $\frac{l(\cdot)}{r(\cdot)}$ is strictly increasing.

**Claim A.3.** *The following properties for $\widetilde{g}_j(\cdot|x;q,n)$ hold:*

1. *Suppose $j' > j$, then $\widetilde{g}_{j'}(\cdot|x;q,n) \succ_{LR} \widetilde{g}_j(\cdot|x;q',n)$ holds if $j > nq+1$ or $j' < nq+1$.*
2. *Suppose $q' > q$, then $\widetilde{g}_j(\cdot|x;q,n) \succ_{LR} \widetilde{g}_j(\cdot|x;q',n)$ holds if $j < nq+1$ or $j > nq'+1$.*

*Proof of Claim A.3.* We first show part (1). Using (A.12) and (A.1), we obtain

$$\frac{\widetilde{g}_{j'}(y|x;q,n)}{\widetilde{g}_j(y|x;q,n)} \propto \begin{cases} \left(\frac{G(y)}{G(x)-G(y)}\right)^{j'-j} & \text{for } y \in [\underline{v},x], \quad \text{if } nq+1 > j' > j \\ \left(\frac{G(y)-G(x)}{1-G(y)}\right)^{j'-j} & \text{for } y \in [x,\bar{v}], \quad \text{if } j' > j > nq+1 \end{cases}$$

In both cases, the likelihood ratio $\frac{\widetilde{g}_{j'}(y|x;q,n)}{\widetilde{g}_j(y|x;q,n)}$ is strictly increasing in $y$ since $j' > j$. To show part (2), suppose $q' > q$ and note that

$$\frac{\widetilde{g}_j(y|x;q,n)}{\widetilde{g}_j(y|x;q',n)} \propto \begin{cases} \left(\frac{G(x)}{G(x)-G(y)}\right)^{n(q'-q)} & \text{for } y \in [\underline{v},x], \quad \text{if } j < nq+1 \\ \left(\frac{G(y)-G(x)}{1-G(y)}\right)^{n(q'-q)} & \text{for } y \in [x,\bar{v}], \quad \text{if } j > nq'+1 \end{cases}$$

In both cases, the likelihood ratio $\frac{\widetilde{g}_j(y|x;q,n)}{\widetilde{g}_j(y|x;q',n)}$ is strictly increasing in $y$ when $q' > q$. □

With all these ingredients, we are ready to prove Proposition A.2. We start with part (1). $\varphi_{nq+1}(x;q,n) = x$ follows immediately from definition. Moreover, (A.13) and the fact that $\underline{t}(x,y) < x < \bar{t}(x,y)$ for $y \in (0,1)$ imply $\varphi_j(x;q,n) > (<) x$ for $j > (<) nq+1$. Hence, $\varphi_{j'}(x;q,n) > \varphi_j(x;q,n)$ holds for $j' \geq nq+1 \geq j$ with at least one inequality holds strictly. Now consider $j' > j > nq+1$ or $nq+1 > j' > j$. Observe that both $\underline{t}(x,y)$ and $\bar{t}(x,y)$ strictly increasing functions of $y$, and $\varphi_j(x;q,n)$ equals the expectation of $\underline{t}(x,y)$ or $\bar{t}(x,y)$ for random variable $y$ under distribution $\widetilde{g}_j(\cdot|x;q,n)$. By Claim A.3.1, $\tilde{g}_{j'}(\cdot|x;q,n) \succ_{LR} \tilde{g}_j(\cdot|x;q,n)$ and strict likelihood ratio dominance implies $\varphi_{j'}(x;q,n) > \varphi_j(x;q,n)$ (see, for instance, Appendix B of Krishna (2009)).

To show part (2), note that both $\underline{t}(x,y)$ and $\bar{t}(x,y)$ strictly increase in $x$ for all $y \in (0,1)$ (cf. Claim A.2). It then follows from (A.13) that $\varphi_j(x;q,n)$ strictly increases in $x$. To show that $\varphi_j(x;q,n)$ decreases in $q$, consider two different $q'$ and $q''$ with $q' < q''$ and both $nq'$ and $nq''$ are integers. If $nq'+1 \leq j \leq nq''+1$ then by (A.13) and Claim A.2 we have $\varphi_j(x;q',n) \leq x \leq \varphi_j(x;q'',n)$ with at least one inequality holds strictly. Now consider $j < nq'+1$ or $j > nq''+1$. In this case it follows from Claim A.3 that $\widetilde{g}_j(\cdot|x;q',n) \succ_{LR} \widetilde{g}_j(\cdot|x;q'',n)$ so that $\varphi_j(x;q',n) < \varphi_j(x;q'',n)$ holds as a standard implication of likelihood ratio dominance.

To show part (3), suppose that $G$ is strictly log-concave so that $\frac{g(\cdot)}{G(\cdot)}$ is strictly increasing. By Claims A.1 and A.2, for $j < nq+1$ we have

$$\varphi_j'(x;q,n) = \int_0^1 \underline{t}_x(x,y)\tau_{nq}\left(y;\frac{j-1}{nq}\right)dy < \int_0^1 \tau_{nq}\left(y;\frac{j-1}{nq}\right)dy = 1$$

The second step follows from part (3) of Claim A.2. The proof for part (4) is analogous.

### A.2.2 Properties of $\phi_n(\cdot)$ for finite $n$ when $\rho > 0$

Proposition A.3 summarizes important properties of $\phi_n(\cdot)$ when $\rho > 0$.

**Proposition A.3.** *Suppose $\rho > 0$. Then $\phi_n(\cdot)$ satisfies the following properties:*

*1. $\phi_n(\cdot)$, $\phi_n'(\cdot)$ and $\phi_n''(\cdot)$ are L-Liptschitz continuous on $[\underline{v},\bar{v}]$ for all $n \geq 0$ and some sufficiently large $L > 0$.*

*2. $\phi_n(x)$ is strictly increasing in $x$.*

*3. For any $x \in (\underline{v},\bar{v})$, $\phi_n(x)$ is strictly decreasing in $q$.*

*4. For any $x \in (\underline{v},\bar{v})$, $\phi_n(x)$ is weakly decreasing as $w(\cdot)$ shifts from $w^I(\cdot)$ to $w^{II}(\cdot)$, where $w^I(\cdot), w^{II}(\cdot) \in \Delta([-1,1])$ and $w^I(\cdot)$ first order stochastically dominates $w^{II}(\cdot)$.*

*5. If both $G$ and $1-G$ are strictly log-concave, then $\phi_n'(x) \leq \rho$ holds for all $x$ and $n \geq 0$. In fact, strict inequality holds whenever $w_{nq+1} < 1$.*

*Proof of Proposition A.3.* We first show part (1) and establish uniform Lipschitz continuity for $\phi_n(\cdot)$, which is three times continuous differentiable. By the Mean Value Theorem, $\forall x, y \in [\underline{v}, \bar{v}]$ we have

$$|\phi_n(x) - \phi_n(y)| = |x-y| \cdot \left|\phi_n'(\xi)\right|$$

for some $\xi$ between $x$ and $y$. Notice that

$$\left|\phi_n'(\xi)\right| = \rho \left|\sum_{j=1}^{n+1} w_j \varphi_j'(\xi; q, n)\right| \leq \max_{j=1,\cdots,n+1} \left|\varphi_j'(\xi; q, n)\right|$$

By (A.13), each $\varphi_j'(\xi; q, n)$ is the expectation of either $\underline{t}_x(\xi, \cdot)$ or $\bar{t}_x(\xi, \cdot)$ under some distribution. Because both $\underline{t}_x(\cdot)$ and $\bar{t}_x(\cdot)$ are uniformly bounded (cf. Claim A.2), there exists $L > 0$ such that $L \geq \max\{\underline{t}_x(x,y), \bar{t}_x(x,y)\}$ for all $(x,y) \in [\underline{v}, \bar{v}] \times [0,1]$. These together imply

$$|\phi_n(x) - \phi_n(y)| = |x-y| \cdot \left|\phi_n'(\xi)\right| < L \cdot |x-y|$$

for all $x, y \in [\underline{v}, \bar{v}]$ and $n \geq 0$. The proofs for uniform Lipschitz continuities for $\phi_n'(\cdot)$ and $\phi_n''(\cdot)$ follow from analogous reasoning and exploit uniform boundedness of $\underline{t}_{xx}(\cdot)$, $\bar{t}_{xx}(\cdot)$, $\underline{t}_{xxx}(\cdot)$ and $\bar{t}_{xxx}(\cdot)$, which are implied by Claim A.2.

Next we show parts (2) and (3). Using (A.11) and the fact that $w_j = w\left(\frac{j}{n+1}\right) - w\left(\frac{j-1}{n+1}\right)$ (cf. (2)), we obtain

$$\phi_n(x) = \rho \sum_{j=1}^{n+1} w_j \varphi_j(x; q, n) + (1-\rho)\chi = \rho \sum_{j=1}^{n+1} \left[ w\left(\frac{j}{n+1}\right) - w\left(\frac{j-1}{n+1}\right)\right] \varphi_j(x; q, n) + (1-\rho)\chi$$

for $j = 1, \cdots, n+1$. By Proposition A.2, for all $j = 1, \cdots, n+1$ it holds that $\varphi_j(x; q, n)$ is strictly increasing in $x$ and decreasing in $q$. Therefore, $\phi_n(x)$ must inherit these properties whenever $\rho > 0$. This proves parts (2) and (3). To show part (4), note that

$$\begin{aligned}
\sum_{j=1}^{n+1} w_j \varphi_j(x; q, n) &= \sum_{j=2}^{n+1} \left[\sum_{l=j}^{n+1} w_l\right] \left(\varphi_j(x; q, n) - \varphi_{j-1}(x; q, n)\right) + \varphi_1(x; q, n) \\
&= \sum_{j=2}^{n+1} \left[1 - w\left(\frac{j-1}{n+1}\right)\right] \left(\varphi_j(x; q, n) - \varphi_{j-1}(x; q, n)\right) + \varphi_1(x; q, n)
\end{aligned}$$

Consider two weighting functions $w^{\mathrm{I}}(\cdot)$ and $w^{\mathrm{II}}(\cdot)$. Let $\phi_n^{\mathrm{I}}(\cdot)$ and $\phi_n^{\mathrm{II}}(\cdot)$ denote function

$\phi_n(\cdot)$ when $w(\cdot)$ equals $w^{\mathrm{I}}(\cdot)$ and $w^{\mathrm{II}}(\cdot)$, respectively. Using the above equation we obtain

$$\phi_n^{\mathrm{I}}(x) - \phi_n^{\mathrm{II}}(x) = \rho \sum_{j=2}^{n+1} \left[ w^{\mathrm{II}}\left(\frac{j-1}{n+1}\right) - w^{\mathrm{I}}\left(\frac{j-1}{n+1}\right)\right] \left(\varphi_j(x;q,n) - \varphi_{j-1}(x;q,n)\right)$$

By Proposition A.2, $\varphi_j(x;q,n) - \varphi_{j-1}(x;q,n) > 0$ holds for all $j > 1$ and $x \in (\underline{v}, \overline{v})$. Suppose $w^{\mathrm{I}}(\cdot)$ first order stochastically dominates $w^{\mathrm{II}}(\cdot)$, then $w^{\mathrm{II}}(y) - w^{\mathrm{I}}(y) \geq 0$ holds for all $y \in (0,1)$ (with strict inequality holds for some $y$). This implies $\phi_n^{\mathrm{I}}(x) - \phi_n^{\mathrm{II}}(x) \leq 0$ for all $n \geq 0$ and $x \in (\underline{v}, \overline{v})$. Finally, we prove part (5). Taking derivative of $\phi_n(x)$ yields

$$\phi_n'(x) = \rho \sum_{j=1}^{n+1} w_j \varphi_j'(x;q,n) \tag{A.18}$$

By Proposition A.2, if both $G$ and $1-G$ are strictly log-concave, then $\varphi_j'(x;q,n) < 1$ for all $j \neq nq+1$ and $\varphi_j'(x;q,n) = 1$ for $j = nq+1$. These together implies ${\phi_n}'(x) \leq \rho$ and the strict inequality must hold whenever $w_{nq+1} < 1$. □

The uniform Liptschitz continuity properties in statement (1) of this lemma shall play important roles in the proofs of Lemmas 2 and 5 below. Statements (2) to (4) of this lemma are consequences of the inference based on the pivotal voter's choice explained in Section 4. The second and third statements say that the indifference curve $\phi_n(\cdot)$ systematically shifts downwards – resulting in a preference shift towards the reform – as $q$ increases or as the weighting function $w(\cdot)$ increases in the first order stochastic dominance ordering. These properties play crucial roles in establishing comparative static results in Section 6.

Moreover, (A.18) and statement (5) of Proposition A.3 imply that Assumption 1 in the main text – $\phi_n'(x) < 2$ for all $x \in [-1,1]$ and $n \geq 0$ – holds under the two sufficient conditions of Lemma 3. First, it follows from (A.18) that $\phi_n'(\cdot) \to 0$ uniformly as $\rho \to 0$. So $\phi_n'(\cdot) < 2$ must hold for $\rho$ close to 0. Second, when both $G$ and $1-G$ are strictly log-concave, statement (5) of this lemma implies $\phi_n'(x) \leq 1$ for all $x$, which ensures Assumption 1.

### A.2.3 Asymptotic properties of $\phi_n(\cdot)$

In this subsection we derive the limit of $\phi_n(\cdot)$ as $n \to \infty$ and prove Lemma 2 and Lemma 3. Given a designer's preference parameters $\rho$, $w(\cdot)$ and $\chi$, define

$$\phi(x) := \rho \left[\int_0^q \underline{t}\left(x, \frac{y}{q}\right) dw(y) + \int_q^1 \overline{t}\left(x, \frac{y-q}{1-q}\right) dw(y)\right] + (1-\rho)\chi \tag{A.19}$$

for $x \in [\underline{v}, \bar{v}]$, where functions $\underline{t}(\cdot)$ and $\bar{t}(\cdot)$ are defined in (A.14) and (A.15), respectively. The first order derivative of $\phi(x)$ is given by

$$\phi'(x) = \rho \left[ \int_0^q \underline{t}_x \left( x, \frac{y}{q} \right) dw(y) + \int_q^1 \bar{t}_x \left( x, \frac{y-1}{1-q} \right) dw(y) \right] \tag{A.20}$$

Moreover, using (A.14), (A.15) and the fact that $v_q^* = G^{-1}(q)$, we obtain that

$$\underline{t}\left( v_q^*, \frac{y}{q} \right) = G^{-1}\left( \frac{y}{q} G(v_q^*) \right) = G^{-1}(y)$$
$$\bar{t}\left( v_q^*, \frac{y-q}{1-q} \right) = G^{-1}\left( \frac{y-q}{1-q} + (1 - \frac{y-q}{1-q}) G(v_q^*) \right) = G^{-1}(y)$$

These together imply

$$\phi^* := \phi(v_q^*) = \rho \int_0^1 G^{-1}(y) dw(y) + (1-\rho)\chi \tag{A.21}$$

Lemma 2 in the main text is then equivalent to that $\phi_n(\cdot)$ and $\phi_n'(\cdot)$ uniformly converge to (A.19) and (A.20), respectively, and $\varphi_n(v)$ converges in probability to (A.21).

*Proof of Lemma 2.* Consider any $z \in (0,1)$. By (A.13) in Claim A.1 we have

$$\varphi_{\lfloor (n+1)z \rfloor}(x; q, n) = \begin{cases} \int_0^1 \underline{t}(x,y) \tau_{nq-1}\left( y; \frac{\lfloor (n+1)z \rfloor - 1}{nq} \right) dy, & \text{if } z < \frac{nq+1}{n+1} \\ \int_0^1 \bar{t}(x,y) \tau_{n(1-q)-1}\left( y; \frac{\lfloor (n+1)z \rfloor - nq - 2}{n(1-q)} \right) dy, & \text{if } z > \frac{nq+1}{n+1} \end{cases}$$

By Remark A.1c, as $n \to \infty$, $\tau_{nq-1}\left( y; \frac{\lfloor (n+1)z \rfloor - 1}{nq} \right)$ concentrates all its probability mass on $\frac{z}{q}$ and $\tau_{n(1-q)-1}\left( y; \frac{\lfloor (n+1)z \rfloor - nq - 2}{n(1-q)} \right)$ concentrates all its mass on $\frac{z-q}{1-q}$ for all $z \neq q$. Therefore,

$$\lim_{n \to \infty} \varphi_{\lfloor (n+1)z \rfloor}(x; q, n) = \begin{cases} \underline{t}\left( x, \frac{z}{q} \right), & \text{if } z < q \\ \bar{t}\left( x, \frac{z-q}{1-q} \right), & \text{if } z > q \end{cases} \tag{A.22}$$

Using the definition of $\phi_n(x)$ and the fact that $w_j = w\left( \frac{j}{n+1} \right) - w\left( \frac{j-1}{n+1} \right)$, we get

$$\phi_n(x) = \rho \sum_{j=1}^{n+1} \left[ w(z_j) - w(z_{j-1}) \right] \varphi_{(n+1)z_j}(x; q, n) + (1-\rho)\chi \tag{A.23}$$

where $z_j := \frac{j}{n+1}$. Taking $n \to \infty$ and using the definition of Riemann integral, we have

$$\lim_{n\to\infty} \sum_{j=1}^{n+1} \left[w(z_j) - w(z_{j-1})\right] \varphi_{(n+1)z_j}(x;q,n) = \int_0^1 \lim_{n\to\infty} \varphi_{\lfloor (n+1)z \rfloor}(x;q,n) dw(z)$$
$$= \int_0^q \underline{t}\left(x, \frac{y}{q}\right) dw(y) + \int_q^1 \bar{t}\left(x, \frac{y-q}{1-q}\right) dw(y)$$

where the last step follows from (A.22). Combining this with (A.23) yields

$$\lim_{n\to\infty} \phi_n(x) = \rho \left[\int_0^q \underline{t}\left(x, \frac{y}{q}\right) dw(y) + \int_q^1 \bar{t}\left(x, \frac{y-q}{1-q}\right) dw(y)\right] + (1-\rho)\chi \qquad \text{(A.24)}$$

Therefore, $\phi_n(x)$ converges point-wise to (A.19) on $[\underline{v}, \bar{v}]$. By statement (1) of Proposition A.3, $\phi_n(x)$ is uniformly $L$-Liptschitz continuous on $[\underline{v}, \bar{v}]$ for some sufficient large $L$. Following the same argument in the proof of Proposition A.3, it can be show that $\phi(x)$ given by (A.19) is also $L$-Liptschitz continuous for sufficiently large $L$. These together imply that the convergence of $\phi_n(x)$ to (A.19) is in fact uniform.[44] The fact that $\phi_n'(x)$ converges uniformly to (A.20) can be proved analogously.

Finally, we prove that $\varphi_n(v) := \rho \sum_{j=1}^{n+1} w_j v^{(j)} + (1-\rho)\chi$ converges in probability to $\phi^*$ given by (A.21). It suffices to show that $\sum_{j=1}^{n+1} w_j v^{(j)}$ converges in probability to $\int_0^1 G^{-1}(y) dw(y)$. We use a result from Van Zwet (1980), who establishes strong law for linear combinations of order statistics, to prove this. Let $U_1, U_2, \cdots, U_{n+1}$ be $n+1$ random variables drawn from a uniform distribution on $(0,1)$, and $U_{1:n+1} \le U_{2:n+1} \le \cdots \le U_{n+1:n+1}$ denote the ordered $U_1, U_2, \cdots, U_{n+1}$. We can therefore rewrite $v^{(j)}$ as $G^{-1}(U_{j:n+1})$ for each $j = 1, \cdots, n+1$. For $t \in (0,1)$, define $\gamma_n(t) := G^{-1}(U_{\lceil (n+1)t \rceil : n+1})$ and $\xi_n(t) := (n+1) \cdot \left[w\left(\frac{\lceil (n+1)t \rceil}{n+1}\right) - w\left(\frac{\lceil (n+1)t \rceil - 1}{n+1}\right)\right]$. We can then rewrite $\sum_{j=1}^{n+1} w_j v^{(j)}$ in an integral

[44] This stems from the following general observation: *For any $L > 0$, let $\{f_n(\cdot)\}_{n\ge 0}$ be a sequence of L-Lipschitz continuous function on a closed interval $[a,b]$ that converges point-wise to a L-Lipschitz continuous function $f(\cdot)$. Then $f_n(\cdot)$ converges uniformly to $f(\cdot)$ on $[a,b]$.* The proof is as follows. Given any $\varepsilon > 0$, consider a pair of $\delta, \eta > 0$ such that $\varepsilon > 2L\delta + \eta$. Partition interval $[a,b]$ into $K+1$ intervals with cutoffs $a = x_0 < x_1 < \cdots < x_{K+1} = b$ such that $|x_i - x_{i-1}| < \delta$ for all $i \in \{1, \cdots, K+1\}$. For this finite set $\{x_i\}_{i=1}^{K+1}$ point-wise convergence implies that there exists a threshold $N$ such that for all $n > N$ we have $|f_n(x_i) - f(x_i)| < \eta$ for all $i \in \{1, \cdots, K\}$. Now consider any $x \in [a,b]$ and let $i$ be such that $x \in [x_{i-1}, x_i]$. We then obtain

$$\begin{aligned} |f_n(x) - f(x)| &= |f_n(x) - f_n(x_i) + f_n(x_i) - f(x_i) + f(x_i) - f(x)| \\ &\le |f_n(x) - f_n(x_i)| + |f_n(x_i) - f(x_i)| + |f(x_i) - f(x)| \\ &< 2L\delta + \eta < \varepsilon \end{aligned}$$

for all $n > N$. This proves uniform convergence on $[a,b]$.

form as

$$\sum_{j=1}^{n+1} w_j v^{(j)} = \sum_{j=1}^{n+1} \left[ w\left(\frac{j}{n+1}\right) - w\left(\frac{j-1}{n+1}\right) \right] G^{-1}(U_{j:n+1}) = \int_0^1 \gamma_n(t)\xi_n(t)dt \quad \text{(A.25)}$$

Our assumptions for $G(\cdot)$ and $w(\cdot)$ ensure that $G^{-1}(\cdot), w(\cdot) \in L_1$, $\sup_n \|\xi_n\|_\infty < \infty$, and $\lim_{n\to\infty} \int_0^t \xi_n(x)dx = \int_0^t w'(x)dx = w(t)$ for all $t \in (0,1)$.[45] It follows from Theorem 2.1 and Corollary 2.1 of Van Zwet (1980) that the integral in (A.25) converges almost surely to $\int_0^1 G^{-1}(y)dw(y)$ as $n \to \infty$, which implies convergence in probability. □

Finally, we establish Proposition A.4, which summarizes useful properties of $\phi(\cdot)$. Statements (1) to (3) of this proposition will be exploited in Appendix D to establish comparative static results. Statement (4) implies Lemma 3 in the main text, because $\phi'(\cdot) < 1$ is sufficient for our single-crossing property to hold.

**Proposition A.4.** *For any $x \in [\underline{v}, \overline{v}]$, the following properties hold:*

1. *If $\rho < 1$, then $\phi(x)$ is strictly increasing in $\chi$.*
2. *If $\rho > 0$, then $\phi(x)$ is strictly decreasing in $q$.*
3. *If $\rho > 0$, then $\phi(x)$ strictly decreases as $w(\cdot)$ shifts from $w^I(\cdot)$ to $w^{II}(\cdot)$, where $w^I(\cdot), w^{II}(\cdot) \in \Delta([-1,1])$ and $w^I(\cdot)$ first order stochastically dominates $w^{II}(\cdot)$. .*
4. *$\phi'(x) < 1$ if either $\rho$ is close to zero or both $G$ and $1-G$ are strictly log-concave.*

*Proof of Proposition A.4.* For all $y \in [0,1]$ Let

$$t(x,y;q) := \begin{cases} \underline{t}\left(x, \frac{y}{q}\right), & \text{if } y \le q \\ \overline{t}\left(x, \frac{y-q}{1-q}\right), & \text{if } y > q \end{cases} \quad \text{(A.26)}$$

Then by (A.19) we can rewrite $\phi(\cdot)$ as

$$\phi(x) = \rho \int_0^1 t(x,y;q)dw(y) + (1-\rho)\chi = \rho \mathbb{E}_w[t(x,\cdot;q)] + (1-\rho)\chi \quad \text{(A.27)}$$

It is obvious from (A.27) that $\phi(x)$ strictly increases in $\chi$ if $\rho < 1$. This proves part (1). Notice that $\phi(x)$ depends on $x$, $q$ and $w(\cdot)$ only through the integral $\int_0^1 t(x,y;q)dw(y)$. Because both $\underline{t}(x,y)$ and $\overline{t}(x,y)$ are strictly increasing in $y$ (cf. Claim A.2), it follows from

[45] Here $L_1$ refers to the space of Lebesgue measurable functions $f : (0,1) \mapsto \mathbb{R}$ with finite $\|\cdot\|_1$ norm. $\|\cdot\|_\infty$ denotes the essential supremum norm. Moreover, the absolute continuity of $w(\cdot)$ ensures that its derivative $w'(\cdot)$ exists almost everywhere and $\int_0^t w'(x)dx = w(t)$ for all $t \in (0,1)$.

(A.26) that $t(x,y;q)$ is strictly decreasing in $q$. $\int_0^1 t(x,y;q)dw(y)$ must inherits the same property and therefore part (2) holds. Next, consider two weighting functions $w^{\mathrm{I}}(\cdot)$ and $w^{\mathrm{II}}(\cdot)$ such that $w^{\mathrm{I}}(\cdot)$ first order stochastically dominates $w^{\mathrm{II}}(\cdot)$. Because $t(x,y;q)$ is strictly increasing in $y$, $\int_0^1 t(x,y;q)dw^{\mathrm{I}}(y) > \int_0^1 t(x,y;q)dw^{\mathrm{II}}(y)$ must hold. Therefore, $\phi(x)$ must be strictly higher under $w^{\mathrm{I}}(\cdot)$ than under $w^{\mathrm{II}}(\cdot)$. This proves part (3).

Finally, we establish part (4). Because both $\underline{t}_x\left(x,\frac{y}{q}\right)$ and $\bar{t}_x\left(x,\frac{y-1}{1-q}\right)$ are uniformly bounded (cf. Claim A.2), it follows from (A.20) that $\phi'(x)$ must converge to 0 uniformly for all $x\in[\underline{v},\overline{v}]$ as $\rho\to 0$. This implies that $\phi'(x)<1$ for all $x\in[\underline{v},\overline{v}]$ when $\rho$ is sufficiently close to zero. When both $G$ and $1-G$ are strictly log-concave, it follows from Claim A.2 that $\underline{t}_x\left(x,\frac{y}{q}\right)<1$ for $y<q$ and $\bar{t}_x\left(x,\frac{y-q}{1-q}\right)<1$ for $y>q$. Therefore, by (A.20), $\phi'(x)<1$ holds if either (i) $\rho<1$ or (ii) $w(\cdot)$ does not put all weights on $y=q$ (i.e., $w(\cdot)$ is not a step function with threshold $q$). The latter condition for $w(\cdot)$ is always satisfied due to our assumption that $w(\cdot)$ is absolutely continuous. □

# Appendix B Proofs of Observations in Section 5.3

In this appendix we prove Observations 1 to 4 – which characterize useful and novel properties of solutions for general linear persuasion problems – in Section 5.3.2. Without loss of generality, we normalize $[\underline{\kappa},\overline{\kappa}]$ to $[-1,1]$ so that problem (MP') in Section 5.3 is equivalent to

$$\max_{H\in\Delta([-1,1])}\int_{-1}^{1}U(\theta)dH(\theta),\ \text{s.t. } F\succeq_{MPS}H \tag{P}$$

Unless explicitly stated otherwise, we assume that utility function $U(\cdot)$ is *regular*: it is piece-wise Lipschitz continuous on $[-1,1]$ and can be partitioned into finitely many intervals on which $U(\cdot)$ is either strictly concave, strictly convex, or affine.[46]

Our proofs for Observations 2 to 4 mainly build on the duality methods for linear persuasion problems, and we shall present some preliminaries in Appendix B.1.

## B.1 The duality method for linear persuasion problems

We present some relevant results – derived using duality methods – to solve the linear persuasion problem (P). These results are mostly due to Dworczak and Martini (2019) (DM

[46] This regularity condition is due to Dworczak and Martini (2019) (see Definition 1 therein).

henceforth) and shall be frequently used in our proofs. The dual problem of (P) is

$$\min_{p(\cdot)} \int_{-1}^{1} p(\theta) dF(\theta), \text{ s.t. } p(\cdot) \text{ being convex and } p(\cdot) \geq U(\cdot) \tag{D}$$

where $p(\cdot)$ is a convex 'price' function on $[-1,1]$ that is everywhere above $U(\cdot)$. DM establishes strong duality for regular $U(\cdot)$.

**Remark B.1.** *(Theorem 2 of DM) Suppose $U(\cdot)$ is regular, then strong duality holds. That is, optimal solutions to both problems* (P) *and* (D) *exist, and their values are identical.*

Moreover, the optimal solution $p(\cdot)$ to the dual problem (D) implies tight constraints on the structure of any solution $H$ to the primal linear persuasion problem (P), through a set of complementary slackness constraints. Remark B.2 (Theorem 1 and Proposition 1 of DM) provides a convenient tool to verify candidate solutions to problems (P) and (D).

**Remark B.2.** *(Theorem 1 and Proposition 1 of DM) Suppose $U(\cdot)$ is regular. If there exists some $H \in \Delta([-1,1])$ and a convex function $p(\cdot)$ on $[-1,1]$ with $p(\cdot) \geq U(\cdot)$ that satisfy*

$$supp(H) \subseteq \{\theta : p(\theta) = U(\theta)\}, \text{ and} \tag{B.1}$$

$$\int_{-1}^{1} p(\theta) dH(\theta) = \int_{-1}^{1} p(\theta) dF(\theta), \text{ and} \tag{B.2}$$

$$F \succeq_{MPS} H, \tag{B.3}$$

*then $H$ is a solution to the primal problem* (P)*, $p(\cdot)$ is a solution to the dual problem* (D)*, and the values of both problems* (P) *and* (D) *are equal to $\int_{-1}^{1} p(k) dF(k)$.*

Since the price function $p(\cdot)$ is convex on $[-1,1]$, $[-1,1]$ can be partitioned into intervals in which it is either strictly convex or affine. Moreover, since $p(\cdot) \geq U(\cdot)$ and $U(\cdot)$ is regular, $[-1,1]$ can also be partitioned into intervals where either $p(\cdot) = U(\cdot)$ or $p(\cdot) > U(\cdot)$. Remark B.3 characterizes the structure of the optimal solution $H$ to (P) in these intervals.

**Remark B.3.** *Suppose (i) the prior $F$ has full support and is continuous on $[-1,1]$, (ii) $U(\cdot)$ is regular, and (iii) $p(\cdot)$ and $H$ satisfies conditions* (B.1) *to* (B.3) *in Remark B.2. Let $[x,y] \subseteq [-1,1]$. The following properties hold:*

1. *If $[x,y]$ is a maximal interval*[47] *on which $p(\cdot)$ is strictly convex, then $p(\cdot) = U(\cdot)$*

[47] Following Dworczak and Martini (2019), we say $[x,y]$ is a maximal interval for some property $E$ if (i) $E$ holds on $(x,y)$, and (ii) $E$ does not hold on any other interval $[c,d]$ that contains $[x,y]$.

*on $[x,y]$ and $H \succeq_{MPS} H_{\mathscr{P}(x,y)}$ (that is, any optimal information policy must be fully revealing on intervals where $p(\cdot)$ is strictly convex);*[48]

2. *If $[x,y]$ is a maximal interval on which $p(\cdot)$ is affine, then $H \succeq_{MPS} H_{\mathscr{P}(x)}$, $H \succeq_{MPS} H_{\mathscr{P}(y)}$, and $p(c) = U(c)$ holds for at least one point $c \in [x,y]$.*

3. *If $[x,y]$ is a maximal interval in which $p(\cdot) > U(\cdot)$ holds, then either*

   *(a) $p(\cdot)$ is affine and $H(\cdot)$ equals a constant on $[x,y]$, or*

   *(b) $p(\cdot)$ is piece-wise affine with a kink $z \in [x,y]$ and $H(\cdot)$ is piece-wise constant with a jump at $z \in [x,y]$ and it satisfies $H \succeq_{MPS} H_{\mathscr{P}(z)}$.*

4. *Let $[x,y]$ be any interval on which $U(\cdot)$ is strictly convex, then either*

   *(a) there exist $[c,d] \in [x,y]$ with $c < d$ such that* $H(\theta) = \begin{cases} F(c), & \text{if } \theta \in [x,c) \\ F(\theta), & \text{if } \theta \in [c,d) \\ F(d), & \text{if } \theta \in [d,y] \end{cases}$*, or*

   *(b) $H(\theta)$ equals a constant on $(x,y)$ or is piece-wise constant with a kink $z \in (x,y)$ and $H \succeq_{MPS} H_{\mathscr{P}(z)}$ holds for that kink.*

*Proof of Remark B.3.* Statements (1) and (2) follow from Proposition 2 of DM. Statement (3) follows from Lemma 4 of DM. In what follows we prove statement (4). Suppose $U(\cdot)$ is strictly convex on $[x,y]$ and let $\xi := \{\theta \in [x,y] | U(\theta) = p(\theta)\}$. By Lemma 5 of DM, $\xi$ must be either empty or a (possibly degenerate) interval. If $\xi$ is empty, then $[x,y]$ is an interval on which $p(\cdot) > U(\cdot)$ so that (4b) follows directly from statement (3) of this remark. In what follows we assume that $\xi = [c,d] \subseteq [x,y]$ for some $x \leq c \leq d \leq y$. In this case $p(\cdot)$ must be strictly convex on $[c,d]$ and, by statement (1), $H(\cdot) = F(\cdot)$ must hold on $[c,d]$. In the meanwhile, by statement (2a), $p(\cdot)$ must be either affine or piece-wise affine with one kink on either $[x,c)$ or $(d,y]$. We continue by showing that $p(\cdot)$ cannot have kink on either interval so that $H(\theta) = F(c)$ for $\theta \in [x,c)$ and $H(\theta) = F(d)$ for $\theta \in (d,y]$ must hold. We show this for $(d,y]$ by contradiction (the proof for this on $[x,c)$ is analogous). Suppose instead there exists a kink $z \in (d,y)$ such that (i) $p(\cdot) > U(\cdot)$ and is affine on $(d,z)$ and $(z,y)$, and (ii) $p'_-(z) < p'_+(z)$.[49] Let $\tilde{p}(\theta) := p(z) + p'_+(z)(\theta - z)$ for $\theta \in [d,y]$. Then, by convexity of $U(\cdot)$ on $[d,y]$, there exists a $w \in (d,z)$ such that $U(\theta) > (<) \tilde{p}(\theta)$ for $\theta < (>) w$. Define $\widehat{p}(\cdot)$ as follows: $\widehat{p}(\theta) = p(\theta)$ for $\theta \notin (d,z)$, $\widehat{p}(\theta) = U(\theta)$ for $\theta \in (d,w]$ and $\widehat{p}(\theta) = \tilde{p}(\theta)$ for

[48] Notice that $H \succeq_{MPS} H_{\mathscr{P}(x,y)}$ if and only if $H(\theta) = F(\theta)$ and $\int_{-1}^{\theta} k dH(k) = \int_{-1}^{\theta} k dF(k)$ for all $\theta \in [x,y]$.

[49] $p'_-(z)$ and $p'_+(z)$ denote, respectively, the left and right derivative of $p(\cdot)$ at $z$.

$\theta \in (w,z)$. It is easy to verify that $\widehat{p}(\cdot)$ is convex on $[x,y]$ and $\widehat{p}(\theta) \leq p(\theta)$ for all $\theta \in [x,y]$ (with strict inequality holds for $\theta \in (d,z)$). Therefore, $\int_{-1}^{1} \widehat{p}(\theta)dF(\theta) < \int_{-1}^{1} p(\theta)dF(\theta)$ must hold, contradicting with the fact that $p(\cdot)$ is the solution to the dual problem (D). □

The final remark, due to Theorem 4 of DM, provides a convenient way to verify whether an induced distribution of posterior means $H$ is unimprovable for a designer with utility function $U(\cdot)$ by solving a monopolistic persuasion problem with his utility function modified by its convex translations.

**Remark B.4.** *(Theorem 4 of DM) Suppose $U(\cdot)$ is continuous and regular. Then $H$ is unimprovable for a designer with utility function $U(\cdot)$ if and only if $H$ is a solution to*

$$\max_{H \in \Delta([-1,1])} \int_{-1}^{1} \widehat{U}(\theta)dH(\theta), \quad s.t.\ F \succeq_{MPS} H$$

*where $\widehat{U}(\cdot) = U(\cdot) + \omega(\cdot)$ for some convex function $\omega(\cdot)$.*

## B.2 Proof of Observation 1

For this Observation we allow $U(\cdot)$ to be any utility function on $[-1,1]$. For any prior $F \in \Delta([-1,1])$ (which need not be continuous and fully supported), let

$$\mathscr{U}_{\pi}(U,F) := \mathbb{E}_{H_{\pi}}[U(\cdot)] = \int_{-1}^{1} U(\theta)dH_{\pi}(\theta)$$

denote the designer's expected payoff under any feasible information policy $\pi \in \Pi$ and prior $F \in \Delta([-1,1])$.[50] Let $\underline{\pi}$ denote the *null* information structure that reveals no information. Then $H_{\underline{\pi}}$ is a degenerate distribution with all mass on prior mean $\mu_F := \mathbb{E}_F[k]$ and therefore

$$\mathscr{U}_{\underline{\pi}}(U,F) = U(\mu_F) \tag{B.4}$$

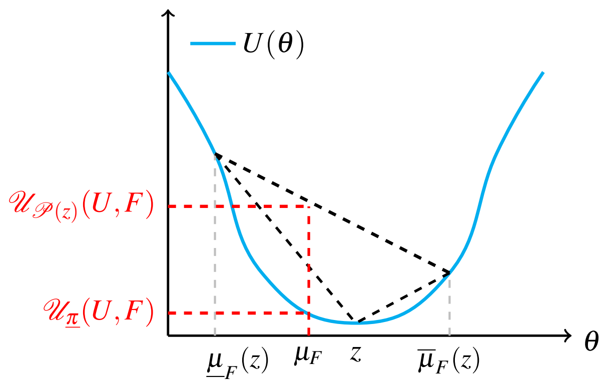
On the other hand, for a cutoff censorship policy $\mathscr{P}(z)$ with $z \in (-1,1)$, we have

$$\mathscr{U}_{\mathscr{P}(z)}(U,F) := F^{-}(z)U(\underline{\mu}_F(z)) + \big(F(z) - F^{-}(z)\big)U(z) + \big(1 - F(z)\big)U(\overline{\mu}_F(z)) \tag{B.5}$$

where $F^{-}(z) := \lim_{x \uparrow z} F(z)$, $\underline{\mu}_F(z) := \mathbb{E}_F[k|k < z]$ and $\overline{\mu}_F(z) := \mathbb{E}_F[k|k > z]$.

[50] Recall that $H_{\pi}$ denotes the distribution of posterior expectations induced by $\pi$ under prior $F$.

Figure B.1: Graphical Illustration for the Proof of Claim B.1




**Claim B.1.** *Suppose $U(\cdot)$ satisfies the increasing slope property at point $z \in (-1,1)$, then $\mathscr{U}_{\mathscr{P}(z)}(U,F) > \mathscr{U}_{\underline{\pi}}(U,F)$ for any $F \in \Delta([-1,1])$ that satisfies $0 < F^-(z) \leq F(z) < 1$.*

*Proof of Claim B.1.* Figure B.1 illustrates $\mathscr{U}_{\mathscr{P}(z)}(U,F)$ and $\mathscr{U}_{\underline{\pi}}(U,F)$ for a function $U(\cdot)$ that satisfies increasing slope property at a point $z \in (-1,1)$ and a prior $F$ with no mass point at $z$. By (B.4) and (B.5), we obtain

$$
\begin{aligned}
\mathscr{U}_{\mathscr{P}(z)}(U,F) - \mathscr{U}_{\underline{\pi}}(U,F) =& F^-(z)\left(\underline{\mu}_F(z) - \mu_F\right)\frac{U(\underline{\mu}_F(z)) - U(\mu_F)}{\underline{\mu}_F(z) - \mu_F} \\
&+ \left(F(z) - F^-(z)\right)\left(z - \mu_F\right)\frac{U(z) - U(\mu_F)}{z - \mu_F} \\
&+ \left(1 - F(z)\right)\left(\overline{\mu}_F(z) - \mu_F\right)\frac{U(\overline{\mu}_F(z)) - U(\mu_F)}{\overline{\mu}_F(z) - \mu_F}
\end{aligned}
$$

On the other hand, by law of iterated expectations, we have

$$
\begin{aligned}
& F^-(z)\underline{\mu}_F(z) + \left(F(z) - F^-(z)\right)z + \left(1 - F(z)\right)\overline{\mu}_F(z) = \mu_F \\
\implies & \left(F(z) - F^-(z)\right)\left(z - \mu_F\right) = -F^-(z)\left(\underline{\mu}_F(z) - \mu_F\right) - \left(1 - F(z)\right)\left(\overline{\mu}_F(z) - \mu_F\right)
\end{aligned}
$$

These together imply

$$
\begin{aligned}
\mathscr{U}_{\mathscr{P}(z)}(U,F) - \mathscr{U}_{\underline{\pi}}(U,F) = & F^-(z)\left(\mu_F - \underline{\mu}_F(z)\right)\left(\frac{U(z) - U(\mu_F)}{z - \mu_F} - \frac{U(\mu_F) - U(\underline{\mu}_F(z))}{\mu_F - \underline{\mu}_F(z)}\right) \\
&+ (1 - F(z))(\overline{\mu}_F(z) - \mu_F)\left(\frac{U(\overline{\mu}_F(z)) - U(\mu_F)}{\overline{\mu}_F(z) - \mu_F} - \frac{U(z) - U(\mu_F)}{z - \mu_F}\right)
\end{aligned}
$$

Since $\underline{\mu}_F(z) < x < \overline{\mu}_F(z)$ for $x \in \{z, \mu_F\}$, increasing slope property at $z$ implies $\frac{U(z)-U(\mu_F)}{z-\mu_F} -$

$\frac{U(\mu_F)-U(\underline{\mu}_F(z))}{\mu_F-\underline{\mu}_F(z)} \geq 0$ and $\frac{U(\overline{\mu}_F(z))-U(\mu_F)}{\overline{\mu}_F(z)-\mu_F} - \frac{U(z)-U(\mu_F)}{z-\mu_F} \geq 0$, with at least one holds with strict inequality.[51] This implies $\mathscr{U}_{\mathscr{P}(z)}(U,F) - \mathscr{U}_{\underline{\pi}}(U,F) \geq 0$ for all $F$. Finally, notice that if $0 < F^-(z) \leq F(z) < 1$ holds, then both $F^-(z)\left(\mu_F - \underline{\mu}_F(z)\right)$ and $(1-F(z))(\overline{\mu}_F(z) - \mu_F)$ are strictly positive so that $\mathscr{U}_{\mathscr{P}(z)}(U,F) - \mathscr{U}_{\underline{\pi}}(U,F) > 0$ must hold. □

We are now ready to prove Observation 1. Suppose $U(\cdot)$ satisfies increasing slope property at point $z$ and let $H$ be any solution to (P) (if it exists). We show that $H \succeq_{MPS} H_{\mathscr{P}(z)}$ must hold by contradiction. Suppose there exists any $H \not\succeq_{MPS} H_{\mathscr{P}(z)}$ that solves (P). Let $\pi = (S, \sigma)$ be an information policy that induces $H$. For each $s \in S$, let $\gamma_s$ denote the posterior distribution induced by $s$ and $h_s$ denote the mean of $\gamma_s$. Finally, let $\delta \in \Delta(S)$ denote the ex-ante distribution of messages $s \in S$ induced by $\pi$. With these we obtain

$$\int_{-1}^{1} U(\theta) dH(\theta) = \int_{s \in S} U(h_s) d\delta(s) = \int_{s \in S} \mathscr{U}_{\underline{\pi}}(U, \gamma_s) d\delta(s)$$

Since $H \not\succeq_{MPS} H_{\mathscr{P}(z)}$, there exists $s \in S$ such that $0 < \gamma_s^-(z) \leq \gamma_s(z) < 1$ holds. Denote by $\widetilde{S} \subseteq S$ the set of all such $s$ and $\widetilde{S}$ must have positive probability measure under $\delta$. Consider the joint information policy $\widetilde{\pi}$ induced by $\pi$ and the cutoff policy $\mathscr{P}(z)$, and let $\widetilde{H} = H_{\widetilde{\pi}}$. For all events $s \in \widetilde{S}$, it follows from Claim B.1 that $\mathscr{U}_{\mathscr{P}(z)}(U, \gamma_s) > \mathscr{U}_{\underline{\pi}}(U, \gamma_s)$. For $s \notin \widetilde{S}$, $\mathscr{U}_{\mathscr{P}(z)}(U, \gamma_s) = \mathscr{U}_{\underline{\pi}}(U, \gamma_s)$ holds trivially. Therefore, we have

$$\begin{aligned}
\int_{-1}^{1} U(\theta) d\widetilde{H}(\theta) &= \int_{s \in \widetilde{S}} \mathscr{U}_{\mathscr{P}(z)}(U, \gamma_s) d\delta(s) + \int_{s \in S/\widetilde{S}} \mathscr{U}_{\mathscr{P}(z)}(U, \gamma_s) d\delta(s) \\
&= \int_{s \in \widetilde{S}} \mathscr{U}_{\mathscr{P}(z)}(U, \gamma_s) d\delta(s) + \int_{s \in S/\widetilde{S}} \mathscr{U}_{\underline{\pi}}(U, \gamma_s) d\delta(s) \qquad \text{(B.6)} \\
&> \int_{s \in \widetilde{S}} \mathscr{U}_{\underline{\pi}}(U, \gamma_s) d\delta(s) + \int_{s \in S/\widetilde{S}} \mathscr{U}_{\underline{\pi}}(U, \gamma_s) d\delta(s) = \int_{-1}^{1} U(\theta) dH(\theta)
\end{aligned}$$

This contradicts that $H$ is a solution to (P).

Next we show that any $H$ that is unimprovable for the designer must also satisfy $H \succeq_{MPS} H_{\mathscr{P}}$. By Remark B.4, $H$ is unimprovable for the designer if and only if $H$ is a solution to (P) with utility function $U(\cdot)$ replaced by $\widehat{U}(\cdot) = U(\cdot) + \omega(\cdot)$ for some convex function $\omega(\cdot)$. Observe that $\frac{\widehat{U}(x)-\widehat{U}(z)}{x-z} = \frac{U(x)-U(z)}{x-z} + \frac{\omega(x)-\omega(z)}{x-z}$ and the latter term is non-decreasing in $x$ because $\omega(\cdot)$ is convex. $\widehat{U}(x)$ thus preserves increasing slope properties of $U(\cdot)$. Therefore, $H \succeq_{MPS} H_{\mathscr{P}(z)}$ must hold following the previous argument.

[51] In case $z = \mu_F$, we can let $\frac{U(z)-U(\mu_F)}{z-\mu_F}$ be any number between $U'_-(z)$ to $U'_+(z)$, which are the left and right derivatives of $U(\cdot)$ at point $z$, respectively. Notice that the increasing slope property at point $z$ implies the existence of both $U'_-(z)$ and $U'_+(z)$ (through the monotone convergence theorem) and that $U'_-(z) \leq U'_+(z)$.

## B.3 Proofs of Observations 2 and 3

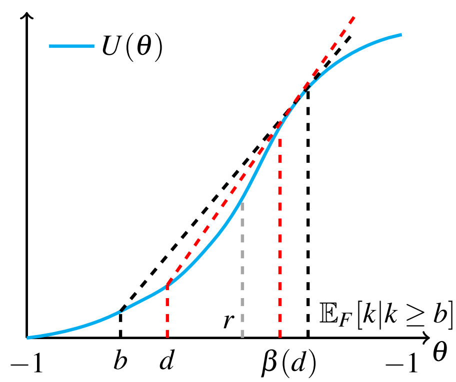
We prove Observation 2, which concerns strictly S-shaped utility functions. The proof for Observation 3 for strictly inverse S-shaped functions is almost identical and thus omitted.

To simplify exposure we continue with the normalization $[\underline{\kappa}, \overline{\kappa}] = [-1, 1]$ and let $U(\cdot)$ be twice continuously differentiable and strictly S-shaped on $[-1, 1]$ with inflection point $r$. Under this normalization, statement (1) of Observation 2 can be simplified to that any solution $H$ to (P) must be induced by an upper censorship policy $\mathscr{P}(-1, b)$ where $b$ satisfies

$$(\mathbb{E}_F[k|k \geq b] - b) U'(\mathbb{E}_F[k|k \geq b]) \leq U(\mathbb{E}_F[k|k \geq b]) - U(b) \tag{B.7}$$

and (B.7) must be binding whenever $b > -1$. This result is equivalent to Theorem 1 of Kolotilin, Mylovanov and Zapechelnyuk (2021) and is proved therein. In what follows we prove statements (2) and (3). To do so we introduce an auxiliary result.

Figure B.2: Graphical Illustration for Function $\beta(d)$ and the Proof of Lemma B.1




For each $d \in [-1, r]$, define

$$\beta(d) := \sup\left\{x \in [d, 1] : (x - d)U'(x) \geq U(x) - U(d)\right\} \tag{B.8}$$

Function $\beta(d)$ is illustrated geometrically in Figure B.2. For any $d \in [-1, r]$ and $\beta(d)$, the straight line passing through $(d, U(d))$ and $(\beta(d), U(\beta(d)))$ must lie above $U(\cdot)$ on $[d, 1]$ (and it must be tangent to $U(\cdot)$ at $\beta(d)$ whenever $\beta(d) < 1$). Claim B.2 summarizes important properties for function $\beta(\cdot)$ that we will use for subsequent proofs.

**Claim B.2.** *Suppose $U(\cdot)$ is strictly S-shaped with inflection point $r \in [-1, 1]$. Let $b$ be given by (B.7). Then function $\beta(d)$ for $d \in [-1, r]$ satisfies the following properties.*

1. *$\beta(d)$ is decreasing on $[-1,r]$, strictly so if $\beta(d)<1$, and $\beta(r)=r$.*
2. *$\beta(d)\le \mathbb{E}_F[k|k\ge d]$ if and only if $d\ge b$.*

*Proof of Claim B.2.* For any $-1\le d\le r$ and $d<x\le 1$, let

$$\xi(d,x) := (x-d)U'(x)-(U(x)-U(d))$$

By (B.8), we have $\beta(d)=\sup\{x\in[d,1]:\xi(d,x)\ge 0\}$. We first show that $\beta(d)\ge r$ for all $d<r$ and $\beta(r)=r$. By the Mean Value Theorem, there exists some $y\in(d,x)$ such that $U(x)-U(d)=(x-d)U'(y)$. We can thus rewrite $\xi(d,x)$ as

$$\xi(d,x)=(x-d)\left(U'(x)-U'(y)\right) \tag{B.9}$$

Suppose $d=r$, then for all $x>r$ we get $\xi(r,x)=(x-r)(U'(x)-U'(y))<0$ because $U'(\cdot)$ is strictly decreasing on $[r,1]$ and $y>r$. Therefore, $\beta(r)=r$ must hold. Suppose $d<r$ and consider any $x\in(d,r]$. Then $\xi(d,\beta(d))=(\beta(d)-d)(U'(x)-U'(y))$ must hold for some $y\in(d,x)$. However, this $\xi(d,x)$ must be strictly positive because $U'(\cdot)$ is strictly increasing on $[d,r]$. This implies $\xi(d,x)>0$ for all $x\in(d,r]$ and hence $\beta(d)\ge r$ must hold.

To complete the remainder of the proof we establish two properties about $\xi(d,x)$:

(i). Given any $d\in[-1,r)$, $\xi(d,x)$ crosses zero at most once and from above for $x\in(d,1]$.

(ii). Suppose $\xi(d',x)=0$ holds for some $d'\in[-1,r)$ and $x\in(d',1]$, then $\xi(d'',x)>0$ must hold for all $d''\in(d',1]$.

To show (i), notice that given any $d\in[-1,r)$, $\xi_x(d,x)=(x-d)U''(x)<0$ holds for all $x\in(r,1]$ because $U(\cdot)$ is strictly concave on $[r,1]$. Moreover, as we already established above, $\beta(d)\ge r$ must hold so that $\xi(d,x)$ must be strictly positive for $x\in(d,r]$. These together imply that $\xi_x(d,x)$ must be strictly decreasing for $x\in[r,1]$ and it can at most cross zero once and from above on $[r,1]$. To show (ii), consider any pair $(d',x)$ with $d'\in[-1,r)$ and $x\in(d',1]$ such that $\xi(d',x)=0$ holds. By (B.9), there exists some $y\in(d',x)$ such that $U'(y)=U'(x)$. In fact, $y\in(d',r)$ must hold because $U'(\cdot)$ is strictly decreasing on $[r,x]$. With these we obtain

$$\xi_d(d,x)=U'(d)-U'(x)=U'(d)-U'(y)$$

for this given $x$. Since $U'(\cdot)$ is strictly increasing on $[-1,r]$, $\xi(d,x)$ is first increasing and then decreasing as $d$ increases from $d'$ to $r$, with a peak at some $y\in(d',r)$. Finally, observe that $\xi(r,x)>0$ holds for sure. These together imply that $\xi(d'',x)>0$ for all $d''\in(d',r]$.

The aforementioned properties (i) and (ii) for $\xi(d,x)$ and $\beta(r) = r$ together imply statement (1) of this claim. To show statement (2), first consider the case $b > -1$. In this case (B.7) must be binding so $\xi(b, \mathbb{E}_F[k|k \geq b]) = 0$ holds. This implies $\beta(b) = \mathbb{E}_F[k|k \geq b]$. The statement then follows from the fact that $\beta(d)$ is decreasing while $\mathbb{E}_F[k|k \geq d]$ is increasing in $d$. If $b = -1$ then (B.7) and (B.8) together imply $\beta(-1) \leq \mathbb{E}_F[k|k > -1] = \mathbb{E}_F[k]$. For any $d > b = -1$, we then have $\beta(d) \leq \beta(-1) \leq \mathbb{E}_F[k] < \mathbb{E}_F[k|k > d]$. $\square$

### B.3.1 Proof for Statement (2) of Observation 2

Consider any designer $m \in M$ whose utility function is $U(\cdot)$. Let $\mathscr{H}_m$ the set of $H$ that is unimprovable to this designer. Under our normalization $[\underline{\kappa}, \overline{\kappa}]$ statement (2) of Observation 2 is equivalent to the following two properties: (here recall that $b$ is characterized by (B.7))

(A). $H \succeq_{MPS} H_{\mathscr{P}(-1,b)}$ for all $H \in \mathscr{H}_m$.

(B). $H_{\mathscr{P}(-1,d)} \in \mathscr{H}_m$ for all $d \in [b,1]$.

As we explain shortly, both properties are implied by Lemma B.1, which characterizes $\mathscr{H}_m$ when $U(\cdot)$ is strictly S-shaped on $[-1,1]$.

**Lemma B.1.** *Suppose $U(\cdot)$ is strictly S-shaped with inflection point $r \in (-1,1)$ and let $b$ be given by (B.7). Then $H \in \mathscr{H}_m$ if and only if (i) $F \succeq_{MPS} H$, and (ii) there exists some $d \in [b,r]$ such that supp$(H) \subseteq [-1,d] \cup [\beta(d),1]$ and $H(\theta) = \min\{F(\theta), F(d)\}$ for $\theta \in [-1, \beta(d))$.*

Lemma B.1 implies that any $H \in \mathscr{H}_m$ must be induced by some information policy $\pi$ that fully reveals states in interval $[-1,d]$ with $d \in [b,d]$. This implies property (A) above, because any such $\pi$ must be Blackwell more informative than $\mathscr{P}(-1,b)$. This feature also implies that $H \in \mathscr{H}_m$ holds if $H$ is implied by a censorship policy $\mathscr{P}(-1,d)$ with $d \in [d,r]$. Finally, by letting $d = r$ and using the fact that $\beta(r) = r$, condition (ii) of Lemma B.1 becomes supp$(H) \subseteq [-1,1]$ and $H(\theta) = F(\theta)$ for all $\theta \in [-1,r]$. This condition is trivially satisfied by all $H_{\mathscr{P}(-1,d)}$ with $d > r$. These together imply property (B). It is therefore sufficient to prove Lemma B.1 to establish statement (2) of Observation 2.

*Proof of Lemma B.1.* We establish the "if" part by construction using duality methods. Consider the following auxiliary price function

$$p(\theta) := \begin{cases} U(\theta), & \text{if } \theta \in [-1,d] \\ U(\beta(d)) + U'(\beta(d))(\theta - \beta(d)), & \text{if } \theta \in (d,1] \end{cases} \tag{B.10}$$

That is, $p(\cdot)$ coincides with $U(\cdot)$ on $[-1,d]$ and it becomes a linear function tangent to $U(\cdot)$ at point $\beta(d)$ on $[d,1]$ (cf. the red dashed line segment in Figure B.2). As is evident graphically, such $p(\cdot)$ is convex and $p(\cdot) \geq U(\cdot)$ holds on $[-1,1]$.[52] Construct function $\omega(\cdot)$ as follows:

$$\omega(\theta) := \begin{cases} 0, & \text{if } \theta \in [-1,\beta(d)) \\ p(\theta) - U(\theta), & \text{if } \theta \in [\beta(d),1] \end{cases} \tag{B.11}$$

Then

$$\omega''(\theta) := \begin{cases} 0, & \text{if } \theta \in [-1,\beta(d)) \\ -U''(\theta), & \text{if } \theta \in (\beta(d),1] \end{cases}$$

Since $U'(\cdot)$ is strictly decreasing on $[r,1]$, $-U''(\theta) > 0$ for all $\theta \in (\beta(d),1]$. Therefore, $\omega(\cdot)$ is convex. Let $\widehat{U}(\theta) := U(\theta) + \omega(\theta)$. Graphically, $\widehat{U}(\cdot)$ coincides with $U(\cdot)$ on $[-1,\beta(d)]$ and it coincides with $p(\cdot)$ on $[\beta(d),1]$. Together with the fact that $p(\cdot) = U(\cdot)$ on $[-1,d]$ and $p(\cdot) > U(\cdot)$ on $(d,\beta(d))$, we obtain

$$\left\{\theta : p(\theta) = \widehat{U}(\theta)\right\} = [-1,d] \cup [\beta(d),1] \tag{B.12}$$

Consider any $H$ that satisfies conditions (i) and (ii) of this lemma. We show that such $H$ must satisfy the following three properties:

$$\text{supp}(H) \subseteq \{\theta : p(\theta) = \widetilde{U}(\theta)\}, \text{ and} \tag{B.13}$$

$$\int_{-1}^{1} p(\theta)dH(\theta) = \int_{-1}^{1} p(\theta)dF(\theta), \text{ and} \tag{B.14}$$

$$F \succeq_{MPS} H, \text{ and} \tag{B.15}$$

with $U(\cdot)$ replaced by $\widehat{U}(\cdot)$. To see this, note that (B.15) follows from condition (i) and (B.13) follows from (B.12) and condition (ii). (B.14) holds because, by condition (ii), $H(\cdot) = F(\cdot)$ holds on $[-1,d]$, where $p(\cdot)$ is strictly convex. Therefore, by Remark B.2, $H$ is a solution to problem (P) for utility function $\widehat{U}(\cdot)$. $H \in \mathscr{H}_m$ thus follows from Remark B.4.

Now we turn to the "only if" part. Consider any $H \in \mathscr{H}_m$. By Remark B.4, there exists a convex function $\widetilde{\omega}(\cdot)$ on $[-1,1]$ such that $H$ is a solution to problem (P) with utility function $\widetilde{U}(\cdot) = U(\cdot) + \widetilde{\omega}(\cdot)$. By Remarks B.2 and B.3, there exists a convex function $p(\cdot)$ that

[52] Formally, notice that $p'(\theta) = U'(\theta)$ for $\theta \in [-1,d)$ and $p'(\theta) = U'(\beta(d)) \geq U'(d)$ for $\theta \in (d,1]$. Because $U(\cdot)$ is strictly convex on $[-1,d]$, $U'(\theta)$ is increasing on $[-1,d]$. Therefore, $p'(\theta)$ is increasing on $[-1,1]$ and $p(\cdot)$ is convex.

satisfies properties (B.13) to (B.15) together with $\widetilde{U}(\cdot)$ and $H$. Moreover, $p(\cdot)$ must also be a solution to

$$\min_{p(\cdot)} \int_{-1}^{1} p(\theta)dF(\theta), \text{ s.t. } p(\cdot) \text{ being convex and } p(\cdot) \geq \widetilde{U}(\cdot) \tag{B.16}$$

We show that for any tuple $(H, \widetilde{U}(\cdot), p(\cdot))$ that satisfies the above conditions, $H$ must satisfy conditions (i) and (ii) of this lemma. (i) is obviously from (B.15). Below we focus on (ii).

Notice that $U(\cdot)$ is strictly convex on $[-1,r]$, this must also hold for $\widetilde{U}(\cdot)$ because $\widetilde{\omega}(\cdot)$ is convex. Therefore, by statement (4) of Remark B.3, we have

$$\left\{\theta \in [-1,r] : \ p(\theta) = \widetilde{U}(\theta)\right\} = [c,d] \subseteq [-1,r]$$

In fact, $c = -1$ must hold for $p(\cdot)$ to be a solution to (B.16).[53] This implies that $p(\cdot)$ is strictly convex on $[-1,d]$ and thus, by statement (1) of Remark B.3, we obtain

$$H(\theta) = F(\theta), \quad \forall \theta \in [-1,d] \tag{B.17}$$

In what follows we show that $d \geq b$ and $H(\theta) = F(d)$ must hold for all $\theta \in [d, \beta(d)]$. These together establish $H(\theta) = \min\{F(\theta), F(d)\}$ for $\theta \in [d, \beta(d)]$ and complete the proof. To do so, let

$$\eta := \inf\left\{\theta \in (r,1] : \ p(\theta) = \widetilde{U}(\theta)\right\}$$

Observe that the set on the right-hand side cannot be empty, because otherwise $p(\theta) > \widetilde{U}(\theta)$ has to hold for all $\theta > d$. This would imply $\text{supp}(H) \cap (d,1] = \emptyset$ through (B.13) and thus $H(\theta) = H(d) = F(d) < 1 = F(1)$ for all $\theta > d$. This is impossible because $H(1) = 1$ must hold for any feasible $H$. Therefore, $\eta$ is well defined. Moreover, the fact that $F$ is a mean-preserving spread of $H$ also implies that

$$\eta \leq \mathbb{E}_F[k|k > d] \tag{B.18}$$

must hold.[54] In this way, $[d, \eta]$ is a maximal interval in which $p(\cdot) > \widetilde{U}(\cdot)$ and this has two

[53] Suppose $c > -1$. Then $p(\cdot) > U(\cdot)$ on $[-1,c]$. Consider an alternative $\widetilde{p}(\cdot)$ which coincides with $p(\cdot)$ on $[c,1]$ but coincides with $\widetilde{U}(\cdot)$ on $[-1,c]$. It is routine to verify that $\widetilde{p}(\cdot)$ is convex and $p(\cdot) \geq \widetilde{p}(\cdot) \geq \widetilde{U}\cdot$. However, since $p(\cdot) > \tilde{p}(\cdot)$ on $[-1,x]$ and prior $F$ has full support on $[-1,1]$, we have $\mathbb{E}_F[p(\theta)] > \mathbb{E}_F[\tilde{p}(\theta)]$. This contradicts with the fact that $p(\cdot)$ solves (B.16).

[54] This is because $F \succeq_{MPS} H$ implies that $F$ and $H$ must have the same mean. Since $F$ and $H$ are identical on $[-1,d]$, they must also have the same conditional expectation on $(d,1]$, that is $\mathbb{E}_F[k|k > d] = \mathbb{E}_H[k|k > d]$.

implications. On the one hand, by (B.13), $\text{supp}(H) \cap (\eta) = \emptyset$ so that

$$H(\theta) = H(d) = F(d), \quad \forall \theta \in [d, \eta) \tag{B.19}$$

On the other hand, by statement (3) of Remark B.3, $p(\cdot)$ must be piece-wise linear with no more than one kink on this interval. We show that having a kink on $(d, \eta]$ is not possible. Suppose to the contrary and there is a kink $z \in (d, \eta]$. Then $H \succeq_{MPS} H_{\mathscr{P}(z)}$ must hold. Because $F$ has full support on $[-1, 1]$, $\text{supp}(H) \cap (d, z)$ cannot be empty. However, by (B.13), this would imply that there exists some $w \in (d, z)$ such that $p(w) = \widetilde{U}(w)$. This contradicts with the fact that $p(\cdot) > \widetilde{U}(\cdot)$ on $(d, \eta)$. Therefore, $p(\cdot)$ must be linear on $[d, \eta]$ and is differentiable at $\eta$. Together with the fact that $p(\cdot)$ coincides with $\widehat{U}(\cdot)$ at both $d$ and $\eta$, we obtain

$$p'(\eta) = \frac{\widetilde{U}(\eta) - \widetilde{U}(d)}{\eta - d}$$

Moreover, $p(\eta) = \widetilde{U}(\eta)$ and $p(\cdot) \geq \widetilde{U}(\cdot)$; this requires

$$\widetilde{U}'_{-}(\eta) \geq p'(\eta) \geq \widetilde{U}'_{+}(\eta)$$

to hold, where $\widetilde{U}'_{-}(\eta)$ and $\widetilde{U}'_{+}(\cdot)$ denote the left and right derivatives of $\widetilde{U}(\cdot)$ at point $\eta$. On the other hand, $\widetilde{U}(\cdot) = U(\cdot) + \omega(\cdot)$ with $U(\cdot)$ continuously differentiable and $\omega(\cdot)$ convex. These imply

$$\widetilde{U}'_{-}(\eta) \leq \widetilde{U}'_{+}(\eta)$$

Combining the three conditions above together we obtain that $\widetilde{U}(\cdot)$ must be differentiable at $\eta$ and $\widetilde{U}'(\eta) = p'(\eta)$ holds. Or equivalently,

$$\widetilde{U}'(\eta) = \frac{\widetilde{U}(\eta) - \widetilde{U}(d)}{\eta - d} \tag{B.20}$$

Plugging $\widehat{U}(\cdot) = U(\cdot) + \widetilde{\omega}(\cdot)$ into (B.20) and exploiting the fact that $\widetilde{\omega}(\cdot)$ is convex, we obtain

$$U'(\eta) - \frac{U(\eta) - U(d)}{\eta - d} = \frac{\widetilde{\omega}(\eta) - \widetilde{\omega}(d)}{\eta - d} - \widetilde{\omega}'(\eta) \leq 0$$

or equivalently

$$(\eta - d) U'(\eta) \leq U(\eta) - U(d) \tag{B.21}$$

This is possible only if $\inf\{\text{supp}(H) \cap (d, 1]\} \leq \mathbb{E}_F[k | k > d]$. (B.18) thus holds because of (B.13) and the fact that $p(\cdot) > \widehat{U}(\cdot)$ on $(d, r]$.

(B.21) and the definition of $\beta(d)$ (cf. (B.8)) together imply that $\eta \geq \beta(d)$ must hold. Combining this with (B.17) and (B.19), we obtain that $H(\theta) = \min\{F(\theta), F(d)\}$ for $\theta \in [-1, \beta(d))$. Finally, $d \geq b$ must be true because $\beta(d) \leq \eta \leq \mathbb{E}_F[k|k > d]$ can possibly hold only if $d \geq b$ (cf. (2) of Claim B.2). This completes the proof. $\square$

### B.3.2 Proof for Statement (3) of Observation 2

**Lemma B.2.** *Suppose $U(\cdot)$ is strictly S-shaped with inflection point $r \in [-1, 1]$. Suppose $H, H' \in \mathscr{H}_m$. Then $H' \succ_{MPS} H$ implies*[55]

$$\int_{-1}^{1} U(\theta) dH(\theta) > \int_{-1}^{1} U(\theta) dH'(\theta)$$

*Proof.* We first discuss two special cases: $r = 1$ and $r = -1$. If $r = 1$ then $U(\cdot)$ is strictly convex on $[-1, 1]$ and so does any $\widehat{U}(\cdot) = U(\cdot) + \omega(\cdot)$ with $\omega(\cdot)$ convex. In this case it is obvious that the only unimprovable $H$ is $F$. Therefore, $\mathscr{H}_m = \{F\}$ is a singleton and this lemma holds trivially. If $r = -1$ then $U(\cdot)$ is strictly concave on $[-1, 1]$. It then follows from the definition of mean-preserving spread that $\mathbb{E}_H[U(\cdot)] > \mathbb{E}_{H'}[U(\cdot)]$ for all $H' \succ_{MPS} H$.

Now consider $r \in (-1, 1)$. Since $H \in \mathscr{H}_m$, by Remark B.4 there exists some $\widehat{U}(\cdot) = U(\cdot) + \omega(\cdot)$ with $\omega(\cdot)$ convex such that

$$\begin{aligned} &\int_{-1}^{1} \widehat{U}(\cdot) dH(\theta) \geq \int_{-1}^{1} \widehat{U}(\cdot) dH'(\theta) \\ \iff &\int_{-1}^{1} U(\theta) dH'(\theta) - \int_{-1}^{1} U(\theta) dH(\theta) \leq \int_{-1}^{1} \omega(\theta) dH(\theta) - \int_{-1}^{1} \omega(\theta) dH'(\theta) \leq 0 \end{aligned} \tag{B.22}$$

holds for all $H' \neq H$. Below we prove that one of the above inequality must hold strictly for all $H' \in \mathscr{H}_m$ that $H' \succ_{MPS} H$, and thus complete the proof.

Because $H \in \mathscr{H}_m$, by Lemma B.1 there exists $d \in [b, r]$ such that $H(\theta) = \min\{F(\theta), F(d)\}$ for all $\theta \in [-1, \beta(d)]$ and $\text{supp}(H) \subseteq [-1, d] \cup [\beta(d), 1]$. As explained in the proof of Lemma B.1, the $p(\cdot)$ and $\omega(\cdot)$ constructed in (B.10) and (B.11) satisfy (B.22) and

$$\int_{-1}^{1} \big(U(\theta) + \omega(\theta)\big) dH(\theta) = \int_{-1}^{1} p(\theta) dH(\theta) = \int_{-1}^{1} p(\theta) dF(\theta) \tag{B.23}$$

Because $H' \in \mathscr{H}_m$ as well, there exists $d' \in [b, r]$ such that $H'(\theta) = \min\{F(\theta), F(d')\}$ for $\theta \in [-1, \beta(d')]$ and $\text{supp}(H') \subseteq [-1, d'] \cup [\beta(d'), 1]$. Moreover, since $H' \succ_{MPS} H$, $d' \geq d$

[55] $H' \succ_{MPS} H$ means $H' \succeq_{MPS} H$ but $H \not\succeq_{MPS} H'$.

must hold. From here on we distinguish between two cases.

*Case 1*: $d' = d$. Let $\pi, \pi' \in \Pi$ be any pair of information policies such that $H_\pi = H$ and $H_{\pi'} = H'$. Then $d' = d$ implies that both $\pi$ and $\pi'$ must be fully revealing on $[-1, d]$. Moreover, $\pi'$ must reveal strictly more information on $[r, 1]$ than $\pi$ because $H' \succ_{MPS} H$. This must lead to a strict lower expected utility because $U(\cdot)$ is strictly concave on $[r, 1]$.

*Case 2*: $d' > d$. In this case all $\theta \in (d, d')$ satisfy $\theta \in \text{supp}(H')$ and $p(\theta) > U(\theta) + \omega(\theta)$. Thus

$$\int_{-1}^{1} \big(U(\theta) + \omega(\theta)\big) dH'(\theta) < \int_{-1}^{1} p(\theta) dH'(\theta) = \int_{-1}^{1} p(\theta) dF(\theta) = \int_{-1}^{1} \big(U(\theta) + \omega(\theta)\big) dH(\theta)$$

The last two steps hold because $p(\cdot)$ is convex, $H' \succ_{MPS} H$ and (B.23). Therefore,

$$\int_{-1}^{1} U(\theta) dH'(\theta) - \int_{-1}^{1} U(\theta) dH(\theta) < \int_{-1}^{1} \omega(\theta) dH(\theta) - \int_{-1}^{1} \omega(\theta) dH'(\theta) \leq 0$$

The last inequality follows from convexity of $\omega(\cdot)$. This completes the proof. □

Now we are ready to prove statement (3) of Observation 2. Let

$$\mathscr{P}_{BR} := \{\mathscr{P}(-1, d) : \; d \in [b, r]\} \; , \tag{B.24}$$

where $b \in [-1, 1]$ and satisfies (B.7). We show that any possible pure strategy profile $\pi_{-m}$ of designers other than $m$, there exists some $\pi_m \in \mathscr{P}_{BR}$ such that $\pi_m$ is designer $m$'s best response to $\pi_{-m}$. Because $\mathscr{P}_{BR}$ is a subset of censorship policies, this result also implies that for any $\pi_{-m}$ there exists a censorship policy that is designer $m$'s best-response to $\pi_{-m}$.

To show this, consider any pure strategy profile $\pi_{-m}$ and let $H_{\pi_{-m}}$ be the distribution of posterior means induced by $\langle \pi_{-m} \rangle$. Suppose $\pi_m$ is a best response to $\pi_{-m}$ and let $H$ denote the distribution of posterior means induced by $\langle \pi_m, \pi_{-m} \rangle$. Feasibility constraint implies that $F \succeq_{MPS} H \succeq_{MPS} H_{\pi_{-m}}$ must hold. Moreover, observe that $H \in \mathscr{H}_m$ must also hold for $\pi_m$ to be a best response to $\pi_{-m}$. This is because otherwise $H_{\pi_{-m}}$ is not unimprovable for designer $m$ and thus he can benefit from unilaterally revealing some more information. This implies

$$H \in \Psi_m := \{H \in \Delta([-1, 1]) : \; H \in \mathscr{H}_m \text{ and } F \succeq_{MPS} H \succeq_{MPS} H_{-m}\} \tag{B.25}$$

whenever $\pi_m$ is a best response to $\pi_{-m}$. Notice that $\Psi_m$ is non-empty because clearly $F$ is both feasible and unimprovable. Finally, by Lemma B.2, for any $H, H' \in \mathscr{H}_m$ designer $m$ strictly prefers $H$ if $H' \succ_{MPS} H$. Hence, there must be no $H' \in \Psi_m$ such that $H \succ_{MPS} H'$.

In what follows we show that $\Psi_m$ in fact has a minimal element $H^*$ such that $H \succ_{MPS} H^*$ for all $H \in \Psi_m$ and $H \neq H^*$. Moreover, $H^*$ can be induced by some $\pi_m \in \mathscr{P}_{BR}$ when combined with $\pi_{-m}$. This therefore implies statement (3) of Observation 2.

We distinguish two cases.

*Case 1*: $H_{\pi_{-m}} \in \mathscr{H}_m$. In this case $H_{\pi_{-m}}$ is umpimprovable for designer $m$ and $H^* = H_{\pi_{-m}}$ is already the minimal element of $\Psi_m$. Moreover, by Lemma B.1, $H_{\pi_{-m}} \succeq_{MPS} H_{\mathscr{P}(-1,b)}$ must hold. The monopolistic optimal censorship policy $\pi_m = \mathscr{P}(-1,b) \in \mathscr{P}_{BR}$ is then already a best response because $H_\pi = H_{\pi_{-m}}$ holds.

*Case 2*: $H_{\pi_{-m}} \notin \mathscr{H}_m$. In this case $H \succ_{MPS} H_{\pi_{-m}}$ must hold for all $H \in \Psi_m$. We represent $\pi_{-m}$ by a tuple $(S, \{\gamma_s\}_{s\in S}, \delta)$, where $S$ is the message space, $\gamma_s$ equals the posterior distribution induced by each $s \in S$, and $\delta \in \Delta(S)$ denotes the induced ex-ante distribution over messages that satisfies the Bayes plausibility constraint: $\int_{s\in S} \gamma_s d\delta(s) = F$ (Kamenica and Gentzkow, 2011). Because $H \in \Psi_m \subseteq \mathscr{H}_m$, by Lemma B.1, any such $H$ must satisfy the following property: there exists $d \geq b$ such that $H(\theta) = \min\{F(\theta), F(d)\}$ for $\theta \in [-1, \beta(d))$ and $\text{supp}(H) \subseteq [-1,d] \cup [\beta(d), 1]$. For each $d \in [b, r]$, let

$$\widetilde{S}_d := \left\{s \in S : \ \mathbb{E}_{\gamma_s}[\theta|\theta > d] < \beta(d)\right\}$$

and

$$\tilde{d} := \inf\left\{d \in [b,r] : \ \Pr_\delta[\widetilde{S}_d] = 0\right\}$$

where $\Pr_\delta[E]$ denotes the probability measure of any $\delta$-measurable $E \subseteq S$ under $\delta$. Consider censorship policy $\pi_m = \mathscr{P}(-1, \tilde{d}) \in \mathscr{P}_{BR}$, which belongs to $\mathscr{P}_{BR}$. Let $\pi = \langle \pi_m, \pi_{-m} \rangle$ denote the joint information policy induced by $\pi_m$ and $\pi_{-m}$. We shall establish that $H_\pi \in \Psi_m$ and it is the minimal element in $\Psi_m$. To show $H_\pi \in \Psi_m$, first observe that $F \succeq_{MPS} H_\pi \succeq_{MPS} H_{\pi_{-m}}$ holds because $\pi$ is clearly feasible. Moreover, note that the choice of $\tilde{d}$ ensures that $\text{supp}(H_\pi) \subseteq [-1, \tilde{d}] \cup [\beta(\tilde{d}), 1]$. Observe that $\pi$ fully reveals states $k \in [-1, \tilde{d}]$ and it puts no mass on any $\text{supp}(H_\pi) \cap (\tilde{d}, \beta(\tilde{d})) = \emptyset$, it holds that $H_\pi(\theta) = \min\{F(\theta), F(\tilde{d})\}$ for $\theta \in [-1, \beta(\tilde{d}))$. Therefore, $H_\pi \in \mathscr{H}_m$ and thus $H_\pi \in \Psi_m$.

Next we show that $H_\pi$ is the minimal element of $\Psi_m$. Suppose instead there exists some $\pi'_m \in \Psi_m$, $\pi'_m \neq \pi_m$ and $\pi' = \langle \pi'_m, \pi_{-m} \rangle$ such that either (i) $H_\pi \succ_{MPS} H_{\pi'}$ or (ii) $H_{\pi'}$ is not comparable with $H_\pi$ under partial order $\succeq_{MPS}$. Let

$$\hat{d} := \sup\{d \in [-1,1] : H_{\pi'}(\theta) = F(\theta) \text{ for all } \theta \in [-1,d]\}$$

Under either (i) or (ii) $\hat{d} < \tilde{d}$ must hold. Then, by construction of $\tilde{d}$, $\widetilde{S}_{\hat{d}}$ must have a strictly positive probability measure under $\delta$. In this case, $\text{supp}(H_{\pi'}) \cap (\hat{d}, \beta(\hat{d}))$ must be non-empty. Therefore, $H_{\pi'} \neq \mathscr{H}_m$ and thus $H_{\pi'} \notin \Psi_m$, leading to a contradiction. This implies that $\pi_m = \mathscr{P}(-1, \tilde{d}) \in \mathscr{P}_{BR}$ is indeed a best response to $\pi_{-m}$.

## B.4 Proof of Observation 4

Suppose $H^*$ is an equilibrium outcome. Then $H^*$ must be unimprovable for all designers. For each designer $m \in M$, Remark B.4 implies that $H^*$ must be a solution to

$$\max_{H \in \Delta([-1,1])} \int_{-1}^{1} \widehat{U}_m(\theta) dH(\theta), \quad \text{s.t. } F \succeq_{MPS} H \tag{B.26}$$

where $\widehat{U}_m(\cdot) = U_m(\cdot) + \omega_m(\cdot)$ for some convex function $\omega_m(\cdot)$. Since strictly convex finite open property (SCFOP) holds on $[-1,1]$ for utility function profile $\{U_m(\cdot)\}_{m \in M}$, there exists a finite collection of open intervals $\{I_j\}_{j=1}^{J}$ that covers $[-1,1]$ and on each $I_j$ there is at least one $m \in M$ such that $U_m(\cdot)$ is strictly convex. Because $\widehat{U}_m(\cdot)$ inherits convexity of $U_m(\cdot)$ for all $m \in M$, the profile $\left\{\widehat{U}_m(\cdot)\right\}_{m \in M}$ inherits SCFOP on $[-1,1]$ with the same collection of open intervals $\{I_j\}_{j=1}^{J}$. For each $I_j$ in this collection, let $m_j$ denote the identify of the designer for whom $\widehat{U}_{m_j}(\cdot)$ is strictly convex on $I_j$. Since $H^*$ is a solution to (B.26) for $m = m_j$, $H^*$ must satisfy statement (4) of Remark B.3 on $I_j$. Moreover, because $[-1,1] \subseteq \left\{I_j\right\}_{j \in J}$, $[-1,1]$ can be partitioned into intervals in which either $H^*$ coincides with $F$ or $H^*$ is piece-wise constant with exactly one jump (and $F$ must be a mean-preserving spread of $H$ when restricted on this interval). Therefore, $H^*$ must be induced by a *monotone partitional information policy* which consists of disjoint pooling and revealing intervals. We conclude the proof by showing that pooling intervals cannot exist. To see this, suppose there is a non-trivial maximal pooling interval let $[l,r] \subset (-1,1)$ and let $\mu = \mathbb{E}_F[k | k \in [l,r]]$. For pooling on $[l,r]$ to be an equilibrium outcome, it must be that no designer can profitably deviate by unilaterally providing more information. By Proposition 3 of Kamenica and Gentzkow (2011), this is true if and only if $\widetilde{U}_m(\mu) = U_m(\mu)$ for all $m \in M$, where $\widetilde{U}_m(\cdot)$ is the concave closure of $U_m(\cdot)$ on $[l,r]$.[56] This can be possible only if all $U_m(\cdot)$ are locally concave at $\mu$, contradicting the strictly finite open cover property.

[56] Formally, $\widetilde{U}_m(\cdot)$ is the smallest concave function restricted on $[l,r]$ that is everywhere above $U_m(\cdot)$.

# Appendix C  Omitted Proofs in Section 5

In this appendix we prove Lemmas 4 and 5, and statement (2) of Theorem 4. Recall from (7) that a designer's utility function is given by

$$W_n^m(\theta) := \int_{\underline{v}}^{\theta} \left(\theta - \phi_n^m(x)\right) \hat{g}_n(x;q)dx = \theta \hat{G}_n(\theta;q) - \int_{\underline{v}}^{\theta} \phi_n^m(x)\hat{g}_n(x;q)dx \tag{C.1}$$

The first and second order derivatives of $W_n^m(\theta)$ are given by

$$W_n^{m\prime}(\theta) = \hat{G}_n(\theta;q) + (\theta - \phi_n^m(\theta))\,\hat{g}_n(\theta;q) \tag{C.2}$$

$$\begin{aligned} W_n^{m\prime\prime}(\theta) &= \hat{g}_n(\theta;q)\left(2 - \phi_n^{m\prime}(\theta)\right) + \hat{g}_n'(\theta;q)\left(\theta - \phi_n^{m\prime}(\theta)\right) \\ &= \hat{g}_n(\theta;q)\left\{2 - \phi_n^{m\prime}(\theta) + (\theta - \phi_n^m(\theta))\,\frac{\hat{g}_n'(\theta;q)}{\hat{g}_n(\theta;q)}\right\} \\ &= \hat{g}_n(\theta;q)\left\{2 - \phi_n^{m\prime}(\theta) + \left(\theta - \phi_n^m(\theta)\right)\left(n\frac{g(\theta)}{G(\theta)}\frac{q - G(\theta)}{1 - G(\theta)} + \frac{g'(\theta)}{g(\theta)}\right)\right\} \end{aligned} \tag{C.3}$$

The last step of (C.3) follows from (A.9).

## C.1  Proof of Lemma 4

By (C.1), for any $\theta \neq z_n^m$ we have

$$\begin{aligned} W_n^m(\theta) - W_n^m(z_n^m) &= \int_{\underline{v}}^{\theta} (\theta - \phi_n^m(x))\,\hat{g}_n(x;q)dx - \int_{\underline{v}}^{z_n^m} (z_n^m - \phi_n^m(x))\,\hat{g}_n(x;q)dx \\ &= \int_{z_n^m}^{\theta} (\theta - \phi_n^m(x))\,\hat{g}_n(x;q)dx + (\theta - z_n^m)\int_{\underline{v}}^{z_n^m} \hat{g}_n(x;q)dx \end{aligned}$$

Therefore,

$$\lambda_n^m(\theta; z_n^m) := \frac{W_n^m(\theta) - W_n^m(z_n)}{\theta - z_n^m} = \int_{z_n^m}^{\theta} \frac{\theta - \phi_n^m(x)}{\theta - z_n^m}\hat{g}_n(x;q)dx + \int_{\underline{v}}^{z_n^m} \hat{g}_n(x;q)dx$$

Taking derivative with respect to $\theta$ yields

$$\lambda_n^{m\prime}(\theta; z_n^m) = \frac{\theta - \phi_n^m(\theta)}{\theta - z_n^m}\hat{g}_n(\theta;q) + \int_{z_n^m}^{\theta} \frac{\phi_n^m(x) - z_n^m}{(\theta - z_n^m)^2}\hat{g}_n(x;q)dx$$

Recall from the premise of this lemma that $\phi_n^m(\cdot)$ crosses zero only once and from above at $z_n^m$. For any $\theta > z_n^m$, $x > \phi_n^m(x) \geq z_n^m$ holds for all $x \in (z_n^m, \theta]$. Therefore, the first term of $\lambda_n^{m\prime}(\theta; z_n^m)$ must be strictly positive and the second term is non-negative. This implies $\lambda_n^{m\prime}(\theta; z_n^m) > 0$ for $\theta > z_n^m$. For any $\theta < z_n^m$, $x < \phi_n^m(x) \leq z_n^m$ holds for all $x \in [\theta, z_n^m)$. So the first term of $\lambda_n^{m\prime}(\theta; z_n^m)$ is strictly positive, and the second term

$$\int_{z_n^m}^{\theta} \frac{\phi_n^m(x) - z_n^m}{(\theta - z_n^m)^2} \hat{g}_n(x;q)dx = \int_{\theta}^{z_n^m} \frac{z_n^m - \phi_n^m(x)}{(\theta - z_n^m)^2} \hat{g}_n(x;q)dx$$

is non-negative. This implies $\lambda_n^{m\prime}(\theta; z_n^m) > 0$ for $\theta < z_n^m$ as well. Taken together, $\lambda_n^{m\prime}(\theta; z_n^m) > 0$ holds for all $\theta \neq z_n^m$. Finally, since $W_n^m(\theta)$ is differentiable, we have that $\lambda_n^m(\theta; z_n^m)$ is continuous at $\theta = z_n^m$ and $\lim_{\theta \to z_n^m} \lambda_n^m(\theta; z_n^m) = W_n^{m\prime}(z_n^m)$. These together establish that $\lambda_n^m(\theta; z_n^m)$ is strictly increasing in $\theta$, which implies increasing slope property at point $z_n^m$.

## C.2 Proof of Lemma 5

By (C.3), $W_n^{m\prime\prime}(\theta) < 0$ if and only if

$$(\theta - \phi_n^m(\theta))(G(\theta) - q) < \frac{G(\theta)(1 - G(\theta))}{ng(\theta)} \left(2 - \phi_n^{m\prime}(\theta) + (\theta - \phi_n^m(\theta)) \frac{g'(\theta)}{g(\theta)}\right) \quad \text{(C.4)}$$

Because $\phi_n^m(\cdot)$, $\phi_n^{m\prime}(\cdot)$ and $\phi_n^{m\prime\prime}(\cdot)$ are uniformly Lipschitz continuous (cf. Proposition A.3) and $g(\cdot)$ is positive and twice continuously differentiable on $[\underline{v}, \overline{v}]$, both the value and the derivative of the right-hand side of (C.4) converge to zero uniformly for all $\theta \in [\underline{v}, \overline{v}]$.[57] Let

$$\zeta_m(\theta) := \lim_{n \to \infty} (\theta - \phi_n^m(\theta))(G(\theta) - q) = (\theta - \phi^m(\theta))(G(\theta) - q)$$

denote the limit of left-hand side (LHS) of (C.4) as $n \to \infty$. On the one hand, $G(\theta) - q$ is increasing in $\theta$ and admits a unique root $\theta = v_q^* := G^{-1}(q)$ at which its derivative equals $g(v_q^*) > 0$. On the other hand, under the single-crossing property, $\theta - \phi^m(\theta) = 0$ admits at most one root on $[\underline{v}, \overline{v}]$. Let

$$z_m^* := \begin{cases} \underline{v} & \text{if } x > \phi^m(x) \text{ for all } x \in [\underline{v}, \overline{v}] \\ x & \text{if } x = \phi^m(x) \text{ for some } x \in [\underline{v}, \overline{v}] \\ \overline{v} & \text{if } x < \phi^m(x) \text{ for all } x \in [\underline{v}, \overline{v}] \end{cases}. \quad \text{(C.5)}$$

[57] This is because both $1 - \phi_n^{m\prime}(\theta)$ and $\frac{g'(\theta)}{G'(\theta)}$ are uniformly bounded on $[\underline{v}, \overline{v}]$ under our assumption for $G$.

Moreover, single-crossing property implies $1-\phi^{m\prime}(z_m^*)>0$ whenever $\phi^m(z_m^*)=z_m^*$. Let

$$\widetilde{\ell}_m^* := \min\left\{z_m^*, v_q^*\right\} \quad \text{and} \quad \widetilde{r}_m^* := \max\left\{z_m^*, v_q^*\right\} \tag{C.6}$$

The results above then imply that $\zeta_m(\theta)<0$ for $\theta \in \left(\widetilde{\ell}_m^*, \widetilde{r}_m^*\right)$ and $\zeta_m(\theta)>0$ for $\theta \in [-1,1]/\left[\widetilde{\ell}_m^*, \widetilde{r}_m^*\right]$. Notice that the value and derivative of the LHS of (C.4) converge uniformly to $\zeta_m(\cdot)$ and $\zeta_m'(\cdot)$, respectively. It follows that there exists some $N\geq 0$ such that for all $n \geq 0$ there exists $-1 \leq \widetilde{\ell}_n^m \leq \widetilde{r}_n^m \leq 1$ such that $W_n^{m\prime\prime}(\theta)>0$ for $\theta \in (\widetilde{\ell}_n^m, \widetilde{r}_n^m)$ and $W_n^{m\prime\prime}(\theta)<0$ for $\theta \in [-1,1]/[\widetilde{\ell}_n^m, \widetilde{r}_n^m]$. In the limit we have $\lim_{n\to\infty} \widetilde{\ell}_n^m = \widetilde{\ell}_m^*$ and $\lim_{n\to\infty} \widetilde{r}_n^m = \widetilde{r}_m^*$.

Next we show $\widetilde{\ell}_n^m < z_n^m < \widetilde{r}_n^m$ for all $n>N$. To do so, let

$$\widetilde{z}_n^m := \begin{cases} \underline{v} & \text{if } x > \phi_n^m(x) \text{ for all } x \in [\underline{v}, \bar{v}] \\ x & \text{if } x = \phi_n^m(x) \text{ for some } x \in [\underline{v}, \bar{v}] \\ \bar{v} & \text{if } x < \phi_n^m(x) \text{ for all } x \in [\underline{v}, \bar{v}] \end{cases}.$$

$z_n^m$ defined in (4) is then equal to $\widetilde{z}_n^m$ when $\widetilde{z}_n^m \in [-1,1]$, and equals to $-1$ (resp. 1) if $\widetilde{z}_n^m < -1$ (resp. $\widetilde{z}_n^m < 1$). In what follows we show $\widetilde{\ell}_n^m < \widetilde{z}_n^m < \widetilde{r}_n^m$ for all $\widetilde{z}_n^m \in (\underline{v}, \bar{v})$, which implies $\widetilde{\ell}_n^m < z_n^m < \widetilde{r}_n^m$ for all $z_n^m \in [-1,1]$. Recall that under single-crossing property $\widetilde{z}_n^m \in (\underline{v}, \bar{v})$ implies $\widetilde{z}_n^m = \phi_n^m(\widetilde{z}_n^m)$ and $1-\phi_n^{m\prime}(\widetilde{z}_n^m)>0$. By (C.3) we have

$$\begin{aligned} W_n^{m\prime\prime}(\widetilde{z}_n^m) &= \hat{g}_n(\widetilde{z}_n^m;q)\left(2-\phi_n^{m\prime}(\widetilde{z}_n^m)\right) + \hat{g}_n'(\widetilde{z}_n^m;q)\left(\widetilde{z}_n^m - \phi_n^m(\widetilde{z}_n^m)\right) \\ &= \hat{g}_n(\widetilde{z}_n^m;q)\left(2-\phi_n^{m\prime}(\widetilde{z}_n^m)\right) > \hat{g}_n(\widetilde{z}_n^m;q) > 0 \end{aligned}$$

Therefore, $\widetilde{z}_n^m \in (\widetilde{\ell}_n^m, \widetilde{r}_n^m)$ must hold because $W_n^{m\prime\prime}(\theta)>0$ only if $\theta \in (\widetilde{\ell}_n^m, \widetilde{r}_n^m)$.

These together imply for all $n>N$ that $W_n^m(\cdot)$ is strictly S-shaped on $[z_n^m, 1]$ with inflection point $r_n^m := \min\{\widetilde{r}_n^m, 1\}$ while strictly inverse S-shaped on $[-1, z_n^m]$ with inflection point $\ell_n^m := \max\{\widetilde{\ell}_n^m, -1\}$. Moreover, in the limit we have

$$\lim_{n\to\infty} \ell_n^m = \max\{\widetilde{\ell}_n^*, -1\} \tag{C.7}$$

$$\lim_{n\to\infty} r_n^m = \min\{\widetilde{r}_n^m, 1\} \tag{C.8}$$

where $\delta(x)$ equals $x$ for $x\in[-1,1]$, equals 1 for $x>1$, and equals $-1$ for $x<-1$.

Finally, we show that $N=0$ if $g(\cdot)$ is log-concave and $\rho$ is equal or sufficient to zero.

By (C.3) we have

$$W_n^{m\prime\prime}(\theta) = \hat{g}_n(\theta;q)\left\{2-\phi_n^{m\prime}(\theta)+(\theta-\phi_n^m(\theta))\frac{\hat{g}_n'(\theta;q)}{\hat{g}_n(\theta;q)}\right\}$$
$$= \hat{g}_n(\theta;q)\,(\theta-\phi_n^m(\theta))\left\{\frac{2-\phi_n^{m\prime}(\theta)}{\theta-\phi_n^m(\theta)}+\frac{\hat{g}_n'(\theta;q)}{\hat{g}_n(\theta;q)}\right\} \tag{C.9}$$

Observe that $\theta-\phi_n^m(\theta)>0$ holds for all $\theta>z_n^m$ due to the single-crossing property. It therefore suffices to show that term in the last curly bracket (C.9) is strictly decreasing. When $g(\cdot)$ is log-concave, it follows from Proposition A.1 in Appendix A.1 that $\hat{g}_n(\cdot;q)$ must be strictly log-concave, and hence $\frac{\hat{g}_n'(\theta;q)}{\hat{g}_n(\theta;q)}$ is strictly decreasing, for all $n>0$. Next we show that $\frac{2-\phi_n^{m\prime}(\theta)}{\theta-\phi_n^m(\theta)}$ is also strictly decreasing in $\theta$ on $(z_n^m,1]$ for all $n>0$. Recall from the definition of $\phi_n^m(\theta)$ (cf. (A.11)) that

$$\phi_n^m(\theta) = \rho_m\sum_{j=1}^{n+1} w_{m,j}\varphi_j(x;q,n)+(1-\rho_m)\chi_m$$
$$\phi_n^{m\prime}(\theta) = \rho_m\sum_{j=1}^{n+1} w_{m,j}\varphi_j'(x;q,n)$$
$$\phi_n^{m\prime\prime}(\theta) = \rho_m\sum_{j=1}^{n+1} w_{m,j}\varphi_j''(x;q,n)$$

As $\rho_m\to 0$, we have $\phi_n^m(\cdot)\to\chi_m$, $\phi_n^{m\prime}(\cdot)\to 0$ and $\phi_n^{m\prime\prime}(\cdot)\to 0$ uniformly for all $\theta\in[\underline{v},\overline{v}]$. Let $\xi(\theta):=\frac{2-\phi_n^{m\prime}(\theta)}{\theta-\phi_n^m(\theta)}$ and simple algebra shows that $\xi'(\theta)<0$ if and only if

$$\phi_n^{m\prime\prime}(\theta)\,(\theta-\phi_n^m(\theta))+\left(2-\phi_n^{m\prime}(\theta)\right)\left(1-\phi_n^{m\prime}(\theta)\right)\geq 0$$

As $\rho_m\to 0$, the left-hand side of the above inequality converges to 2 for all $\theta\in[-1,1]$. This implies that $\xi'(\theta)<0$ for all $\theta\in[-1,1]$ and $n>0$ if $\rho_m$ is sufficiently close to zero. These together show that $W_n^{m\prime\prime}(\theta)$ crosses zero at most once and if so from above on $(z_n^m,1]$ – which implies the strict S-shape property on $[z_n^m,1]$ – for all $n>0$. The strict inverse S-shape property on $[-1,z_n^m]$ can be proved using analogous arguments.

## C.3 Proof for statement (2) of Theorem 4

Because single-crossing property holds for all designers and $n>\max_{m\in M}N_m$, it follows from Lemma 5 and Theorem 1 that for each $m\in M$

(i). $W_n^m(\cdot)$ is strictly convex on $[\ell_n^m, r_n^m]$ and strictly concave elsewhere; and

(ii). censorship policy $\mathscr{P}(a_n^m, b_n^m)$ with $\ell_n^m \le a_n^m \le z_n^m \le b_n^m \le r_n^m$ is optimal under monopolistic persuasion.

Throughout the proof we take the above properties and notations as given. Let $\mathscr{P}$ be the set of all censorship policies. Then for each designer $m \in M$ his initial set of pure strategies is $\mathscr{P}_m^{(0)} = \mathscr{P}$. Let $\mathscr{P}_m^{(1)}$ for each $m \in M$ denote the set of censorship policies that survives the first round of deletion of weakly dominated strategies, given restriction $\pi_{-m} \in \times \mathscr{P}^{|M|-1}$. The following two lemmas hold.

**Lemma C.1.** *If $|M| = 2$, then for each $m \in M$ we have*

$$\mathscr{P}_m^{(1)} = \{\mathscr{P}(c,d) : \ c \in [\ell_n^m, a_n^m] \text{ and } d = b_n^m, \text{ or } c = a_n^m \text{ and } d \in [b_n^m, r_n^m]\} \tag{C.10}$$

*If $|M| \ge 3$, then for each $m \in M$ we have*

$$\mathscr{P}_m^{(1)} = \{\mathscr{P}(c,d) : \ [a_n^m, b_n^m] \subseteq [c,d] \subseteq [\ell_n^m, r_n^m]\} \tag{C.11}$$

**Lemma C.2.** *Suppose $|M| \ge 2$ and each designer $m \in M$ is restricted to use the subset of censorship policies characterized by $\mathscr{P}_m^{(1)}$. Then any joint information policy $\pi$ induced in any pure strategy equilibrium in weakly undominated strategies must be outcome-equivalent to $\mathscr{P}\left(a_n^{\min}, b_n^{\max}\right)$, where $a_n^{\min} = \min_{m \in M}\{a_n^m\}$ and $b_n^{\max} = \max_{m \in M}\{b_n^m\}$.*

Lemmas C.1 and C.2 together imply statement (2) of Theorem 4.

In what follows we prove Lemmas C.1 and C.2. To do so we establish an auxiliary result. Let $[\underline{\kappa}, \overline{\kappa}] \subseteq [-1,1]$ be a maximal pooling interval.[58] Let $\widetilde{F}$ denote the cdf of the distribution of $k$ conditional on $k \in [\underline{\kappa}, \overline{\kappa}]$.[59] For any censorship policy $\mathscr{P}(c,d)$ with $[c,d] \subseteq [\underline{\kappa}, \overline{\kappa}]$, the designer's expected payoff under this policy conditional on event $k \in [\underline{\kappa}, \overline{\kappa}]$ is given by

$$\begin{aligned}\mathbb{E}_{\mathscr{P}(c,d)}\left[W_n^m(\cdot)|\underline{\kappa}, \overline{\kappa}\right] =& \widetilde{F}(c) W_n^m\left(\mu\left(\underline{\kappa}, c\right)\right) + \int_c^d W_n^m(k) d\widetilde{F}(k) \\ &+ \left(1 - \widetilde{F}(d)\right) W_n^m\left(\mu\left(d; \overline{\kappa}\right)\right)\end{aligned} \tag{C.12}$$

where $\mu(x,y) := \mathbb{E}_F\left[k | k \in [x,y]\right]$ for all $-1 \le x \le y \le 1$.

[58] This means voters precisely learn whether the realized state $k$ lies in $[\underline{\kappa}, \overline{\kappa}]$, but are not able to further distinguish state realizations within this interval.

[59] That is, $\widetilde{F}(x)$ equals $\frac{F(x)-F(\underline{\kappa})}{F(\overline{\kappa})-F(\underline{\kappa})}$ for $x \in [\underline{\kappa}, \overline{\kappa}]$, and equals 0 (resp. 1) for $x \le \underline{\kappa}$ (resp. $x \ge \overline{\kappa}$).

**Claim C.1.** *Suppose* $[\underline{\kappa},\overline{\kappa}]\cap[\ell_n^m,r_n^m]\neq\emptyset$. *Then*

1. $\frac{\partial\mathbb{E}_{\mathscr{P}(c,d)}[W_n^m(\cdot)|\underline{\kappa},\overline{\kappa}]}{\partial d}>(<0)$ *for* $d<(>)d^*$, *where*

$$d^*=\sup\left\{d\in[-1,1]:\ \frac{W_n^m(\mu(d,\overline{\kappa}))-W_n^m(d)}{\mu(d,\overline{\kappa})-d}-W_n^{m\prime}(\mu(d,\overline{\kappa}))\geq 0\right\}. \tag{C.13}$$

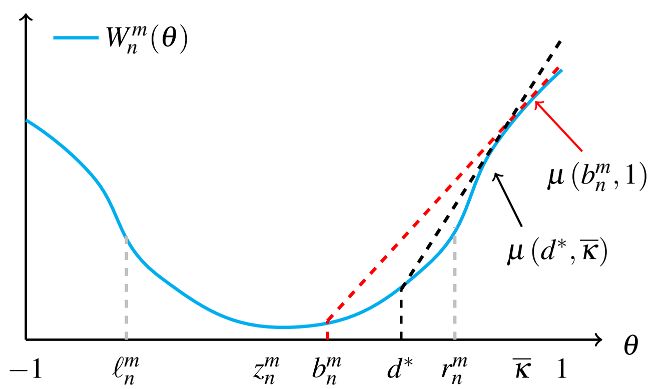
*Moreover,* $d^*=\overline{\kappa}$ *for* $\overline{\kappa}\in[\ell_n^m,r_n^m]$, $d^*$ *is decreasing in* $\overline{\kappa}$ *for* $\overline{\kappa}>r_n^m$ *and* $d^*=b_n^m$ *if* $\overline{\kappa}=1$.

2. $\frac{\partial\mathbb{E}_{\mathscr{P}(c,d)}[W_n^m(\cdot)|\underline{\kappa},\overline{\kappa}]}{\partial c}>(<)0$ *for* $c>(<)c^*$, *where*

$$c^*=\inf\left\{c\in[-1,1]:\ W_n^{m\prime}(\mu(\underline{\kappa},c))-\frac{W_n^m(c)-W_n^m(\mu(\underline{\kappa},c))}{c-\mu(c;\underline{\kappa})}\leq 0\right\}. \tag{C.14}$$

*Moreover,* $c^*=\underline{\kappa}$ *for* $\underline{\kappa}\in[\ell_n^m,r_n^m]$, $c^*$ *is decreasing in* $\underline{\kappa}$ *for* $\underline{\kappa}<\ell_n^m$ *and* $c^*=a_n^m$ *if* $\underline{\kappa}=-1$.

Figure C.1: Graphical Illustration for the Proof of Claim C.1




*Proof of Claim C.1.* Taking derivatives of (C.12) with respect to $c$ and $d$ yield[60]

$$\begin{aligned}\frac{\partial\mathbb{E}_{\mathscr{P}(c,d)}[W_n^m(\cdot)|\underline{\kappa},\overline{\kappa}]}{\partial d}&=\widetilde{f}(d)\left(W_n^m(d)-W_n^m(\mu(d,\overline{\kappa}))\right)+\left(1-\widetilde{F}(d)\right)W_n^{m\prime}(\mu(d,\overline{\kappa}))\,\mu_x(d,\overline{\kappa})\\&=\widetilde{f}(d)\cdot(\mu(d,\overline{\kappa})-d)\cdot\left[\frac{W_n^m(\mu(d,\overline{\kappa}))-W_n^m(d)}{\mu(d,\overline{\kappa})-d}-W_n^{m\prime}(\mu(d,\overline{\kappa}))\right]\\\frac{\partial\mathbb{E}_{\mathscr{P}(c,d)}[W_n^m(\cdot)|\underline{\kappa},\overline{\kappa}]}{\partial c}&=\widetilde{f}(c)\left(W_n^m(\mu(\underline{\kappa},c))-W_n^m(c)\right)+\widetilde{F}(c)\cdot W_n^{m\prime}(\mu(\underline{\kappa},c))\,\mu_y(\underline{\kappa},c)\\&=\widetilde{f}(c)\cdot(c-\mu(\underline{\kappa},c))\cdot\left[W_n^{m\prime}(\mu(\underline{\kappa},c))-\frac{W_n^m(c)-W_n^m(\mu(\underline{\kappa},c))}{c-\mu(c;\underline{\kappa})}\right]\end{aligned}$$

[60] In the derivation we exploit the fact that $\mu_x(x,y):=\frac{\partial\mu}{\partial x}=\frac{\widetilde{f}(x)(\mu(x,y)-x)}{\widetilde{F}(y)-\widetilde{F}(x)}$ and $\mu_y(x,y):=\frac{\partial\mu}{\partial y}=\frac{\widetilde{f}(y)(y-\mu(x,y))}{\widetilde{F}(y)-\widetilde{F}(x)}$.

Because both $\tilde{f}(d)$ and $\mu(d,\overline{\kappa}) - d$ are positive, $\frac{\partial \mathbb{E}_{\mathscr{P}(c,d)}[W_n^m(\cdot)|\underline{\kappa},\overline{\kappa}]}{\partial d}$ is sign-equivalent to

$$\frac{W_n^m(\mu(d,\overline{\kappa})) - W_n^m(d)}{\mu(d,\overline{\kappa}) - d} - W_n^{m\prime}(\mu(d,\overline{\kappa})) \tag{C.15}$$

Recall that $W_n^m(\cdot)$ is strictly convex on $[\ell_n^m, r_n^m]$ and strictly concave on $[r_n^m, 1]$. If $\overline{\kappa} \in [z_n^m, r_n^m]$, then (C.15) is strictly positive for all $d \in [\ell_n^m, \overline{\kappa}]$ because of convexity. Therefore, by (C.13) we have $d^* = \overline{\kappa}$ and $\mathbb{E}_{\mathscr{P}(c,d)}[W_n^m(\cdot)|\underline{\kappa},\overline{\kappa}]$ is increasing in $d$ on $[\ell_n^m, \overline{\kappa}]$. If $\overline{\kappa} > r_n^m$, then (C.15) crosses 0 at most once and if so from above at some point $d^*$ (cf. black dashed line Figure C.1). As is clear graphically, this $d^*$ is decreasing in $\overline{\kappa}$ on $(r_n^m, 1]$ and equals $b_n^m$ if $\overline{\kappa} = 1$ (cf. red dashed line).[61] $\mathbb{E}_{\mathscr{P}(c,d)}[W_n^m(\cdot)|\underline{\kappa},\overline{\kappa}]$ is then strictly increasing in $d$ on $[\ell_n^m, d^*]$ while strictly decreasing on $[d^*, \overline{\kappa}]$. This proves part (1) of this claim. The proof for part (2) is analogous and thus omitted. □

Claim C.1 shows that $\mathbb{E}_{\mathscr{P}(c,d)}[W_n^m(\cdot)|\underline{\kappa},\overline{\kappa}]$ is single-peaked in both thresholds $c$ and $d$.

### C.3.1 Proof of Lemma C.1

Throughout the proof we let $\pi_{-m} \in \times \mathscr{P}^{|M|-1}$ denote any profile of censorship policies by designers $m$. We first show that, for each $m \in M$, a censorship policy $\mathscr{P}(c,d)$ is not weakly dominated for designer $m$ must satisfy $[a_n^m, b_n^m] \subseteq [c,d] \subseteq [\ell_n^m, r_n^m]$. That is,

$$\mathscr{P}_m^{(1)} \subseteq \{\mathscr{P}(c,d) : \; [a_n^m, b_n^m] \subseteq [c,d] \subseteq [\ell_n^m, r_n^m]\} \tag{C.16}$$

must hold. To prove this we distinguish between two cases.

**Case 1**: $[c,d] \cap [\ell_n^m, r_n^m] \neq \emptyset$. For this case, we start by establishing that any $\mathscr{P}(c,d)$ with $c \leq r_n^m < d$ is weakly dominated by $\mathscr{P}(c, r_n^m)$. This is because $W_n^m(\cdot)$ is strictly concave on $[r_n^m, 1]$ so that any mean-preserving spread of posterior means on $[r_n^m, 1]$ strictly decreases the designer's payoff. By replacing $d$ with $r_n^m$, designer $m$ reveals less information and avoid inducing any mean-preserving spread of posterior means on $[r_n^m, 1]$. Therefore, her payoff cannot be lower after this replacement and hence $\mathscr{P}(c,d)$ is weakly dominated by $\mathscr{P}(c, r_n^m)$ for all $d > r_n^m$. By analogous reasoning, it is also true that any $\mathscr{P}(c,d)$ with $c < \ell_n^m \leq d$ is weakly dominated by $\mathscr{P}(\ell_n^m, d)$. These together imply that any $\mathscr{P}(c,d)$ that is weakly undominated for designer $m$ must satisfy $[c,d] \subset [\ell_n^m, r_n^m]$ whenever $[c,d] \cap [\ell_n^m, r_n^m] \neq \emptyset$.

[61] Notice that when $\overline{\kappa} = 1$ the definition of $d^*$ (cf. (C.13)) is equivalent to optimality condition for $b_n$; that is, (FOC: b) with $U(\cdot) = W_n^m(\cdot)$ and $\overline{\kappa} = 1$.

Next we show that any $\mathscr{P}(c,d)$ with $\ell_n^m \leq d < b_n^m$ is weakly dominated by $\mathscr{P}(c,b_n^m)$. If $\pi_{-m}$ reveals whether the state is above or below $e$ for some $e \in [b_n^m, r_n^m]$. Then replacing $d$ with $b_n^m$ cannot decrease designer $m$'s payoff because it only reveals states in $[d, b_n^m]$, where $W_n^m(\cdot)$ is strictly convex. Now suppose instead there exists a maximal pooling interval $[\underline{\kappa}, \overline{\kappa}]$ under $\pi_{-m}$ such that $\underline{\kappa} < b_n^m$ and $\overline{\kappa} > r_n^m$. Then, by Claim C.1, the designer's expected utility is strictly increasing in $d$ as long as $d < d^*$, and $d^* \geq b_n^m$ holds for all $\overline{\kappa} > r_n^m$. Therefore, replacing $d$ by $b_n^m$ must strictly increase designer $m$'s expected payoff. These imply $d \geq b_n^m$ must hold for any weakly undominated $\mathscr{P}(c,d)$. Analogously, $c \leq a_n^m$ must also hold. These together show that $[a_n^m, b_n^m] \subseteq [c,d]$ for any weakly undominated $\mathscr{P}(c,d)$ whenever $[c,d] \cap [\ell_n^m, r_n^m] \neq \emptyset$.

**Case 2**: $[c,d] \cap [\ell_n^m, r_n^m] = \emptyset$. Without loss of generality, we focus on $r_n^m < c \leq d$. We will show that any such $\mathscr{P}(c,d)$ is weakly dominated by $\mathscr{P}(a_n^m, \beta)$, where $\beta$ is given by (C.13) with $\overline{\kappa} = c$, by proving the following chain of inequalities for any $\pi_{-m} \in \times \mathscr{P}^{|M|-1}$:

$$\mathbb{E}_{H_{\pi_1}}[W_n^m(\cdot)] \leq \mathbb{E}_{H_{\pi_2}}[W_n^m(\cdot)] \leq \mathbb{E}_{H_{\pi_3}}[W_n^m(\cdot)] \leq \mathbb{E}_{H_{\pi_4}}[W_n^m(\cdot)] \tag{C.17}$$

where $\pi_1 = \langle \pi_{-m}, \mathscr{P}(c,d) \rangle$, $\pi_2 = \langle \pi_{-m}, \mathscr{P}(c) \rangle$, $\pi_3 = \langle \pi_{-m}, \mathscr{P}(c), \mathscr{P}(a_n^m, \beta) \rangle$ and $\pi_4 = \langle \pi_{-m}, \mathscr{P}(a_n^m, \beta) \rangle$.

*Step 1*: $\mathbb{E}_{H_{\pi_1}}[W_n^m(\cdot)] \leq \mathbb{E}_{H_{\pi_2}}[W_n^m(\cdot)]$. That is, any $\mathscr{P}(c,d)$ with $r_n^m < c < d$ is weakly dominated by $\mathscr{P}(c)$. This is because $c > r_n^m$ and $W_n^m(\cdot)$ is strictly concave on $[r_n^m, 1]$. Lowering the upper threshold from $d$ to $c$ thus reduces information disclosure in this concave region and hence cannot decrease the designer's payoff.

*Step 2*: $\mathbb{E}_{H_{\pi_2}}[W_n^m(\cdot)] \leq \mathbb{E}_{H_{\pi_3}}[W_n^m(\cdot)]$. That is, given that $\pi_{-m}$ and $\mathscr{P}(c)$ are already implemented, designer $m$'s payoff would be weakly higher if he can further reveal states in $[a_n^m, \beta]$. This is equivalent to show that $\mathscr{P}(a_n^m, \beta)$ weakly dominates $\underline{\pi}$ (i.e., providing no information) when the state space is restricted to $[-1, c]$ only.[62] Below we show that this is indeed true. On the one hand, recall that $W_n^m(\cdot)$ satisfies single-crossing property at point $z_n^m$ (cf. Lemma 4). It then follows from Claim B.1 in Appendix B.2 that $\mathscr{P}(z_n^m)$ must weakly dominates $\underline{\pi}$.[63] On the other hand, $\mathscr{P}(a_n^m, \beta)$ is in fact the unique solution to designer $m$'s monopolistic persuasion problem when the state space is restricted to $[-1, c]$. Together with the fact that $z_n^m \in [a^m, \beta] \subseteq [\ell_n^m, r_n^m] \neq \emptyset$, the arguments for Case 1 imply that $\mathscr{P}(a_n^m, \beta)$ weakly dominates $\mathscr{P}(z_n^m)$. $\mathscr{P}(a_n^m, \beta)$ thus weakly dominates $\underline{\pi}$ by transitivity.

[62] Because $\beta \leq r_n^m < c$, inducing $\mathscr{P}(a_n^m, \beta)$ has no effect on outcomes for any state $k \geq c$.

[63] This argument holds for $z_n^m \in (-1, 1]$. If instead $z_n^m = -1$ then $\mathscr{P}(z_n^m)$ is outcome equivalent to $\underline{\pi}$ so the weakly dominance trivially holds.

*Step 3*: $\mathbb{E}_{H_{\pi_3}}[W_n^m(\cdot)] \leq \mathbb{E}_{H_{\pi_4}}[W_n^m(\cdot)]$. That is, given that $\pi_{-m}$, $\mathscr{P}(a_n^m, e)$ and $\mathscr{P}(c)$ are all implemented, designer $m$ would be doing better by removing $\mathscr{P}(c)$. This is because our choice of $\beta$ ensures that for every possible maximal pooling interval $[x,y] \subset [\beta, 1]$ under $\pi_3$ that contains $c$ in its interior, the concave closure of $W_n^m(\cdot)$ restricted on $[x,y]$ surely coincides with $W_n^m(\cdot)$ at the posterior expected state $\mu(x,y)$ in this pooling interval.[64] Therefore, by Proposition 3 of Kamenica and Gentzkow (2011), the optimal response is no disclosure and designer $m$ weakly benefits from removing $\mathscr{P}(c)$. This, together with the results in steps 1 and 2, shows that $\mathscr{P}(z_n^m, \beta)$ weakly dominates $\mathscr{P}(c,d)$ whenever $r_n^m < c \leq d$.

All the above arguments together show that (C.16) must hold for any $\mathscr{P}(c,d)$ that is weakly undominated. Finally, we pin down $\mathscr{P}_m^{(1)}$ depending on the size of $M$. If $|M| \geq 3$, $\mathscr{P}_m^{(1)}$ in fact coincides with $\{\mathscr{P}(c,d) : [a_n^m, b_n^m] \subseteq [c,d] \subseteq [\ell_n^m, r_n^m]\}$ for any $m \in M$. To see why, take any $c \in [\ell_n^m, a_n^m]$ and $d \in [b_n^m, r_n^m]$. Consider any two designers $l, r$ other than $m$ and the following feasible joint information policy profile: $l$ implements a cutoff policy $\mathscr{P}(c)$, $r$ implements $\mathscr{P}(d)$, and all other designers (if any) remain silent (e.g., implementing $\mathscr{P}(-1)$). This strategy profile of designers other than $m$ essentially partitions the state space into (at most) three pooling intervals: $[-1,c]$, $[c,d]$, and $[d,1]$. In this case $\mathscr{P}(c,d)$ is the unique best response of designer $m$. This is on the one hand because $W_n^m(\cdot)$ is strictly convex on $[c,d]$ so it is optimal for designer $m$ to fully reveal states therein. On the other hand, the single-peakedness properties in Claim C.1 imply that it is optimal for the designer not to further expand his revelation interval beyond $[c,d]$.

If $|M| = 2$, we show that $\mathscr{P}_m^{(1)}$ is given by (C.10), which is in general a proper subset of $\{\mathscr{P}(c,d) : [a_n^m, b_n^m] \subseteq [c,d] \subseteq [\ell_n^m, r_n^m]\}$. This is because under $|M| = 2$ each designer $m$ only faces one opponent who can only use censorship policies. Let $\mathscr{P}(l,r)$ be the opponent's strategy. If $[l,r] \cap [a_n^m, b_n^m] \neq \emptyset$, then $\mathscr{P}(a_n^m, b_n^m)$ is already designer $m$'s best response. If instead $b_n^m < l \leq r$, then designer $m$'s best response is $\mathscr{P}(a_n^m, \beta)$, where $\beta$ is given by (C.13) with $\overline{\kappa} = l$. Notice that such $\beta$ can cover the entire interval $[b_n^m, r_n^m]$ as $l$ varies from $b_n^m$ to 1. Finally, if $l \leq r < a_n^m$. Then designer $m$'s best response is $\mathscr{P}(\alpha, b_n^m)$, where $\alpha$ is given by (C.14) with $\underline{\kappa} = l$. Again, $\alpha$ can cover the entire interval $[\ell_n^m, a_n^m]$ as $r$ varies from $a_n^m$ to $-1$. These together generate (C.10).

[64] To see why this is true, let $\widehat{W}_n^m(\cdot)$ be the concave closure of $W_n^m(\cdot)$ restricted on $[x,y]$. If $x \geq r_n^m$, the $\widehat{W}_n^m(\cdot) = W_n^m(\cdot)$ because $W_n^m(\cdot)$ is strictly concave on $[x,y]$. If instead $x < r_n^m$ and $y > \beta > r_n^m$, then there exists a point $p > r_n^m$ such that $\widehat{W}_n^m(\theta) = W_n^m(\theta)$ for $\theta \in [p,y]$. On $[x,y]$, $\widehat{W}_n^m(\theta)$ is an affine function tangent to $W_n^m(\cdot)$ at $p$. Our choice of $\beta$ ensures that $p \leq \mu(\beta, c)$. Moreover, since $\mu(x,y) > \mu(\beta,c)$ for all $x \geq \beta$ and $y > c$, it follows that $\widehat{W}_n^m(\mu(x,y)) = W_n^m(\mu(x,y))$ always holds.

#### C.3.2 Proof of Lemma C.2

Using the same argument the proof of statement (1) of Theorem 4 in the main text, we can show that $\mathscr{P}(a_n^{\min}, b_n^{\max})$ is still an equilibrium outcome in this modified game where designers can only use censorship policies. In what follows we establish that any equilibrium outcome in pure and weakly undominated strategies must be both no more and no less informative than $\mathscr{P}(a_n^{\min}, b_n^{\max})$. These together imply the uniqueness of $\mathscr{P}(a_n^{\min}, b_n^{\max})$.

We first show that for any $m \in M$ and any $\mathscr{P}(c,d)$ with $d > b_n^{\max}$ is weakly dominated by $\mathscr{P}(c, b_n^{\max})$, provided that any other designer $i \neq m$ chooses strategies from $\mathscr{P}_i^{(1)}$. Let $\pi_{-m}$ denote a strategy profile by other designers and under $\pi_{-m}$ let $\eta \in [b_n^{\max}, 1]$ be the threshold such that $k \in [b_n^{\max}, \eta]$ are revealed, while $k > \eta$ are pooled together. Replacing $\mathscr{P}(c,d)$ with $\mathscr{P}(c, b_n^{\max})$ can only make a difference in states $k \in [b_n^{\max}, 1]$. If $d \leq \eta$, then such replacement has no effect on the joint information policy so designer $m$ is indifferent with it. If instead $d > \eta$, then such replacement lowers the threshold of upper pooling interval and it reduces the informativeness of the joint policy. By Claim C.1, for sufficiently large $n$ and each $m \in M$, $\mathbb{E}_{\mathscr{P}(c,d)}[W_n^m(\cdot)]$ is single-peaked at $b_n^m$. Since $\eta \geq b_n^{\max} > b_n^m$, it follows that any designer $m$'s expected payoff would increase were $\mathscr{P}(c,d)$ replaced by $\mathscr{P}(c, b_n^{\max})$. Hence, any $\mathscr{P}(c,d)$ with $d > b_n^{\max}$ is weakly dominated by $\mathscr{P}(c, b_n^{\max})$. Using analogous argument we can also show that any $\mathscr{P}(c,d)$ with $c < a_n^{\min}$ is weakly dominated by $\mathscr{P}(a_n^{\min}, d)$. Together these imply that any $\mathscr{P}(c,d)$ with $d > b_n^{\max}$ or $c < a_n^{\min}$ is weakly dominated. This shows that any outcome induced by a pure-strategy equilibrium with undominated strategies must be weakly less informative than $\mathscr{P}(a_n^{\min}, b_n^{\max})$.

Next we show that no equilibrium outcome can be strictly less informative than censorship policy $\mathscr{P}(a_n^{\min}, b_n^{\max})$.[65] Observe that the structure of $\mathscr{P}_m^{(1)}$ implies that any equilibrium outcome must be weakly more informative than $\mathscr{P}(a_n^m, b_n^m)$ for all $m \in M$. Therefore, if $\cup_{m \in M}[a_n^m, b_n^m] = \left[a_n^{\min}, b_n^{\max}\right]$ the claim trivially holds. In what follows we assume that $\cup_{m \in M}[a_n^m, b_n^m]$ is a proper subset of $\left[a_n^{\min}, b_n^{\max}\right]$. In this case, there must be at least one pair of designers $l, r \in M$ such that (i) $b_n^l < a_n^r$, and (ii) for all $m \in M \setminus \{l, r\}$ there are $[a_n^m, b_n^m] \cap \left(b_n^l, a_n^r\right) = \emptyset$.[66] By the construction of $\mathscr{P}_m^{(1)}$ for all $m \in M$, there could be at most one nontrivial pooling interval $[x,y] \subseteq \left[b_n^l, a_n^r\right]$. Given $[x,y]$ and let $\mu(x,y) := \mathbb{E}_F[k | k \in [x,y]]$,

[65] Notice that the information environment is no longer Blackwell-connected when each designer $m$ is restricted to choose information policies from $\mathscr{P}_m^{(1)}$. Therefore, Proposition 2 of Gentzkow and Kamenica (2016*a*) no longer applies (i.e., a feasible outcome being unimprovable to all designers is no longer necessary for that outcome to be sustained in equilibrium).

[66] In fact, there could be at most $|M| - 1$ such pairs. The argument presented below holds for any such pair.

the expected utility of designer $m = \{l, r\}$ conditional on event $k \in \left[b_n^l, a_n^r\right]$ is given by

$$V_m = \int_{b_n^l}^{x} W_n^m(k) d\widetilde{F}(k) + \left(\widetilde{F}(y) - \widetilde{F}(x)\right) W_n^m(\mu(x,y)) + \int_{y}^{a_n^r} W_n^m(k) d\widetilde{F}(k)$$

where $\widetilde{F}(\cdot)$ denote the cdf of the distribution of $k$ conditional on $k \in \left[b_n^l, a_n^r\right]$. Taking derivatives of $V_m$ with respect to $x$ and $y$ yields

$$\begin{aligned}
\frac{\partial V_m}{\partial x} &= \widetilde{f}(x)\left(W_n^m(x) - W_n^m(\mu(x,y))\right) + \left(\widetilde{F}(y) - \widetilde{F}(x)\right) W_n^{m\prime}(\mu(x,y))\, \mu_x(x,y) \\
&= \widetilde{f}(x)\left(\mu(x,y) - x\right)\left[W_n^{m\prime}(\mu(x,y)) - \frac{W_n^m(\mu(x,y)) - W_n^m(x)}{\mu(x,y) - x}\right]
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial V_m}{\partial y} &= \widetilde{f}(y)\left(W_n^m(\mu(x,y)) - W_n^m(y)\right) + \left(\widetilde{F}(y) - \widetilde{F}(x)\right) W_n^{m\prime}(\mu(x,y))\, \mu_y(x,y) \\
&= \widetilde{f}(x)\left(y - \mu(x,y)\right)\left[W_n^{m\prime}(\mu(x,y)) - \frac{W_n^m(y) - W_n^m(\mu(x,y))}{y - \mu(x,y)}\right]
\end{aligned}$$

For both $l$ and $r$ to have no incentive to reveal any extra information, it is necessary that $\frac{\partial V_l}{\partial x} \leq 0$ and $\frac{\partial V_r}{\partial y} \geq 0$, or equivalently[67]

$$W_n^{l\prime}(\mu(x,y)) \leq \frac{W_n^l(\mu(x,y)) - W_n^l(x)}{\mu(x,y) - x} \tag{C.18}$$

and

$$W_n^{r\prime}(\mu(x,y)) \geq \frac{W_n^r(y) - W_n^l(\mu(x,y))}{y - \mu(x,y)} \tag{C.19}$$

Recall that $W_n^l(\cdot)$ is strictly S-shaped on $[z_n^l, 1]$ with inflection point $r_n^l$, and $W_n^r(\cdot)$ is strictly inverse S-shaped on $[-1, z_n^r]$ with inflection point $\ell_n^r$. Both $[z_n^l, 1]$ and $[-1, z_n^r]$ contain $[x, y]$ their interior. Therefore, for (C.18) to hold, $\mu(x,y) > r_n^l$ must be true so that $\mu(x,y)$ falls into the concave region of $W_n^l(\cdot)$. Similarly, for (C.19) to hold, $\mu(x,y) < \ell_n$ must hold for $\mu(x,y)$ to fall into the concave region of $W_n^r(\cdot)$. These together imply that both $W_n^l(\cdot)$ and $W_n^r(\cdot)$ have to be strictly concave at $\mu(x,y)$. This, however, is impossible because

[67] This is because each $m \in \{l, r\}$ can only choose censorship policies from $\mathscr{P}_m^{(1)}$, whose revelation interval must contain $[a_n^m, b_n^m]$. Therefore, since $[x, y] \subset \left(b_n^l, a_n^r\right)$, only designer $l$ can marginally increase $x$ while only designer $r$ can marginally decrease $y$. These two inequalities are necessary to insure that such marginal deviations are not profitable for either designer.

strictly convex open cover property holds for $\{W_n^m(\cdot)\}_{m\in\{l,r\}}$ on $[z_n^l, z_n^r]$, which contains $[x,y]$.[68] Therefore, the incentive compatibility conditions for designers $l$ and $r$ cannot be simultaneously satisfied and hence it is impossible to have any non-trivial pooling interval $[x,y]$ in equilibrium. This in turn implies that no equilibrium outcome can be strictly less informative than $\mathscr{P}(a_n^{\min}, b_n^{\max})$ and thus completes the proof.

# Appendix D Omitted Materials for Section 6

In this appendix we prove the comparative static results presented in Section 6. Throughout we assume there is a monopoly designer for whom the single-crossing property is satisfied and omit index $m$. We combine two different approaches to establish our results.

## D.1 First-order approach and proofs of Propositions 1 and 2

In this subsection we use the first-order approach to prove Propositions 1 and 2. We only prove the statements concerning threshold $b_n$; the proofs for claims concerning $a_n$ are similar and thus omitted.

Recall from (FOC: b) that the optimality condition for $b_n$ is given by

$$\left(\tilde{b}_n - b_n\right) W_n'(\tilde{b}_n) \leq W_n(\tilde{b}_n) - W_n(b_n) \tag{D.1}$$

where $\tilde{b}_n = \mathbb{E}_F[k|k \geq b_n]$ and this condition is binding whenever $b_n \in (-1,1)$. By (C.1) and (C.2) we have

$$\begin{aligned}
W_n(\tilde{b}_n) - W_n(b_n) &= \int_{\underline{v}}^{\tilde{b}_n} \left(\tilde{b}_n - \phi_n(x)\right) \hat{g}_n(x;q)dx - \int_{\underline{v}}^{b_n} \left(b_n - \phi_n(x)\right) \hat{g}_n(x;q)dx \\
&= \int_{b_n}^{\tilde{b}_n} \left(\tilde{b}_n - \phi_n(x)\right) \hat{g}_n(x;q)dx + \left(\tilde{b}_n - b_n\right) \int_{\underline{v}}^{b_n} \hat{g}_n(x;q)dx \\
\left(\tilde{b}_n - b_n\right) W_n'(\tilde{b}_n) &= \left(\tilde{b}_n - b_n\right) \left[ \left(\tilde{b}_n - \phi_n^m(\tilde{b}_n)\right) \hat{g}_n(\tilde{b}_n;q) + \int_{\underline{v}}^{\tilde{b}_n} \hat{g}_n(x;q)dx \right]
\end{aligned}$$

[68] This is because Assumption 1 holds for $M = \{l,r\}$ and $a_n^m \leq z_n^m \leq b_n^m$ has to hold for each $m \in M$, as implied by Theorem 1.

Plugging these into (D.1), we obtain that (D.1) is equivalent to

$$\tilde{b}_n - b_n \le \int_{b_n}^{\tilde{b}_n} \frac{b_n - \phi_n(x)}{\tilde{b}_n - \phi_n(\tilde{b}_n)} \frac{\hat{g}_n(x;q)}{\hat{g}_n(\tilde{b}_n;q)} dx \tag{D.2}$$

By Lemma 5, for sufficiently large $n$ it holds that $W_n(\cdot)$ is strictly S-shaped on $[z_n, 1]$ with some inflection point $r_n \in [z_n, 1]$. This implies that

$$(\mathbb{E}_F[k|k \ge x] - x) W_n'(\mathbb{E}_F[k|k \ge x]) - [W_n(\mathbb{E}_F[k|k \ge x]) - W_n(x)]$$

can cross zero at most once and from above as $x$ increases from $z_n$ to 1. In particular, suppose $b_n$ satisfies (D.2) with equality and hold it fixed, then if any parameter change increases the value of the right-hand side of (D.2), then $\left(\tilde{b}_n - b_n\right) W_n'(\tilde{b}_n) - \left[W_n(\tilde{b}_n) - W_n(b_n)\right]$ will be negative following this parameter change. $b_n$ must therefore decrease to regain equality. Comparative static analyses thus can done with the right-hand side of (D.2) only. With this we can prove Propositions 1 and 2.

*Proof of Proposition 1.* Let $\gamma_n(x) := \sum_{j=1}^{n+1} w_j \varphi_j(x;q,n)$. If $\rho < 1$, it follows from (3) that

$$\phi_n(\cdot) = \rho \gamma_n(x) + (1-\rho)\chi$$

As is explained in the proof of Lemma 3 in Appendix A.2, under either condition (i) or (ii) of Lemma 3 it holds that $1 - \phi'(x) > 0$ for all $x \in [\underline{v}, \overline{v}]$. Since $\phi_n(\cdot)$ converges uniformly to $\phi'(\cdot)$ (cf. Lemma 2), $1 - \phi_n'(\cdot) > 0$ on $[\underline{v}, \overline{v}]$ must hold for sufficiently large $n$. This implies that $x - \phi_n(x)$ is strictly increasing. Moreover, because $\tilde{b}_n = \mathbb{E}_F[k|k \ge b_n] > b_n$ and $\phi_n(x)$ is non-decreasing (cf. Proposition A.3), for all $x \in (b_n, \tilde{b}_n)$ we have

$$1 > \frac{b_n - \phi_n(b_n)}{\tilde{b}_n - \phi_n(\tilde{b}_n)} \ge \frac{b_n - \phi_n(x)}{\tilde{b}_n - \phi_n(\tilde{b}_n)} = \frac{b_n - \rho\gamma_n(x) - (1-\rho)\chi}{\tilde{b}_n - \rho\gamma_n(\tilde{b}_n) - (1-\rho)\chi}$$

Consider any $\chi_{\text{I}} > \chi_{\text{II}}$. Observe that a decrease of $\chi$ from $\chi_{\text{I}}$ to $\chi_{\text{II}}$ induces a common increase on both the nominator and the denominator of $\frac{b_n - \phi_n(x)}{\tilde{b}_n - \phi_n(\tilde{b}_n)}$, which is smaller than one for all $x \in \left(b_n, \tilde{b}_n\right)$. This shift of $\chi$ therefore strictly increases the value of $\frac{b_n - \phi_n(x)}{\tilde{b}_n - \phi_n(\tilde{b}_n)}$ for all $x \in \left(b_n, \tilde{b}_n\right)$.[69] On the other hand, the term $\frac{\hat{g}_n(x;q)}{\hat{g}_n(\tilde{b}_n;q)}$ is independent of $\chi$ for all $x \in \left(b_n, \tilde{b}_n\right)$. These together implies that a shift of $\chi$ from $\chi_{\text{I}}$ to $\chi_{\text{II}}$ strictly increases the right-hand side of (D.2). Therefore, if $b_n \in (-1, 1)$ under $\chi_{\text{I}}$ so that (D.2) is binding, such shift of $\chi$ will

[69] This follows from the fact that $\frac{a+c}{b+c} > \frac{a}{b}$ for all $b, c > 0$ and $b > a$.

make the right-hand side of (D.2) strictly higher than the left-hand side. $b_n$ must strictly decrease to make (D.2) binding again or drop to $-1$. If $b_n = -1$ $\chi_I$ so that (D.2) holds with '$\leq$', then it must remain to be the case after the shift of $\chi$ so that the optimal $b_n$ remains to be $-1$. These together show that $b_n$ is non-increasing as $\chi$ decreases and thus prove the claim for $b_n$ in Proposition 1. The claim for $a_n$ can be proved analogously. □

Next we prove Proposition 2. To do so we need to introduce an auxiliary result.

**Claim D.1.** *For any $0 < y < z < 1$, $\int_y^z \frac{\tau_n(x;q)}{\tau_n(z;q)} dx$ is strictly decreasing in $q$, where $\tau_n(x;q)$ is defined by (A.1) in Appendix A.*[70]

*Proof of Claim D.1.* For any pair of $(x,y) \in (0,1)^2$ and $q \in (0,1)$, define

$$\Delta\psi(x,y;q) := q \ln \frac{x}{y} + (1-q) \ln \frac{1-x}{1-y} = \ln \frac{1-x}{1-y} + q \left( \ln \frac{x}{1-x} - \ln \frac{y}{1-y} \right) \tag{D.3}$$

It then follows from the definition of $\tau_n(\cdot;q)$ that

$$\ln \frac{\tau_n(x;q)}{\tau_n(y;q)} = n \left( q \ln \frac{x}{y} + (1-q) \ln \frac{1-x}{1-y} \right) = n\Delta\psi(x,y;q)$$

We can thus rewrite $\int_y^z \frac{\tau_n(x;q)}{\tau_n(z;q)} dx$ as

$$\int_y^z \frac{\tau_n(x)}{\tau_n(z)} dx = \int_y^z e^{n\Delta\psi(x,z;q)} dx$$

Using (D.3) and the fact that $\ln \frac{x}{1-x}$ is strictly increasing in $x$, we obtain for all $y < z$ that

$$\frac{\partial}{\partial q} \int_y^z \frac{\tau_n(x;q)}{\tau_n(z;q)} dx = n \int_y^z e^{n\Delta\psi(x,z;q)} \left( \ln \frac{x}{1-x} - \ln \frac{z}{1-z} \right) dx < 0$$

This implies the strict decreasing property stated in this claim. □

*Proof of Proposition 2.* Recall from (A.5) that $\hat{g}_n(x;q) = \tau_n(G(x);q)\, g(x)$ for all $x \in [\underline{v}, \overline{v}]$.

[70] Here we implicitly assume $nq$ is an integer for ease of exposure. If this is not the case, then just replace $q$ with $\hat{q} = \lfloor nq \rfloor / n$ and the all arguments hold for $\hat{q}$.

Plugging this to (D.2), we obtain

$$\begin{aligned}\int_{b_n}^{\tilde{b}_n} \frac{b_n-\phi_n(x)}{\tilde{b}_n-\phi_n(\tilde{b}_n)} \frac{\hat{g}_n(x;q)}{\hat{g}_n(\tilde{b}_n;q)} dx &= \int_{b_n}^{\tilde{b}_n} \frac{b_n-\phi_n(x)}{\tilde{b}_n-\phi_n(\tilde{b}_n)} \frac{\tau_n\left(G(x);q\right)g(x)}{\tau_n\left(G(\tilde{b}_n);q\right)g(\tilde{b}_n)} dx \\ &= \frac{1}{g(\tilde{b}_n)} \int_{G(b_n)}^{G(\tilde{b}_n)} \frac{b_n-\phi_n\left(G^{-1}(y)\right)}{\tilde{b}_n-\phi_n(\tilde{b}_n)} \frac{\tau_n\left(y;q\right)}{\tau_n\left(G(\tilde{b}_n);q\right)} dy\end{aligned} \tag{D.4}$$

For $\rho = 0$ we have $\phi_n(x) = \chi$ for all $x \in [\underline{v}, \overline{v}]$ and therefore $W_n(\theta) = (\theta-\chi)\hat{G}_n(\theta;q)$ (cf. (3) and (7)). Plugging $W_n(\theta) = (\theta-\chi)\hat{G}_n(\theta;q)$ into (D.4), we obtain

$$\int_{b_n}^{\tilde{b}_n} \frac{b_n-\phi_n(x)}{\tilde{b}_n-\phi_n(\tilde{b}_n)} \frac{\hat{g}_n(x;q)}{\hat{g}_n(\tilde{b}_n;q)} dx = \frac{1}{g(\tilde{b}_n)} \left(\frac{b_n-\chi}{\tilde{b}_n-\chi}\right) \int_{G(b_n)}^{G(\tilde{b}_n)} \frac{\tau_n(x)}{\tau_n(G(\tilde{b}_n))} dx \tag{D.5}$$

Because $\tilde{b}_n > b_n > \chi$ and $G(\tilde{b}_n) > G(b_n)$, Claim D.1 implies that (D.5) is strictly decreasing in $q$. Therefore, the right-hand side of (D.2) strictly increases as $q$ rises from $q_{\text{I}}$ to $q_{\text{II}}$ for all $q_{\text{I}} < q_{\text{II}}$. If $b_n \in (-1,1)$ under $q_{\text{I}}$ so that (D.2) is binding, then such a shift of $q$ will make (D.2) hold with '$>$' so that $b_n$ must strictly increase to regain equality or up to 1. If $b_n = -1$ under $q_{\text{I}}$, then (D.2) holds with '$\leq$'. The shift of $q$ either (i) retains (D.2) with '$\leq$' so that the optimal $b_n$ is still $-1$, or it shifts '$\leq$' to '$>$' so that the optimal $b_n > -1$. These together show that $b_n$ is non-decreasing as $q$ increases and thus prove the claim for $b_n$ in Proposition 2. The claim for $a_n$ can be proved analogously. □

## D.2 Limiting approach for comparative statics

In this subsection we characterize the structure of the asympotitically optimal censorship policy as $n \to \infty$. These asymptotic results are used to establish Proposition 3 and an analog of Proposition 1 for weighting function $w(\cdot)$ when $\rho > 0$.

Under single-crossing property, it follows from Theorem 1 that there exists a cutoff $N \geq 0$ such that for all $n > N$ the optimal information policy is uniquely given by a censorship policy $\mathscr{P}(a_n, b_n)$ with $a_n \leq z_n \leq b_n$. Let $\{b_n\}_{n \geq N}$ and $\{a_n\}_{n \geq N}$ denote the sequences of the cutoff points of the optimal censorship policies. Theorem D.1 shows that both sequences converge and explicitly characterizes their limits.

**Theorem D.1.** *Suppose that the single-crossing property holds for the monopoly designer,*

$v_q^* := G^{-1}(q) \in (-1,1)$, *and* $\phi^* \in [-1,1]$.[71] *Let*

$$z^* := \begin{cases} -1 & \text{if } x > \phi(x) \text{ for all } x \in [-1,1] \\ x & \text{if } x = \phi(x) \text{ for some } x \in [-1,1] \\ 1 & \text{if } x < \phi(x) \text{ for all } x \in [-1,1] \end{cases}. \tag{D.6}$$

*Then both* $a^* := \lim_{n\to\infty} a_n$ *and* $b^* := \lim_{n\to\infty} b_n$ *exist and they are characterized as follows:*

1. *If* $\underline{\phi}(v_q^*) \le \phi^* \le \overline{\phi}(v_q^*)$, *then* $a^* = \min\{\phi^*, z^*\}$ *and* $b^* = \max\{\phi^*, z^*\}$.
2. *If* $v_q^* > \mathbb{E}_F[k]$ *and* $\phi^* < \underline{\phi}(v_q^*)$, *then* $a^* = z^* \le \phi^*$ *and* $b^* = \underline{\phi}(v_q^*) > \phi^*$.
3. *If* $v_q^* > \mathbb{E}_F[k]$ *and* $\phi^* > \overline{\phi}(v_q^*)$, *then* $a^* = \overline{\phi}(v_q^*) < \phi^*$ *and* $b^* = z^* \ge \phi^*$.

Functions $\overline{\phi}(\cdot)$ and $\underline{\phi}(\cdot)$ are given by (11) and (12) in Section 7 of the main text:

$$\overline{\phi}(v_q^*) := \sup\left\{y \in \mathbb{R} : \mathbb{E}_F\left[k | k \le y\right] \le v_q^*\right\}$$
$$\underline{\phi}(v_q^*) := \inf\left\{y \in \mathbb{R} : \mathbb{E}_F\left[k | k \ge y\right] \ge v_q^*\right\}$$

To prove Theorem D.1 we need the following Lemma (D.1), which characterizes the set of censorship policies $\mathscr{P}^*$ that are asymptotically optimal for the designer as $n \to \infty$. The precise formulation of the asymptotic persuasion problem and the proof for Lemma (D.1) are provided in Appendix E.

**Lemma D.1.** *Suppose* $v_q^* \in (-1,1)$ *and* $\phi^* \in [-1,1]$. *Then* $\mathscr{P}^*$ *is characterized as follows.*
1. *If* $\phi^* > \overline{\phi}(v_q^*)$, *then* $\mathscr{P}^* = \left\{\mathscr{P}(a,b) : a = \overline{\phi}(v_q^*) \text{ and } b \in \left[\overline{\phi}(v_q^*), 1\right]\right\}$.
2. *If* $\phi^* \in (v_q^*, \overline{\phi}(v_q^*)]$, *then* $\mathscr{P}^* = \{\mathscr{P}(a,b) : a = \phi^* \text{ and } b \in [\phi^*, 1]\}$.
3. *If* $\phi^* = v_q^*$, *then* $\mathscr{P}^* = \{\mathscr{P}(a,b) : -1 \le a \le \phi^* \le b \le 1\}$.
4. *If* $\phi^* \in [\underline{\phi}(v_q^*), v_q^*)$, *then* $\mathscr{P}^* = \{\mathscr{P}(a,b) : a \in [-1, \phi^*] \text{ and } b = \phi^*\}$.
5. *If* $\phi^* < \underline{\phi}(v_q^*)$, *then* $\mathscr{P}^* = \left\{\mathscr{P}(a,b) : a \in \left[-1, \underline{\phi}(v_q^*)\right] \text{ and } b = \underline{\phi}(v_q^*)\right\}$.

Now we prove Theorem D.1.

*Proof of Theorem D.1.* Because both sequences $\{b_n\}_{n\ge N}$ and $\{a_n\}_{n\ge N}$ are bounded on a closed interval, by Bolzano–Weierstrass Theorem they must contain at least one convergent

[71] Again, assuming $\phi^* \in [-1,1]$ simplifies exposure and is without loss of generality. For this theorem, any $\phi^* < -1$ (resp. $\phi^* > 1$) is equivalent to the case $\phi^* = -1$ (resp. $\phi^* = 1$). The same comment applies to Lemma D.1 below.

subsequence each. Let $b^*$ and $a^*$ denote the limits of these convergent subsequences. In what follows we shall explicitly characterize $b^*$ and $a^*$, and then show that all sub-sequences of $\{a_n\}_{n\geq N}$ and $\{b_n\}_{n\geq N}$ converge to them so that $\{a_n\}_{n\geq N}$ and $\{b_n\}_{n\geq N}$ indeed converge.[72]

On the one hand, asymptotic optimality requires that $\mathscr{P}(a^*,b^*)\in\mathscr{P}^*$ must hold, where $\mathscr{P}^*$ is characterized in Lemma D.1. On the other hand, by Lemma 5, single-crossing property implies for sufficiently large $n$ that there exists $z_n\in[-1,1]$ and $-1\leq \ell_n\leq z_n\leq r_n\leq 1$ such that the following conditions must hold:

$$\ell_n \leq a_n \leq z_n \leq b_n \leq r_n$$

and $z_n \to z^*$, $\ell_n \to \min\left\{z^*, v_q^*\right\}$ and $r_n \to \max\left\{z^*, v_q^*\right\}$ for $v_q^*\in(-1,1)$.[73] We now distinguish three cases. For all these cases recall that $\phi(v_q^*)=\phi^*$ and $\phi(\cdot)$ is non-decreasing

**Case 1**. $\phi^*=v_q^*$. In this case $z^*=v_q^*$ because $\phi(v_q^*)=\phi^*=v_q^*$. Therefore, both $\ell_n$ and $r_n$ converges to $\phi^*$. By squeeze theorem both $a_n$ and $b_n$ must also converge to $\phi^*$ and hence $a^*=b^*=\phi^*$.

**Case 2**. $\phi^*<v_q^*$. In this case $z^*\leq\phi^*<v_q^*$ so that $\ell_n\to z^*$ and hence $a^*=z^*<v_q^*$. On the other hand, it follows from Lemma D.1 that $b^*=\phi^*$if $\phi^*\in[\underline{\phi}(v_q^*),v_q^*)$ and $b^*=\underline{\phi}(v_q^*)$ if $\phi^*<\underline{\phi}(v_q^*)$.

**Case 3**. $\phi^*>v_q^*$. In this case $z^*\geq\phi^*>v_q^*$ so that $r_n\to z^*$ and hence $b^*=z^*>v_q^*$. On the other hand, it follows from Lemma D.1 that $a^*=\phi^*$if $\phi^*\in(v_q^*,\overline{\phi}(v_q^*)]$ and $a^*=\overline{\phi}(v_q^*)$ if $\phi^*>\overline{\phi}(v_q^*)$.

These complete the characterization for $a^*$and $b^*$ and this characterization applies for any convergent subsequences of $a_n$ and $b_n$. Therefore, the limits of $\{a_n\}$ and $\{b_n\}_n$ exist and are equal to $a^*$ and $b^*$, respectively. □

Figure D.1 illustrates Theorem D.1 when $v_q^*\geq\phi^*$ holds. In this case, Theorem D.1 implies that $a^*=z^*\leq\phi^*$ and $b^*=\max\left\{\phi^*,\underline{\phi}(v_q^*)\right\}\geq\phi^*$.

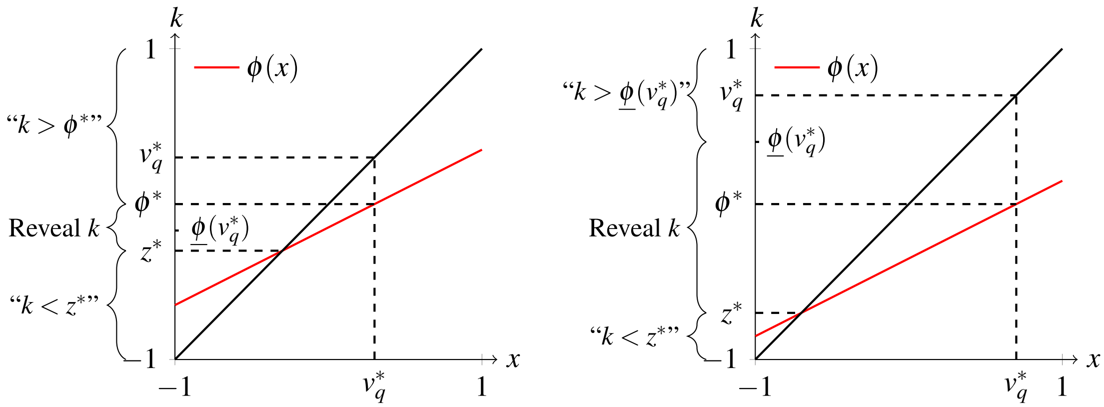
In what follows we analyze how do the asymptotic thresholds $a^*$ and $b^*$ vary with a pro-social designer's weighting function $w(\cdot)$ and voting rule $q$. By Theorem D.1, all these boil down to understand how these factors affect $z^*$, $\phi^*$ and $v_q^*$.

[72] Here we explore the following observation: let $\{x_n\}$ be a sequence on a bounded closed interval and suppose all its convergent subsequences have the same limit $x^*$, then $x_n$ converges to $x^*$. To see this, suppose instead that $x_n$ does not converge to $x^*$. Then there exists some $\varepsilon>0$ and a subsequence $\left\{x_{n_j}\right\}$ indexed by $j=1,2,\cdots$ such that $\left|x_{n_j}-x^*\right|>\varepsilon$ holds for all $n_j$. Since $x_{n_j}$ is bounded in a closed interval, by Bolzano-Weierstrass Theorem it must contain a convergent subsequence. Yet this subsequence does not converge to $x^*$, leading to a contradiction.

[73] The limiting results for $\ell_n$ and $r_n$ follow from (C.5), (C.6), (C.7) and (C.8) in Appendix C.2.

Figure D.1: Asymptotically Optimal Censorship Policy (for the case $\phi^* \leq v_q^*$)

(a) $\underline{\phi}(v_q^*) \leq \phi^* \leq v_q^*$ (b) $\phi^* < \underline{\phi}(v_q^*) < v_q^*$




The following Lemma D.2 characterizes how $z^*$, $\phi^*$ and $v_q^*$ vary with model parameters $w(\cdot)$ and $q$. For any two weighting functions $w^{\text{I}}(\cdot)$ and $w^{\text{II}}(\cdot)$ that are absolutely continuous cdfs on $[-1,1]$, we use $w^{\text{I}}(\cdot) \succeq_{FOSD} w^{\text{II}}(\cdot)$ to denote that $w^{\text{I}}(\cdot)$ first order stochastically dominates $w^{\text{II}}(\cdot)$.

**Lemma D.2.** *Suppose $\rho > 0$. The following comparative statics hold:*

*1. $v_q^* = G^{-1}(q)$ is increasing in $q$.*

*2. $\phi^*$ is invariant in $q$ and it strictly decreases as $w(\cdot)$ shifts from $w^I(\cdot)$ to $w^{II}(\cdot)$, where $w^I(\cdot) \succeq_{FOSD} w^{II}(\cdot)$.*

*3. Suppose both $G$ and $1-G$ are strictly log-concave. Then $z^*$ is decreasing in $q$ and $z^* = v_q^*$ if $\phi^* = v_q^*$. Moreover, $z^*$ also decreases as $w(\cdot)$ shifts from $w^I(\cdot)$ to $w^{II}(\cdot)$, where $w^I(\cdot) \succeq_{FOSD} w^{II}(\cdot)$.*

*Proof.* Part (1) is obvious from the definition of $v_q^*$. For part (2), recall from (A.21) that

$$\phi^* = \rho \int_0^1 G^{-1}(y)dw(y) + (1-\rho)\chi$$

It is clear from its expression that $\phi^*$ is independent of $q$. Consider any $w^{\text{I}}(\cdot)$ and $w^{\text{II}}(\cdot)$ with $w^{\text{I}}(\cdot) \succeq_{FOSD} w^{\text{II}}(\cdot)$. Then $\int_0^1 G^{-1}(y)dw^{\text{I}}(y) > \int_0^1 G^{-1}(y)dw^{\text{II}}(y)$ must hold because $G^{-1}(y)$ is strictly increasing. Since $\rho > 0$, it follows that $\phi^*$ strictly decreases as $w(\cdot)$ shifts from $w^{\text{I}}(\cdot)$ to $w^{\text{II}}(\cdot)$. These together prove part (2).

Next we show part (3). By Lemma 3, strict log-concavity of $G$ and $1-G$ ensures single-crossing property and hence existence of a unique $z^*$ for all $w(\cdot)$ and $q$. By (D.6), it is obvious $z^*$ must decrease if function $\phi(\cdot)$ systematically shifts down – $\phi(x)$ strictly decreases for all $x \in [\underline{v}, \overline{v}]$ – after some shift of either $w(\cdot)$ or $q$. The decreasing properties in part (3) then follow from Proposition A.4, which claims that $\phi(\cdot)$ shifts downwards if $q$ increases or if $w(\cdot)$ varies from some $w^{\text{I}}(\cdot)$ to $w^{\text{II}}(\cdot)$ with $w^{\text{I}}(\cdot) \succeq_{FOSD} w^{\text{II}}(\cdot)$, when $\rho > 0$. Finally, by (A.21) we have $\phi^* = \phi(v_q^*)$. Therefore, $\phi^* = v_q^*$ implies $v_q^* = \phi(v_q^*)$. Since $v_q^* \in (-1,1)$, it follows from (D.6) that $z^* = v_q^*$ must also hold in this case. $\square$

With the help of Theorem D.1 and Lemma D.2, we are ready to prove Proposition 3 and establish an analog of Proposition 1 for weighting function $w(\cdot)$ when $\rho > 0$.

*Proof of Proposition 3.* We prove Proposition 3 by construction. For any pair of $q_{\text{I}}$ and $q_{\text{II}}$, we use $a_i^*$ and $b_i^*$ to denote the thresholds of the asymptotically optimal censorship policy under $q = q_i$ for $i \in \{\text{I}, \text{II}\}$. We assume $\phi^* \in (-1,1)$ and let $\widehat{q} := G^{-1}(\phi^*)$.[74]

First, suppose $q_{\text{I}}$ and $q_{\text{II}}$ satisfy (i) $\widehat{q} \leq q_{\text{I}} < q_{\text{II}}$, (ii) $\phi^* \in \left[\underline{\phi}(v_{q_{\text{I}}}^*), \overline{\phi}(v_{q_{\text{I}}}^*)\right]$ (e.g., panel (a) of Figure D.1), and (iii) $\phi^* < \underline{\phi}(v_{q_{\text{II}}}^*)$ (e.g., panel (b) of Figure D.1). Then, by Theorem D.1, we have $(a_{\text{I}}^*, b_{\text{I}}^*) = (z_{\text{I}}^*, \phi^*)$ and $(a_{\text{II}}^*, b_{\text{II}}^*) = \left(z_{\text{II}}^*, \underline{\phi}(v_{q_{\text{II}}}^*)\right)$. Since $\underline{\phi}(v_{q_{\text{I}}}^*) > \phi^*$ and $z_{\text{II}}^* < z_{\text{I}}^*$ (cf. part (3) of Lemma D.2), we get $a_{\text{II}}^* < a_{\text{I}}^* \leq b_{\text{I}}^* < b_{\text{II}}^*$. This implies for sufficiently large $n$ that $a_n$ decreases and $b_n$ increases as $q$ shifts from $q_{\text{I}}$ to $q_{\text{II}}$. This proves case 1 of Proposition 3.

To show that case 2 of Proposition 3 is also possible, consider any $q_{\text{I}}$ and $q_{\text{II}}$ that satisfy (i) $q_{\text{I}} < q_{\text{II}} \leq \widehat{q}$, (ii) $\phi^* > \overline{\phi}(v_{q_{\text{I}}}^*)$, and (iii) $\phi^* \in \left[\underline{\phi}(v_{q_{\text{II}}}^*), \overline{\phi}(v_{q_{\text{II}}}^*)\right]$. By Theorem D.1, $(a_{\text{I}}^*, b_{\text{I}}^*) = \left(\overline{\phi}(v_{q_{\text{I}}}^*), z_{\text{I}}^*\right)$ and $(a_{\text{II}}^*, b_{\text{II}}^*) = (\phi^*, z_{\text{II}}^*)$. In this case we have $a_{\text{I}}^* < a_{\text{II}}^* \leq b_{\text{II}}^* < b_{\text{I}}^*$. So, for sufficiently large $n$, $a_n$ increases while $b_n$ decreases as $q$ varies from $q_{\text{I}}$ to $q_{\text{II}}$. $\square$

**Proposition D.1.** *Suppose $\rho > 0$ and $v_q^* \in (-1,1)$. Let $w^I(\cdot)$ and $w^{II}(\cdot)$ be two absolutely continuous cdfs on $[-1,1]$ that satisfy (i) $w^I(\cdot) \succeq_{FOSD} w^I(\cdot)$, and (ii) for $w^I(\cdot)$ or $w^{II}(\cdot)$ there are $\phi^* \in (-1,1)$ and $\phi^* \in \left(\underline{\phi}(v_q^*), \overline{\phi}(v_q^*)\right)$. Then, for sufficiently large $n$, both $a_n$ and $b_n$ decrease as $w(\cdot)$ shifts from $w^I(\cdot)$ to $w^{II}(\cdot)$.*

*Proof of Proposition D.1.* We use $a_i^*$ and $b_i^*$ to denote the thresholds of the asymptotically optimal censorship policy under $w(\cdot) = w^i(\cdot)$ for $i \in \{\text{I}, \text{II}\}$. For ease of exposure we focus on the case where $\phi^* \in (-1,1)$ and $\phi^* \in \left(\underline{\phi}(v_q^*), \overline{\phi}(v_q^*)\right)$ hold for both $w^{\text{I}}(\cdot)$ or $w^{\text{II}}(\cdot)$.[75] In this case, Theorem D.1 implies $a_i^* = \min\{z_i^*, \phi_i^*\}$ and $b_i^* = \max\{z_i^*, \phi_i^*\}$ for both $i \in \{\text{I}, \text{II}\}$. By Lemma D.2, we have $\phi_{\text{I}}^* > \phi_{\text{II}}$ and $z_{\text{I}}^* > z_{\text{II}}^*$ (equality holds only if both values are $-1$). These together imply (i) $b_{\text{I}}^* > b_{\text{II}}^*$ and (ii) $a_{\text{I}}^* \geq a_{\text{II}}^*$ (equality holds only if both values are $-1$). Hence, for sufficiently large $n$, both $a_n$ and $b_n$ decrease as $q$ shifts from $w^{\text{I}}(\cdot)$ to $w^{\text{II}}(\cdot)$. $\square$

[74] Under $q = \widehat{q}$ we have $\phi^* = G^{-1}(q) = v_q^*$.

[75] The proof is almost identical for the case where only one weighting function satisfies this condition.

# Appendix E Omitted Materials for Section 7

In this appendix we prove Theorem 5 in Section 7 and characterize the set of all censorship policies that are asymptotically optimal.

We start by proving a useful auxiliary lemma. In the statement recall that $W_n(\theta)$ equals the designer's expected utility under a common posterior expected state $\theta$.

**Lemma E.1.** *For all $\theta \in [-1,1]$, $W_n(\theta)$ converges point-wise to*

$$W(\theta) := \begin{cases} \theta - \phi^*, & \text{if } \theta > v_q^* \\ (\theta - \phi^*)/2, & \text{if } \theta = v_q^* \\ 0, & \text{if } \theta < v_q^* . \end{cases} \tag{E.1}$$

*Proof.* Recall that

$$W_n(\theta) = \int_{\underline{v}}^{\theta} (\theta - \phi_n(x))\, \hat{g}_n(x;q)dx = \theta \hat{G}_n(\theta;q) - \int_{\underline{v}}^{\theta} \phi_n(x)\hat{g}_n(x;q)dx$$

As $n \to \infty$, it follows from (A.7) (in the proof of Proposition A.1 in Appendix A.1) that $\theta \hat{G}_n(\theta;q)$ converges to $\theta$ for $\theta > v_q^*$, to $\theta/2$ for $\theta = v_q^*$, and to 0 for $\theta < v_q^*$. We now show

$$\lim_{n\to\infty} \int_{\underline{v}}^{\theta} \phi_n(x)\hat{g}_n(x;q)dx = \begin{cases} \phi^* & \text{if } \theta > v_q^* \\ \phi^*/2 & \text{if } \theta = v_q^* \\ 0 & \text{if } \theta < v_q^* \end{cases} \tag{E.2}$$

so that $W_n(\theta)$ indeed converges to $W(\theta)$ for all $\theta \in [-1,1]$. Suppose first $\theta < v_q^*$. Since $\phi_n(\cdot)$ is non-decreasing and $\hat{G}_n(\theta;q) \to 0$ for $\theta < v_q^*$, we have

$$\int_{\underline{v}}^{\theta} \phi_n(x)\hat{g}_n(x;q)dx \leq \int_{\underline{v}}^{\theta} \phi_n(\theta)\hat{g}_n(x;q)dx = \phi_n(\theta)\hat{G}_n(\theta;q) \to 0$$

Now consider $\theta > v_q^*$ and observe that

$$\int_{\underline{v}}^{\theta} \phi_n(x)\hat{g}_n(x;q)dx = \int_{\underline{v}}^{\overline{v}} \phi_n(x)\hat{g}_n(x;q)dx - \int_{\theta}^{\overline{v}} \phi_n(x)\hat{g}_n(x;q)dx$$

The first integral on the right-hand side converges to $\phi(v_q^*) = \phi^*$ because $\phi_n(\cdot)$ converges uniformly to $\phi(\cdot)$ on $[\underline{v}, \overline{v}]$ (cf. Lemma 2) and distribution $\hat{G}_n(\cdot;q)$ puts all its mass on

$v_q^* = G^{-1}(q)$ as $n \to \infty$ (cf. Lemma 1). Regarding the second integral, we have

$$\phi_n(\theta)\left(1-\hat{G}_n(\theta;q)\right) \leq \int_\theta^{\overline{v}} \phi_n(x)\hat{g}_n(x;q)dx \leq \phi_n(\overline{v})\left(1-\hat{G}_n(\theta;q)\right)$$

because $\phi_n(\cdot)$ is non-decreasing and $\int_\theta^{\overline{v}} \hat{g}_n(x;q)dx = 1-\hat{G}_n(\theta;q)$. As $\hat{G}_n(\theta;q) \to 1$ for any $\theta > v_q^*$, we obtain $\int_\theta^{\overline{v}} \phi_n(x)\hat{g}_n(x;q)dx \to 0$ and thus $\int_{\underline{v}}^\theta \phi_n(x)\hat{g}_n(x;q)dx \to \phi^*$ for $\theta > v_q^*$. Finally, consider $\theta = v_q^*$. Take any $\varepsilon > 0$ and observe that

$$\int_{\underline{v}}^{v_q^*} \phi_n(x)\hat{g}_n(x;q)dx = \int_{v_q^*-\varepsilon}^{v_q^*} \phi_n(x)\hat{g}_n(x;q)dx + \int_{\underline{v}}^{v_q^*-\varepsilon} \phi_n(x)\hat{g}_n(x;q)dx$$

Since $v_q^* - \varepsilon < v_q^*$, it follows from the previous argument that the second integral goes to 0 as $n \to \infty$. The non-decreasing property of $\phi_n(\cdot)$ implies for the first integral that

$$\phi_n\left(v_q^*-\varepsilon\right)\left(\hat{G}_n(v_q^*)-\hat{G}_n(v_q^*-\varepsilon)\right) \leq \int_{v_q^*-\varepsilon}^{v_q^*} \phi_n(x)\hat{g}_n(x;q)dx \leq \phi_n\left(v_q^*\right)\left(\hat{G}_n(v_q^*)-\hat{G}_n(v_q^*-\varepsilon)\right)$$

Because $\phi_n(\cdot)$ converges uniformly to $\phi(\cdot)$ and $\hat{G}_n(v_q^*)-\hat{G}_n(v_q^*-\varepsilon) \to 1/2$ for all $\varepsilon > 0$, the limit of $\int_{v_q^*-\varepsilon}^{v_q^*} \phi_n(x)\hat{g}_n(x;q)dx$ must be bounded between $\phi\left(v_q^*-\varepsilon\right)/2$ and $\phi^*(v_q^*)/2$. Since $\phi(\cdot)$ is continuous and $\varepsilon$ can be arbitrarily small, we have $\int_{\underline{v}}^{v_q^*} \phi_n(x)\hat{g}_n(x;q)dx \to \phi(v^*)/2 = \phi^*/2$. These observations together imply (E.2) and hence show that $W_n(\theta)$ converges point-wise to $W(\theta)$. □

## E.1 Proof of Theorem 5

Observe that the objective function $W(\theta)$ is not upper semi-continuous at $\theta = v_q^*$, we slightly modify it by

$$\widetilde{W}(\theta) = \begin{cases} W(\theta), & \text{if } \theta \neq v_q^* \\ \max\{v_q^*-\phi^*, 0\}, & \text{if } \theta = v_q^* \end{cases}$$

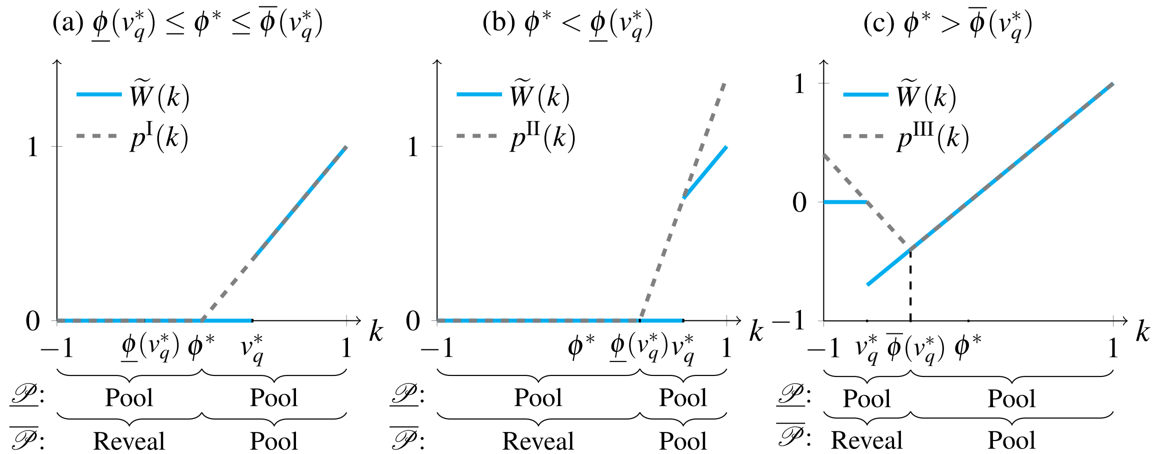
and consider the auxiliary persuasion problem

$$\max_{H\in\Delta([-1,1])} \int_{-1}^{1} \widetilde{W}(\theta)dH(\theta), \text{ s.t. } F \succeq_{MPS} H \tag{E.3}$$

Note that $\widetilde{W}(\theta)$ is upper semi-continuous so that problem (E.3) always admits a solution (Kamenica and Gentzkow, 2011; Dworczak and Martini, 2019). We denote the value to problem (E.3) by $W^*$. The dual problem of (E.3) is

$$\min_{p(\cdot)} \int_{-1}^{1} p(\theta)dF(\theta), \text{ s.t. } p(\cdot) \text{ is convex and } p(\cdot) \geq \widetilde{W}(\cdot) \tag{E.4}$$

Figure E.1: Illustrate for the proof of Theorem 5




Note: In all three panels, the blue solid lines denote $\widetilde{W}(\cdot)$ and the gray dashed lines denote the auxiliary price functions. Moreover, $\underline{\mathscr{P}}$ and $\overline{\mathscr{P}}$ denote, respectively, the least and most informative censorship policies that solves problem (E.3).

Since $\widetilde{W}(\theta)$ is regular, we can apply the duality method in Appendix B.1 to solve (E.3). We distinguish between three cases.

**Case 1**: $\underline{\phi}(v_q^*) \leq \phi^* \leq \overline{\phi}(v_q^*)$. In this case $\mathbb{E}_F[k|k \geq \phi^*] \geq v_q^*$ and $\mathbb{E}_F[k|k \leq \phi^*] \leq v_q^*$. Let $p^{\mathrm{I}}(\theta) := \max\{\theta - \phi^*, 0\}$ for $\theta \in [-1,1]$. $p^{\mathrm{I}}(\cdot)$ is illustrated in panel (a) of Figure E.1. Consider the cutoff policy $\mathscr{P}(\phi^*)$ and let $\underline{H}^{\mathrm{I}} = H_{\mathscr{P}(\phi^*)}$. The following conditions are straightforward to verify: (i) $p^{\mathrm{I}}(\cdot)$ is convex and $p^{\mathrm{I}}(\cdot) \geq \widetilde{W}(\cdot)$ on $[-1,1]$; (ii) $F \succeq_{MPS} \underline{H}^{\mathrm{I}}$ and $\int_{-1}^{1} p^{\mathrm{I}}(\theta)d\underline{H}^{\mathrm{I}}(\theta) = \int_{-1}^{1} p^{\mathrm{I}}(\theta)dF(\theta)$, and (iii)

$$\begin{aligned}
\text{supp}(\underline{H}^{\mathrm{I}}) &= \{\mathbb{E}_F[k|k \leq \phi^*], \mathbb{E}_F[k|k \geq \phi^*]\} \\
&\subset \left\{\theta \middle| p^{\mathrm{I}}(\theta) = \widetilde{W}(\theta)\right\} = \begin{cases} [-1,1], & \text{if } v_q^* = \phi^* \\ [-1,v_q^*] \cup [\phi^*,1], & \text{if } v_q^* < \phi^* \\ [-1,\phi^*] \cup [v_q^*,1], & \text{if } v_q^* > \phi^* \end{cases}
\end{aligned}$$

Therefore, by Remark B.2, $\underline{H}^{\mathrm{I}}$ solves the primal problem (E.3), $p^{\mathrm{I}}(\cdot)$ solves the dual problem (E.4), and by strong duality the values to both problems are identical and equal to

$$\int_{-1}^{1} p^{\mathrm{I}}(k)dF(k) = \int_{-1}^{1} \max\{k-\phi^*,0\}dF(k) = \int_{\phi^*}^{1}(k-\phi^*)dF(k)$$

In fact, for $\underline{\phi}(v_q^*) \le \phi^* \le \overline{\phi}(v_q^*)$, the set of solutions to problem (E.3) is characterized by

$$\mathscr{H}^{\mathrm{I}} = \left\{H \in \Delta(-1,1):\ F \succeq_{MPS} H \succeq_{MPS} \underline{H}^{\mathrm{I}} \text{ and } \mathrm{supp}(H) \subset \left\{\theta \middle| p^{\mathrm{I}}(\theta) = \widetilde{W}(\theta)\right\}\right\} \tag{E.5}$$

**Case 2**: $v_q^* > \mathbb{E}_F[k]$ and $\phi^* < \underline{\phi}(v_q^*)$. In this case we have $\underline{\phi}(v_q^*) \in (-1,1)$ and $\mathbb{E}_F[k|k \ge \underline{\phi}(v_q^*)] = v_q^*$. Let

$$p^{\mathrm{II}}(\theta) := \begin{cases} \frac{v_q^*-\phi^*}{v_q^*-\underline{\phi}(v_q^*)}\left(\theta - \underline{\phi}(v_q^*)\right), & \text{if } \theta \in \left[\underline{\phi}(v_q^*),1\right] \\ 0, & \text{if } \theta \in \left[-1,\underline{\phi}(v_q^*)\right) \end{cases}.$$

$p^{\mathrm{II}}(\cdot)$ is plotted in panel (b) of Figure E.1. Consider cutoff policy $\mathscr{P}\left(\underline{\phi}(v_q^*)\right)$ and let $\underline{H}^{\mathrm{II}} = H_{\mathscr{P}(\underline{\phi}(v_q^*))}$. The following conditions are easy to verify: (i) $p^{\mathrm{II}}(\cdot)$ is convex and $p^{\mathrm{II}}(\cdot) \ge \widetilde{W}(\cdot)$ on $[-1,1]$; (ii) $F \succeq_{MPS} \underline{H}^{\mathrm{II}}$ and $\int_{-1}^{1} p^{\mathrm{II}}(\theta)d\underline{H}^{\mathrm{II}}(\theta) = \int_{-1}^{1} p^{\mathrm{II}}(\theta)dF(\theta)$; and (iii)

$$\mathrm{supp}(\underline{H}^{\mathrm{II}}) = \left\{\mathbb{E}_F\left[k|k<\underline{\phi}(v_q^*)\right], v_q^*\right\} \subset \left\{\theta \middle| p^{\mathrm{II}}(\theta) = \widetilde{W}(\theta)\right\} = \left[-1,\underline{\phi}(v_q^*)\right] \cup \{v_q^*\}.$$

Hence, $\underline{H}^{\mathrm{II}}$ solves the primal problem (E.3), $p^{\mathrm{II}}(\cdot)$ solves the dual problem (E.4), and the values of both problems are identical and equal to

$$\begin{aligned}\int_{-1}^{1} p^{\mathrm{II}}(k)dF(k) &= \left(1-F\left(\underline{\phi}(v_q^*)\right)\right)\frac{v_q^*-\phi^*}{v_q^*-\underline{\phi}(v_q^*)}\left(\mathbb{E}_F\left[k|k>\underline{\phi}(v_q^*)\right]-\underline{\phi}(v_q^*)\right) \\ &= \left(1-F\left(\underline{\phi}(v_q^*)\right)\right)(v_q^*-\phi^*) = \int_{\underline{\phi}(v_q^*)}^{1}(k-\phi^*)dF(k)\end{aligned}$$

The second step follows from piece-wise linearity of $p^{\mathrm{II}}(\cdot)$ and the third step follows from $\mathbb{E}_F[k|k>\underline{\phi}(v_q^*)] = v_q^*$, as implied by the definition of $\underline{\phi}(v_q^*)$. For $\phi^* < \underline{\phi}(v_q^*)$, the set of solutions to problem (E.3) is characterized by

$$\mathscr{H}^{\mathrm{II}} = \left\{H \in \Delta(-1,1):\ F \succeq_{MPS} H \succeq_{MPS} \underline{H}^{\mathrm{II}} \text{ and } \mathrm{supp}(H) \subset \left[-1,\underline{\phi}(v_q^*)\right] \cup \{v_q^*\}\right\}. \tag{E.6}$$

**Case 3**: $v_q^* < \mathbb{E}_F[k]$ and $\phi^* > \overline{\phi}(v_q^*)$. In this case we have $\overline{\phi}(v_q^*) \in (-1,1)$ and $\mathbb{E}_F\left[k|k \leq \overline{\phi}(v_q^*)\right] = v_q^*$. Let

$$p^{\text{III}}(\theta) = \begin{cases} \theta - \phi^*, & \text{if } \theta \in \left[\overline{\phi}(v_q^*), 1\right] \\ \frac{\overline{\phi}(v_q^*) - \phi^*}{\overline{\phi}(v_q^*) - v_q^*}\left(\theta - v_q^*\right), & \text{if } \theta \in \left[-1, \overline{\phi}(v_q^*)\right) \end{cases}.$$

$p^{\text{III}}(\cdot)$ is plotted in panel (c) of Figure E.1.Consider cutoff policy $\mathscr{P}\left(\overline{\phi}(v_q^*)\right)$ and let $\underline{H}^{\text{III}} = H_{\mathscr{P}(\overline{\phi}(v_q^*))}$. The following conditions are again easy to verify: (i) $p^{\text{III}}(\cdot)$ is convex and $p^{\text{III}}(\cdot) \geq \widetilde{W}(\cdot)$; (ii) $F \succeq_{MPS} \underline{H}^{\text{III}}$ and $\int_{-1}^{1} p^{\text{III}}(\theta) d\underline{H}^{\text{III}}(\theta) = \int_{-1}^{1} p^{\text{III}}(\theta) dF(\theta)$; and (iii)

$$\text{supp}(\underline{H}^{\text{III}}) = \left\{\mathbb{E}_F\left[k|k > \overline{\phi}(v_q^*)\right], v_q^*\right\} \subset \left\{\theta \middle| p^{\text{III}}(\theta) = \widetilde{W}(\theta)\right\} = \left\{v_q^*\right\} \cup \left[\overline{\phi}(v_q^*), 1\right]$$

Following analogous arguments as in previous cases, we can establish that $\underline{H}^{\text{III}}$ solves the primal problem (E.3), $p^{\text{III}}(\cdot)$ solves the dual problem (E.4), and the values of both problems are identical and equal to

$$\begin{aligned} \int_{-1}^{1} p^{\text{III}}(k) dF(k) &= F\left(\overline{\phi}(v_q^*)\right) \frac{\overline{\phi}(v_q^*) - \phi^*}{\overline{\phi}(v_q^*) - v_q^*} \left(\mathbb{E}\left[k \middle| k < \overline{\phi}(v_q^*)\right] - v_q^*\right) + \int_{\overline{\phi}(v_q^*)}^{1} (k - \phi^*) dF(k) \\ &= \int_{\overline{\phi}(v_q^*)}^{1} (k - \phi^*) dF(\theta) \end{aligned}$$

This follows from the piece-wise linearity of $p^{\text{III}}(\cdot)$ and the fact that $\mathbb{E}\left[k \middle| k < \overline{\phi}(v_q^*)\right] = v_q^*$. For $\phi^* > \overline{\phi}(v_q^*)$, the set of solutions to problem (E.3) is characterized by

$$\mathscr{H}^{\text{III}} = \left\{H \in \Delta(-1,1) : F \succeq_{MPS} H \succeq_{MPS} \underline{H}^{\text{III}} \text{ and supp}(H) \subset \left\{v_q^*\right\} \cup \left[\overline{\phi}(v_q^*), 1\right]\right\}. \tag{E.7}$$

These together complete the proof.

## E.2 Proof of Lemma D.1

The proof of Theorem 5 above allows us to easily characterize the set $\mathscr{P}^*$ of censorship policies that solve problem (E.3). Such $\mathscr{P}^*$ is characterized by Lemma D.1 under the same assumption that $v_q^* \in (-1,1)$ and $\phi^* \in [-1,1]$.

Lemma D.1 follows directly from the solution sets (E.5), (E.6) and (E.7) characterized above. The least and most informative censorship policies of $\mathscr{P}^*$, denoted by $\underline{\mathscr{P}}$ and $\overline{\mathscr{P}}$, respectively, are demonstrated in the three panels of Figure E.1.

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